

# A Spatial Autoregressive Production Frontier Model for Panel Data: With an Application to European Countries

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## Abstract

In this paper we merge techniques from the efficiency literature with spatial econometric techniques. In particular, we combine calculation of efficiency from the unit specific effects with the spatial autoregressive model to develop a spatial autoregressive frontier model for panel data. Features of the modeling include time-varying efficiency and estimation of own and spillover returns to scale. The model is applied to aggregate production in European countries over the period 1995 – 2008. Among other things, we find that production in the sample average country is characterized by increasing returns to scale when we allow for returns to scale spillovers from other countries, and constant returns when these spillovers are ignored.

**Keywords:** Spatial Autoregression; Panel Data; Time-varying Efficiency; Returns to Scale

**JEL Classification:** C23; C51

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# 1 Introduction

The omitted variable bias from overlooking the spatial autoregression between neighbors has long since been recognized. This motivated the development of the spatial autoregressive model in the seminal contributions by Cliff and Ord (1973; 1981). Other spatial models with a spatial autoregressive variable which have since been proposed include the spatial Durbin model which also includes spatially lagged independent variables (Anselin, 1988) and a model which also includes a spatial autocorrelation term (Drukker *et al.*, 2012; Kapoor *et al.*, 2007; Kelejian and Prucha, 1998; 1999; 2010). In the context of frontier models, biased parameter estimates because the spatial autoregression between neighboring cross-sectional units is overlooked can also have implications for the efficiency scores. We therefore blend techniques used in parametric frontier modeling with applied spatial econometric techniques to develop a spatial autoregressive production frontier model for panel data where technical efficiency is time-variant. The model is then applied to a classic case of aggregate production for 40 European countries over the period 1995 – 2008.

To date there is one key study by Druska and Horrace (2004) in the fledgling literature on spatial frontier modeling. In this key study the authors develop a GMM frontier model which they estimate using panel data on production for a sample of Indonesian rice farms. Specifically, they develop a spatial error production frontier model by including the spatial autocorrelation term as an exogenous variable which shifts the frontier technology. They then calculate time-invariant inefficiencies from the random effects using the approach proposed by Schmidt and Sickles (1984) (SS from hereon). The marginal effect of an explanatory variable from such a model is not a function of the spatial autocorrelation term so the coefficients on the inputs and the exogenous variables can be interpreted as elasticities in the usual way. The spillover marginal effect from such a model relates to the disturbance. This spillover marginal effect, however, is not as informative as the spillover effects for the explanatory variables which we report in the application section of this paper.<sup>1</sup>

LeSage and Pace (2009) demonstrate that the coefficients on the explanatory variables from a fitted spatial autoregressive model cannot be interpreted as elasticities. This is because the marginal effect of an explanatory variable is a function of the spatial autoregressive variable. LeSage and Pace (2009) therefore propose Bayesian Markov Chain Monte Carlo (MCMC) simulation of the own (i.e. direct), spillover (i.e. indirect)

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<sup>1</sup>Schmidt *et al.* (2009) also incorporate spatial dependence into their parametric frontier analysis of Brazilian farm production. However, they account for the spatial dependence by allowing spatially lagged latent regional effects (i.e. not farm effects) to affect the inefficiency distribution or shift the frontier technology. In contrast, in the spatial error frontier model in Druska and Horrace (2004) and also in the spatial autoregressive frontier model which we present, the spatial dependence is explicitly modeled.

and total marginal effects of the explanatory variables.<sup>2</sup> Using the direct, indirect and total marginal effects from a fitted production frontier model with a spatial autoregressive exogenous variable we develop a new line of enquiry for parametric productivity analysis. In particular, we calculate direct, indirect and total returns to scale.

The direct marginal effect estimates the effect of changing an explanatory variable in a particular cross-sectional unit on that unit's dependent variable and includes feedback effects i.e. effects which pass through first order and higher order neighbors via the spatial multiplier matrix and back to the unit which initiated the change. The indirect marginal effect can be interpreted in two ways. The first interpretation estimates the impact of changing an explanatory variable in a particular unit on the dependent variables of all the other units in the sample. The second interpretation estimates the change in the dependent variable for one particular unit following a change in an explanatory variable in all the other units. Further in the paper we explain why numerically both interpretations of the indirect marginal effect are the same. To estimate indirect and total returns to scale in the application of our spatial autoregressive frontier model to aggregate production in 40 European countries we must use the second interpretation of the indirect marginal effect.

Two features of the application are firstly, rather than assume that efficiency is time-invariant à la Druska and Horrace (2004), we allow efficiency to be time-variant using the Cornwell *et al.* (1990) (CSS from hereon) estimator. Secondly, we use ten specifications of the spatial weights matrix, where the specifications are weighted by various proxies for economic distance or various proxies for composite geographical-economic distance. Economic distance between two countries will differ depending on the direction so we choose a direction. Specifically, our proxies for economic distance are a country's biggest 3–7 import flows. And our proxies for geographical and economic distance are a country's nearest 3 – 7 import flows. The range 3 – 7 is chosen to capture the effect of assuming that the spatial dependence is highly concentrated around production in a small number of near/big import partners and then is assumed to be progressively less concentrated. Further in the paper we discuss the specifications of the spatial weights matrices in detail.

To provide an insight into the type of conclusions which can be made from the spatial autoregressive frontier model which we develop, two of the key empirical findings from the application are as follows. Firstly, we find that production in the sample average country is characterized by constant own returns to scale, but when returns to scale spillovers from other countries are taken into account they are sufficient for production in the sample average country to exhibit increasing total returns to scale. Secondly, we find that over the entire sample the mean annual efficiency score when the spatial weights matrix is weighted by a country's biggest 3 – 7 import flows is smaller than the mean

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<sup>2</sup>The total marginal effect of an explanatory variable is the sum of the corresponding direct and indirect marginal effects.

annual score when the matrix is weighted by a country’s nearest 3 – 7 import flows.

The remainder of this paper is organized as follows. In section 2 we formally present a Cliff-Ord type production function. In section 3 we follow CSS by firstly, showing how we move from the Cliff-Ord type production function in section 2 to the associated frontier model. Secondly, we explain how we use the fixed effects to calculate time-varying efficiency. The steps involved in the estimation of the frontier model are set out in section 4. Section 5 discusses how we estimate the direct, indirect and total elasticities for the inputs and exogenous variables, and also how these estimates for the inputs are used to calculate direct, indirect and total returns to scale. In section 6, we apply the frontier model to aggregate production in 40 European countries. In section 7 we conclude with a summary of the main contributions of the paper and suggest a worthwhile area for further work.

## 2 Cliff-Ord Type Production Function

Consider the following Cliff-Ord type production function for panel data:

$$\begin{aligned}
 y_{it} &= X_{it}\beta + \lambda \sum_{j=1}^N w_{ij}y_{jt} + \varepsilon_{it} \\
 i &= 1, \dots, N; \quad t = 1, \dots, T,
 \end{aligned}
 \tag{1}$$

where  $N$  is a cross-section of economic units operating over a fixed time dimension  $T$ ;  $y_{it}$  is a positive observation for the output of the  $i$ -th unit at time  $t$ ;  $X_{it}$  is a  $(1 \times K)$  vector of positive observations for the  $K$  inputs of the  $i$ -th unit at time  $t$ ;  $w_{ij}$  is a known non-negative element of the  $(N \times N)$  spatial weights matrix,  $W$ ;  $\beta$  is the  $(K \times 1)$  vector of fixed parameters to be estimated;  $\lambda$  is the spatial autoregressive parameter;  $\varepsilon_{it}$  is an i.i.d. disturbance for  $i$  and  $t$  with zero mean and variance  $\sigma^2$ .<sup>3</sup>

$\sum_{j=1}^N w_{ij}y_{jt}$ , which is an exogenous variable in (1) and therefore shifts the production technology, is typically referred to as the spatial lag of  $y_{it}$ .  $W$  captures the spatial interaction of  $y_{it}$  in the cross-section and must be specified prior to estimation according to some measure of proximity e.g. contiguity or physical, economic or climatic distance between the units. If a cross-sectional unit  $j$  is related to  $i$ , the pre-specified spatial weight  $w_{ij}$  will be non-zero and units  $i$  and  $j$  are described as neighbors. We discuss in detail the specification of the spatial weights matrices for the application to aggregate production in 40 European countries further in the paper.

For our asymptotic analysis of the estimator which we employ, the following underlying assumptions with regards to the Cliff-Ord type production function in (1) are made.

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<sup>3</sup>Using the CSS estimator we transform the Cliff-Ord type production function in (1) into the associated frontier model by introducing unit specific effects to (1). For the moment, however, we postpone the introduction of unit specific effects.

**Assumption 1.** *All the diagonal elements of the non-stochastic spatial weights matrix,  $W$ , are zero.*

The zero diagonal assumption is a normalization of the model. It implies that no cross-sectional unit is described as its own neighbor. In other words, spatial self-influence of the units is excluded.

**Assumption 2.** *The matrix  $(I_N - \lambda W)$  is non-singular for all values of  $\lambda$ , where  $I_N$  is the identity matrix of dimension  $N$  and the parameter space of  $\lambda$  is taken to be  $1/r_{\min} < \lambda < 1$ .*

$r_{\min}$  denotes the most negative real characteristic root of  $W$  and, as is common in spatial econometrics, we use a row-normalized  $W$  so 1 is the largest real characteristic root of  $W$ . It is also assumed that the parameter space of  $\lambda$  does not depend on the sample size, which is also a common assumption in the spatial econometrics literature.<sup>4</sup> As a result of Assumption 2,  $y_{it}$  is complete and uniquely defined by (1), and (1) has the following reduced form, where the subscript  $i$ 's are dropped to denote successive stacking of cross-sections.

$$y_t = (I_N - \lambda W)^{-1} X_t \beta + (I_N - \lambda W)^{-1} \varepsilon_t, \quad (2)$$

where  $y_t$  is an  $(N \times 1)$  vector;  $X_t$  is an  $(N \times K)$  matrix of positive observations for the inputs; and  $\varepsilon_t$  is an  $(N \times 1)$  vector

**Assumption 3.** *The row and column sums of  $W$  and  $(I_N - \lambda W)$  are bounded uniformly in absolute value.*

Assumption 3 limits the spatial correlation of the cross-sectional observations for the dependent variable to a manageable degree. As a result, the spatial correlation has a ‘fading’ memory (Kelejian and Prucha, 1998; 1999; 2010). Assumption 3 therefore plays an important role in the asymptotic properties of the estimators for spatial models because if row and column sums are bounded uniformly in absolute value then the row and column sums of products of matrices have the same property (Kelejian and Prucha, 2004; Lee, 2004). Hence, the row and column sums of the variance-covariance matrix are bounded uniformly as  $N$  goes to infinity.

### 3 Fixed Effects and Technical Efficiency

The Cliff-Ord type production function in (1) can be transformed into the associated frontier model by introducing unit specific time-invariant effects. This is because the SS and CSS estimators use these effects to calculate unit specific technical inefficiencies. We

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<sup>4</sup>See Kelejian and Prucha (2010) for a detailed discussion of the parameter space for spatial autoregressive parameter.

proceed along these lines by introducing fixed effects to (1).

$$\begin{aligned}
y_{it} &= \alpha_i + X_{it}\beta + \lambda \sum_{j=1}^N w_{ij}y_{jt} + \varepsilon_{it}; \\
i &= 1, \dots, N; \quad t = 1, \dots, T,
\end{aligned} \tag{3}$$

where  $\alpha_i$  is a dummy variable for each unit.

Following SS, the  $N$  estimated unit specific effects can be used to calculate time-invariant technical efficiency for each unit,  $TE_i$ , as follows:

$$TE_i = \hat{\alpha}_i - \max_i(\hat{\alpha}_i), \quad i = 1, \dots, N. \tag{4}$$

Rather than estimate the technical efficiencies of the units relative to an absolute standard, (4) estimates the efficiencies relative to the most efficient unit in the sample. Accordingly, the unit with largest unit specific effect is assumed to lie on the frontier. Estimating the efficiencies from the unit specific effects ensures that the efficiencies are not correlated with the input levels, and an *a priori* assumption does not need to be made about the inefficiency distribution.

CSS extend the SS approach by using the unit specific effects in conjunction with a unit specific flexibly parameterized function of time to calculate time-varying efficiencies.<sup>5</sup> We follow this approach by replacing  $\alpha_i$  in (3) with  $\delta_{it}$  from (5) to obtain (6):

$$\delta_{it} = \alpha_i + \eta_i t + \rho_i t^2 \quad i = 1, \dots, N. \tag{5}$$

$$\begin{aligned}
y_{it} &= \delta_{it} + X_{it}\beta + \lambda \sum_{j=1}^N w_{ij}y_{jt} + \varepsilon_{it} \\
&= \alpha_i + \eta_i t + \rho_i t^2 + X_{it}\beta + \lambda \sum_{j=1}^N w_{ij}y_{jt} + \varepsilon_{it} \\
&= \alpha_i + X_{it}\beta + \lambda \sum_{j=1}^N w_{ij}y_{jt} + v_{it},
\end{aligned} \tag{6}$$

where (6) is the specification of the frontier model which we estimate in the application and  $v_{it} = \eta_i t + \rho_i t^2 + \varepsilon_{it}$ . We obtain the estimates of  $\eta_i$  and  $\rho_i$  as in CSS by regressing the residuals from the estimate of (6),  $v_{it}$ , on time and time-squared for each unit.<sup>6</sup> The

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<sup>5</sup>Recently, Knip, Sickles and Song (2012) have showed that the CSS and many other common specifications of temporal heterogeneity can be considered special cases of their general factor model. Thus one could view our model set up here as providing a link between the factor model literature and the spatial correlation literature.

<sup>6</sup>In the next section we describe how we obtain the fixed effects from the fitted frontier model.

three components in (5) are then summed to obtain  $\delta_{it}$ . Finally, the estimates of  $\delta_{it}$  are used to calculate time-varying technical efficiency as follows:

$$TE_{it} = \widehat{\delta}_{it} - \max_i \left( \widehat{\delta}_{it} \right) \quad i = 1, \dots, N. \quad (7)$$

The time-varying efficiencies are calculated relative to the most efficient unit in the sample in each time period. The most efficient unit in the sample in each time period can of course change across time periods.

## 4 Estimation of the Frontier Model

Models with spatial interaction effects can be estimated using maximum likelihood (ML), instrumental variables or generalized method of moments (IV/GMM), or the Bayesian MCMC approach. In this paper (6) is estimated using ML. Assuming that the panel is balanced, the log-likelihood function associated with (6) is as follows:

$$\text{Log}L = -\frac{NT}{2} \log(2\pi\sigma^2) + T \log |I_N - \lambda W| - \frac{1}{2\sigma^2} \sum_{i=1}^N \sum_{t=1}^T \left( y_{it} - \lambda \sum_{j=1}^N w_{ij} y_{jt} - X_{it}\beta - \alpha_i \right)^2. \quad (8)$$

Since the spatial autoregressive variable in (6) is endogenous the assumption of the standard regression,  $E\left[\left(\sum_{j=1}^N w_{ij} y_{jt}\right) v_{it}\right] = 0$ , is violated. We ensure that  $\lambda$  lies within its parameter space, adjust for the endogeneity of the spatial autoregressive variable and also the fact that  $v_t$  is not observed in the usual way by including the scaled logged determinant of the Jacobian transformation of  $v_t$  to  $y_t$  (i.e.  $T \log |I_N - \lambda W|$ ) in the log-likelihood function (see Anselin, 1988, and Elhorst, 2009). Solving the partial derivatives of (8) with respect to  $\alpha_i$  for  $\alpha_i$  yields (9)

$$\alpha_i = \frac{1}{T} \sum_{t=1}^T \left( y_{it} - \lambda \sum_{j=1}^N w_{ij} y_{jt} - X_{it}\beta \right) \quad i = 1, \dots, N. \quad (9)$$

As Elhorst (2009) notes, it is evident from (9) that the fixed effects adjust for the spatial dependence in the cross-section at each point in time. Therefore no further adjustment to account for the spatial interaction in the cross-section in each time period is necessary.

The concentrated log-likelihood function with respect to  $\beta$ ,  $\lambda$  and  $\sigma^2$  in (10) is obtained by substituting (9) into (8), and to circumvent the incidental parameter problem when estimating (6)  $y_{it}$  and  $X_{it}$  are demeaned.

$$\text{Log}L = -\frac{NT}{2} \log(2\pi\sigma^2) + T \log |I_N - \lambda W| - \frac{1}{2\sigma^2} \sum_{i=1}^N \sum_{t=1}^T \left[ y_{it}^* - \lambda \left( \sum_{j=1}^N w_{ij} y_{jt} \right)^* - X_{it}^* \beta \right]^2, \quad (10)$$

where  $y_{it}^* = y_{it} - \frac{1}{T} \sum_{t=1}^T y_{it}$  and  $X_{it}^* = X_{it} - \frac{1}{T} \sum_{t=1}^T X_{it}$ . By demeaning  $y_{it}$  and  $X_{it}$  the intercept and the fixed effects drop out of the fitted models. Usually interest does not center on the fixed effects so removing them from the fitted model does not pose a problem. This is not the case, however, when SS and CSS type frontier models are estimated because the fixed effects are needed to calculate the efficiencies. Having estimated (6) the fixed effects are retrieved using (9).

Dropping the subscript  $i$ 's and  $t$ 's from  $y_{it}^*$  and  $X_{it}^*$  to denote an  $(NT \times 1)$  vector and an  $(NT \times K)$  matrix, respectively, of stacked cross-sectional observations for  $t = 1, \dots, T$ . The estimate of  $\lambda$  is obtained by maximizing the concentrated log-likelihood function in (11) and then  $\lambda$  is used in (12) and (13) to obtain the estimates of  $\beta$  and  $\sigma^2$ .

$$\text{Log}L = C - \frac{NT}{2} \log [(e_0^* - \lambda e_1^*)'(e_0^* - \lambda e_1^*)] + T \log |I_N - \lambda W|, \quad (11)$$

$$\beta = b_0 - \lambda b_1 = (X^{*'} X^*)^{-1} X^{*'} [y^* - \lambda (I_T \otimes W) y^*], \quad (12)$$

$$\sigma^2 = \frac{1}{NT} (e_0^* - \lambda e_1^*)'(e_0^* - \lambda e_1^*), \quad (13)$$

where  $I_T$  is the identity matrix of dimension  $T$ ;  $\otimes$  is the Kronecker product;  $C$  is a constant which does not depend on  $\lambda$ ;  $b_0$  and  $b_1$  are the OLS parameters from successively regressing  $y^*$  and  $(I_T \otimes W)y^*$  on  $X^*$ ; and  $e_0^*$  and  $e_1^*$  are the residuals from these OLS regressions, respectively.<sup>7</sup>

The asymptotic variance-covariance matrix for the estimates of  $\beta$ ,  $\lambda$  and  $\sigma^2$ , which Elhorst and Freret (2009) show has following form, is computed to obtain the associated standard errors and  $t$ -values.

$$\text{Asy. Var}(\beta, \lambda, \sigma^2) = \begin{bmatrix} \frac{1}{\sigma^2} X^{*'} X^* & - & - \\ \frac{1}{\sigma^2} X^{*'} (I_T \otimes \widetilde{W}) X^{*'} \beta & T^* \text{tr}(\widetilde{W} \widetilde{W} + \widetilde{W}' \widetilde{W}) + \frac{1}{\sigma^2} \beta' X^{*'} (I_T \otimes \widetilde{W}' \widetilde{W}) X^{*'} \beta & - \\ 0 & \frac{T}{\sigma^2 \text{tr}(\widetilde{W})} & \frac{NT}{2\sigma^4} \end{bmatrix}^{-1}, \quad (14)$$

where  $\widetilde{W} = W(I_N - \lambda W)^{-1}$  and  $\text{tr}$  denotes the trace of the relevant matrix. Because (14)

<sup>7</sup>The maximization problem in (11) can only be solved numerically as a closed form solution for  $\lambda$  does not exist. See Elhorst (2009) for details of the numerical approach which is used to maximize (11).



is a symmetric matrix we omit the upper diagonal elements. Lee and Yu (2010), however, show that using the demeaning procedure to estimate a spatial model with fixed effects such as (6) results in a biased estimate of  $\sigma^2$  if  $N$  is large and  $T$  is fixed. Following Lee and Yu (2010) and Elhorst (2011) we correct for this bias by replacing the biased  $\sigma^2$  in (14) with the bias corrected estimate of  $\sigma^2$ ,  $\sigma_{BC}^2 = T\sigma^2/(T - 1)$ , which will change the standard errors and  $t$ -values.

## 5 Direct, Indirect and Total Returns to Scale

As we noted above, the coefficients on the inputs and exogenous variables from a fitted Cliff-Ord type production frontier cannot be interpreted as elasticities. This is because the marginal effect of an independent variable is a function of the spatial autoregressive variable. LeSage and Pace (2009) therefore suggest the following approach to calculate the direct, indirect and total marginal effects of the independent variables and the associated significance levels having estimated a model such as (6). Using estimates of the direct, indirect and total marginal effects for the inputs we calculate direct, indirect and total returns to scale.

Differentiating (2) with respect to the  $k$ -th input,  $x_{k,t}$ , yields the following vector of partial derivatives:

$$\begin{aligned} \left[ \frac{\partial y}{\partial x_{k,1}} \quad \frac{\partial y}{\partial x_{k,N}} \right]_t &= \begin{bmatrix} \frac{\partial y_1}{\partial x_{k,1}} & \cdot & \frac{\partial y_1}{\partial x_{k,N}} \\ \cdot & \cdot & \cdot \\ \frac{\partial y_N}{\partial x_{k,1}} & \cdot & \frac{\partial y_N}{\partial x_{k,N}} \end{bmatrix}_t \\ &= (I_N - \lambda W)^{-1} \begin{bmatrix} \beta_k & 0 & \cdot & 0 \\ 0 & \beta_k & \cdot & 0 \\ \cdot & \cdot & \cdot & \cdot \\ 0 & 0 & \cdot & \beta_k \end{bmatrix}, \end{aligned} \quad (15)$$

where the right-hand side of (15) is independent of the time index. (15) will yield different direct and indirect marginal effects on each unit so to facilitate interpretation LeSage and Pace (2009) suggest reporting a mean direct marginal effect (average of the diagonal elements on the right-hand side of (15)) and a mean indirect marginal effect (average column or row sum of the non-diagonal elements on the right-hand side of (15) since the magnitude of these two calculations are the same). The average column and row sums on the right-hand side of (15) relate to the first and second interpretations of the indirect marginal effect, respectively, as defined in the opening section of this paper. The mean total marginal effect is simply the sum of the mean direct and indirect marginal effects.

To compute  $t$ -statistics for the average direct, indirect and total marginal effects, LeSage and Pace (2009) propose Bayesian MCMC simulation of the distributions of the

effects using the variance-covariance matrix associated with the ML estimates. 1,000 parameter combinations of the  $\beta$ ,  $\lambda$  and  $\sigma_{BC}^2$  estimates are drawn from the variance-covariance matrix such that each combination is a vector of length  $2 + K$  (number of parameters estimated excluding the intercept and the fixed effects) consisting of random values drawn from a normal distribution with mean zero and standard deviation one. Mean direct, indirect and total marginal effects are calculated for each parameter combination. The mean direct, indirect and total marginal effects which we report are the averages over the 1,000 draws. The associated  $t$ -statistics are obtained by dividing the reported mean direct, indirect and total marginal effects by the standard deviation across the corresponding 1,000 mean marginal effects.

For the  $i$ -th unit at time  $t$  which uses inputs to produce a single output, own (i.e. direct) returns to scale is the percentage change in in the  $i$ -th unit's output due to a one percent increase in all the  $i$ -th unit's inputs. The estimates of direct returns to scale also include feedback effects i.e. effects which pass through first order neighbors and higher order neighbors via the spatial multiplier matrix and back to the  $i$ -th unit which initiated the change. Spillover (i.e. indirect) returns to scale for the  $i$ -th unit at time  $t$  is the percentage change in the  $i$ -th unit's output due to a one percent increase in the inputs for all the other  $J$  units. Total returns to scale is the sum of the direct and indirect returns to scale. Total returns to scale for the  $i$ -th unit at time  $t$  is therefore the percentage change in the  $i$ -th unit's output due to a one percent increase in the inputs for all  $N$  units in the sample. Direct, indirect and total returns to scale for the  $i$ -th unit at time  $t$ , which are denoted  $RTS_{it}^{Direct}$ ,  $RTS_{it}^{Indirect}$  and  $RTS_{it}^{Total}$ , respectively, can be calculated as follows:

$$RTS_{it}^{Direct} + RTS_{it}^{Indirect} = RTS_{it}^{Total} \quad (16)$$

$$\sum_{k=1}^{k=K} ex_{k,it}^{Direct} + \sum_{k=1}^{k=K} ex_{k,it}^{Indirect} = \sum_{k=1}^{k=K} ex_{k,it}^{Total}, \quad (17)$$

where  $ex_{k,it}^{Direct}$ ,  $ex_{k,it}^{Indirect}$  and  $ex_{k,it}^{Total}$  are column vectors of direct, indirect and total elasticities for the  $k$ -th input, respectively. Production in the  $i$ -th unit is characterized by total decreasing returns to scale if  $RTS_{it}^{Total} < 1$ , total increasing returns to scale if  $RTS_{it}^{Total} > 1$  and total constant returns to scale if  $RTS_{it}^{Total} = 1$ . Using the estimates of  $RTS_{it}^{Direct}$  and  $RTS_{it}^{Indirect}$ , direct and indirect returns to scale are classified in the same way.

## 6 An Application to Aggregate Production in 40 European Countries

In this section we use data for 40 European countries for the period 1995 – 2008 and ten specifications of  $W$ . Specifically, in this application the production structure takes the form of a single output translog function and so the specification of the Cliff-Ord type production frontier which we estimate is as follows.

$$y_{it} = \tau + \alpha_i + TL(X, t)_{it} + z_{it}\phi + \lambda \sum_{j=1}^N w_{ij}y_{jt} + v_{it} \quad (18)$$

$$v_{it} = \eta_i t + \rho_i t^2 + \varepsilon_{it}, \quad (19)$$

where  $\tau$  is the intercept;  $TL(X, t)_{it}$  represents the technology as the non-constant returns to scale translog approximation of the log of the production function;  $X_{it}$  is a  $(1 \times 2)$  vector of input levels where the elements are denoted  $x_{1,it}$  and  $x_{2,it}$ ;  $z_{it}$  is a  $(1 \times 3)$  vector of country specific exogenous characteristics for  $i$ , where the elements are denoted  $z_{1,it}, \dots, z_{3,it}$ ;  $\phi$  is the associated vector of parameters to be estimated. All the other variables, matrices and parameters in (18) and (19) are as described in previous sections.

### 6.1 Data and Specification of the Spatial Weights Matrix

The data is a balanced panel of macroeconomic variables which are logged where it is appropriate. All the variables with the exception of the dummy variables are then normalized around their mean values so the first order input and time parameters can be interpreted as elasticities at the sample mean.

The output is real GDP in 2005 international dollars,  $y$ . The inputs are number of workers,  $x_1$ , and real capital stock in 2005 international dollars,  $x_2$ . Data was extracted from the Penn World Table Version 7.0, *PWT7* (Heston *et al.*, 2011), to calculate  $y$ ,  $x_1$  and  $x_2$ . The variables which we extracted are as follows: real GDP per capita calculated using the Laspeyres index and the chain method, denoted as *rgdpl* and *rgdpch* in *PWT7* (both of which are in 2005 international dollars); population, *pop*; real GDP per worker calculated using the chain method, *rgdpwok*; and investment as a share of *rgdpl*, *ki*.

$x_1 = (rgdpch * pop)/rgdpwok$ ,  $y = x_1 * rgdpwok$  and we calculate  $x_2$  in two steps. Firstly, we calculate real aggregate investment which is  $rgdpl * pop * ki$ . Secondly, real capital stock in 1995 is assumed to be depreciated real aggregate investment in 1994 because of concerns about the number of observations in the sample, and we follow much of the literature on estimating capital stock and use a 6% depreciation rate.<sup>8</sup> Observations

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<sup>8</sup>We thank Joe Pearlman for suggesting this approach to obtain a starting value for real capital stock. Although this is not the usual approach to obtain a starting value for capital stock it will become apparent that the capital elasticities using this approach are sensible. A more conventional approach to obtain the

for real capital stock for the remainder of the sample are estimated using the perpetual inventory method.  $z_1$  is trade openness the data for which was taken from *PWT7*,  $z_2$  is government spending as a share of *rgdpl* where the data is again from *PWT7* and  $z_3$  is a dummy variable for EU membership.

(18) is estimated using ten specifications of  $W$ . The weights in all ten specifications are calculated using data from the IMF Direction of Trade Statistics database on import flows in 2000 US dollars for the period 2000 – 2008. The proxies for composite geographical distance and economic distance are a country’s average real imports over period 2000 – 2008 from the nearest 3 – 7 countries according to distances between capital cities. These proxies are used to weight and row normalize the first five specifications of  $W$  (denoted  $W_{3Near}, \dots, W_{7Near}$ ). The proxies for economic distance are a country’s average real imports over the period 2000 – 2008 on its biggest 3 – 7 real import flows. These proxies are used to weight and row normalize the remaining specifications of  $W$  (denoted  $W_{3Big}, \dots, W_{7Big}$ ). The descriptive statistics for the continuous variables are presented in Table 1 and are for the raw data.

[Insert Table 1]

## 6.2 Estimation Results

We estimate (18) using all ten specification of  $W$  with and without fixed effects. To test the null hypothesis that the fixed effects are not jointly significant (i.e.  $\alpha_i = \dots = \alpha_N = \tau$ ) we perform a likelihood ratio (LR) test on each of the ten fitted Cliff-Ord type frontier models against the corresponding pooled model. The associated test statistic is chi-squared distributed with degrees of freedom equal to the number of restrictions which must be imposed on the unrestricted model to obtain the restricted model, which in this case is  $N - 1$ . On each occasion we reject the null hypothesis at the 1% level, thereby justifying the inclusion of fixed effects in (18) for all ten specifications of  $W$ .<sup>9</sup> The fixed effects from the ten Cliff-Ord type frontier models are then used to compute the efficiencies. To enable comparisons we also use the same approach to calculate the efficiencies from the non-spatial frontier model which CSS fit. This simply involves omitting  $\lambda \sum_{j=1}^N w_{ij}y_{jt}$  from (18) and fitting the standard fixed effects model using the Within estimator.

In Tables 2 and 3 we present the estimation results for the non-spatial frontier model and selected Cliff-Ord type frontier models. Before we discuss the estimation results we once again define direct, indirect and total elasticities in the context of our results. The

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starting value is to use fully depreciated GDP but this would require several years of additional data, which was not available for all the countries in the sample.

<sup>9</sup>The test statistics range from 1174.29 – 1185.53 for model specifications using  $W_{3Near} - W_{7Near}$  and from 1207.35 – 1227.26 for model specifications using  $W_{3Big} - W_{7Big}$ .  $W_{3Near}$  ( $W_{3Big}$ ) and  $W_{7Near}$  ( $W_{7Big}$ ) do no relate to the lower and upper limits of these ranges, respectively, or any other ranges which we report in this application.

$\beta_1 - \beta_9$  and  $\phi_1 - \phi_3$  parameters are the usual own elasticities from the non-spatial frontier model.  $\beta_1^{Direct} - \beta_9^{Direct}$  and  $\phi_1^{Direct} - \phi_3^{Direct}$  are also own elasticities from the Cliff-Ord type frontier models but different notation is used to denote an own elasticity from the non-spatial frontier model and an own elasticity from a Cliff-Ord type frontier model. We refer to own elasticities from a Cliff-Ord type frontier model as direct elasticities because they take into account feedback effects to the  $i$ -th unit (i.e. the effect of a change in an explanatory variable in the  $i$ -th unit which affects a neighboring unit's dependent variable which in turn affects the  $i$ -th unit's dependent variable), whereas by construction the own elasticities from the non-spatial frontier model ignore such effects.  $\beta_1^{Indirect} - \beta_9^{Indirect}$  and  $\phi_1^{Indirect} - \phi_3^{Indirect}$  denote the spillover elasticities from a Cliff-Ord type frontier model which, using what we earlier described as the second interpretation of a spillover elasticity, refers to the effect on the dependent variable of the  $i$ -th unit following a change in an explanatory variable in all the other  $J$  units.  $\beta_1^{Total} - \beta_9^{Total}$  and  $\phi_1^{Total} - \phi_3^{Total}$  denote the sum of the direct and indirect elasticities from a Cliff-Ord type frontier model and therefore refer to the effect on the dependent variable in the  $i$ -th unit following a change in an explanatory variable in all units in the sample.

[Insert Tables 2 and 3]

Since  $\lambda$  is assumed to lie in the parameter space  $(1/r_{\min}, 1)$  it cannot be interpreted as an elasticity. However, the estimates of  $\lambda$  indicate how the spatial dependence of  $y$  is affected by the specification of  $W$ . All the estimates of  $\lambda$  are positive and significant at the 1% level or lower and when  $W$  is weighted by the  $i$ -th country's nearest (biggest) 3 – 7 import flows the estimates range from 0.122 – 0.287 (0.472 – 0.579). These ranges suggest that the output of the  $i$ -th country depends much more on the output of its biggest 3 – 7 import partners than the output of its nearest 3 – 7 import partners. Moreover, we find that  $\lambda$  tends to increase as  $W$  is weighted by more big and, in particular, more near import flows (see Table 2).

A production function assumes that the  $i$ -th unit's output is monotonically increasing in the  $i$ -th unit's inputs. A new line of enquiry which follows from our spatial autoregressive production function is whether the  $i$ -th unit's output is monotonically increasing/decreasing in the inputs of the other  $J$  units and monotonically increasing/decreasing in the inputs of all  $N$  units in the sample. The own labor and capital elasticities from the non-spatial frontier model, and the direct, indirect and total labor and capital elasticities from the Cliff-Ord type frontier models are all positive and significant at the 1% level or lower. This indicates that at the sample mean the  $i$ -th unit's output is monotonically increasing in the  $i$ -th unit's inputs, the inputs of the other  $J$  units and the inputs of all  $N$  units in the sample.<sup>10</sup> Furthermore, the own labor and capital elasticities from the

<sup>10</sup>In Tables 2 and 3  $\beta_1$ ,  $\beta_1^{Direct}$ ,  $\beta_1^{Indirect}$  and  $\beta_1^{Total}$  correspond to labor and  $\beta_2$ ,  $\beta_2^{Direct}$ ,  $\beta_2^{Indirect}$  and  $\beta_2^{Total}$  relate to capital.

non-spatial frontier model indicate that, on average, a country’s output is monotonically increasing in its own labor and capital for 100% of the sample. For the ten Cliff-Ord type frontier models, the direct, indirect and total labor and capital elasticities suggest that the  $i$ -th unit’s output is: monotonically increasing in the  $i$ -th unit’s labor and capital, monotonically increasing in the labor and capital of the other  $J$  units and monotonically increasing in the labor and capital of all  $N$  units for 100% of the sample.

Furthermore, a production function assumes that the  $i$ -th unit’s output is concave in the  $i$ -th unit’s inputs. From our spatial autoregressive production function there is the added issues of whether the  $i$ -th unit’s output is concave/convex in the inputs of the other  $J$  units and concave/convex in the inputs of all  $N$  units in the sample. If a fitted production function is concave we will observe a particular sign pattern for the principal minors in the Hessian matrix: all the odd numbered principal minors must be non-positive and all the even numbered principal minors must be non-negative. Applying this test to the non-spatial production frontier model reveals that for 47.9% of the sample the  $i$ -th unit’s output is concave in the  $i$ -th unit’s inputs. When we apply this test to the ten Cliff-Ord type production frontier models, on average, we observe concavity of the  $i$ -th unit’s output in the  $i$ -th unit’s inputs for a larger percentage of the sample than is the case for the non-spatial frontier model. Specifically, we find that, on average, the  $i$ -th unit’s output is: concave in the  $i$ -th unit’s inputs for 59.7% of the sample; concave in the inputs of the other  $J$  units for 77.3% of the sample; and concave in the inputs of all  $N$  units for 59.2% of the sample.

Despite the own elasticities from the non-spatial frontier model ignoring feedback effects, whereas the direct elasticities from the Cliff-Ord type frontier models take account of these effects, the own and direct labor/capital elasticities are similar. With respect to labor we observe an estimate of  $\beta_1$  of 0.669 and the estimates of  $\beta_1^{Direct}$  are of the order 0.630 – 0.703. These estimates of  $\beta_1$  and  $\beta_1^{Direct}$  are therefore robust to model specification and are also in line with evidence on the labor income share of GDP for the EU–15 member states (for verification see Table 1 in Arpaia *et al.*, 2009). Moreover, with respect to capital the estimates of  $\beta_2$  and  $\beta_2^{Direct}$  are essentially the same (0.261 – 0.280) and are therefore also robust to model specification.

All the indirect labor and capital elasticities from the Cliff-Ord type frontier models are positive and significant. This indicates that the evidence of positive labor and capital spillovers across Europe is robust across the ten specifications of  $W$ . Moreover, since all the direct and indirect labor and capital elasticities are positive and significant it follows that all the total labor and capital elasticities are positive and significant. Turning our attention now to the magnitude of the indirect labor and capital elasticities. Interestingly, the average estimate of  $\beta_1^{Indirect}$  when  $W$  is specified according to the  $i$ -th country’s 3 – 7 biggest import flows (0.710) is much larger than the corresponding average estimate when the specification of  $W$  is  $W_{3Near} - W_{7Near}$  (0.201). Furthermore, the average estimate of

$\beta_2^{Indirect}$  when the specification of  $W$  is  $W_{3Big} - W_{7Big}$  (0.287) is greater than the average estimate of  $\beta_2^{Indirect}$  when  $W$  is weighted by the  $i$ -th country's 3–7 nearest import flows (0.078). This evidence suggests that, on average, there are greater labor and capital spillovers across Europe over economic distance than there are over a combination of geographical distance and economic distance.

Comparing the direct labor and capital elasticities with the corresponding indirect elasticities when the specification of  $W$  is  $W_{3Near} - W_{7Near}$ , we find that the direct labor and capital elasticities are always greater than the corresponding indirect elasticity. When the specification of  $W$  is  $W_{3Near} - W_{7Near}$ , on average, the direct labor (capital) elasticity is 0.484 (0.191) larger than the corresponding indirect elasticity. In contrast, when the specification of the spatial interaction is  $W_{3Big} - W_{7Big}$ , we find that, in general, the indirect labor and capital elasticities are as least as large as the corresponding direct elasticity. For example, it is evident from Table 3 that the direct and indirect labor elasticities from Model 5 ( $W_{3Big}$ ) are very similar, whereas in Model 6 ( $W_{5Big}$ ) the indirect labor elasticity is noticeably greater than the direct labor elasticity.

Considering now the average direct, indirect and total labor and capital elasticities outside the sample mean from firstly, models where the specification of  $W$  is  $W_{3Near} - W_{7Near}$  and secondly, models when the spatial interaction is  $W_{3Big} - W_{7Big}$  (See Panels A and B in Figure 1, respectively).<sup>11</sup> From Panel A in Figure 1 it is evident that over the entire sample period the average direct labor (capital) elasticity across the  $W_{3Near} - W_{7Near}$  models is a lot larger than the corresponding average indirect labor (capital) elasticity, which also what we observed at the sample mean from the individual  $W_{3Near} - W_{7Near}$  models. Interestingly Panel B in Figure 1 indicates that over the entire sample period the average direct labor (capital) elasticity across the  $W_{3Big} - W_{7Big}$  models is approximately the same as the corresponding average indirect labor (capital) elasticity. As we noted above this is also the case at the sample mean for some of the individual  $W_{3Big} - W_{7Big}$  models. Moreover, we can see from Panels A and B in Figure 1 that over the entire sample period, the non-spatial frontier model yields, on average, own labor and capital elasticities which are a good approximation of the average direct labor and capital elasticities.

[Insert Figure 1]

The parameter on time from the non-spatial frontier model ( $\beta_6$ ) and the direct, indirect and total time parameters from the Cliff-Ord type frontier models ( $\beta_6^{Direct}$ ,  $\beta_6^{Indirect}$  and  $\beta_6^{Total}$ ) are all negative and significant at the 5% level or lower. Our total time parameters may appear large but this is because they are the sum of the direct and indirect time elasticities (see Tables 2 and 3). Moreover, negative own and direct time effects

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<sup>11</sup>To enable comparisons we also include in Panels A and B in Figure 1 the labor and capital elasticities outside the sample mean from the non-spatial frontier model.

is in line with the negative and significant own time parameter from Kumbhakar and Wang’s (2005) stochastic production frontier analysis for a sample of 87 countries over the period 1960 – 1987. In our case we almost certainly observe negative time effects for the hypothetical sample average country because our sample contains a relatively large number of Eastern European countries which were in the early stages of their transition to market economies at the beginning of the study period.

Finally, we examine the findings for the exogenous  $z$ -variables (i.e. the  $\phi$  parameters). The only exogenous  $z$ -variable where the parameter is significant in the non-spatial frontier model, and the direct and indirect parameters from the Cliff-Ord type models are significant is the public sector size variable,  $z_2$ . We find that the own public sector size parameters ( $\phi_2$  and  $\phi_2^{Direct}$  in the non-spatial and Cliff-Ord type frontier models, respectively) and the estimates of the spillover public sector size parameter ( $\phi_2^{Indirect}$ ) are all negative. A significant negative own public sector size parameter is consistent with the robust negative relationship between government expenditure and growth which Folster and Henrekson (2001) observe. When the specification of  $W$  is  $W_{3Near} - W_{7Near}$ , the estimates of  $\phi_2^{Direct}$  and  $\phi_2^{Indirect}$  range from  $-1.479 - (-1.343)$  and  $-0.541 - (-0.202)$ , respectively. However, when the specification of the spatial interaction is  $W_{3Big} - W_{7Big}$ , the estimates of  $\phi_2^{Direct}$  range from  $-1.532 - (-1.268)$  and the estimates of  $\phi_2^{Indirect}$  are of the order  $-1.710 - (-1.112)$ . This suggests firstly, that the estimates of  $\phi_2^{Direct}$  are robust to the specification of  $W$ . Secondly, it is apparent that the estimates of  $\phi_2^{Indirect}$  very much depend upon whether the spatial dependence in production is based on economic distance or a combination of geographical distance and economic distance.

### 6.3 Estimates of Direct, Indirect and Total Returns to Scale

Before we report and discuss our returns to scale estimates we define direct, indirect and total returns to scale in the context of our results. Using the own labor and capital elasticities at the sample mean from the non-spatial frontier model ( $\beta_1$  and  $\beta_2$ , respectively) we compute own returns to scale in the usual way. For the  $i$ -th unit which uses its own inputs to produce a single output at time  $t$ , own returns to scale at the sample mean,  $RTS$ , can be calculated as follows using the relevant parameters from a non-spatial production function.

$$RTS_{it} = \frac{\partial \ln y_{it}}{\sum_{k=1}^K \partial \ln x_{k,it}} \equiv \sum_{k=1}^{k=K} ex_{k,it}, \quad (20)$$

where  $ex_{k,it}$  denotes the  $k$ -th input elasticity at the sample mean. Using the direct, indirect and total labor and capital elasticities at the sample mean from the Cliff-Ord type frontier models ( $\beta_1^{Direct}$  and  $\beta_2^{Direct}$ ,  $\beta_1^{Indirect}$  and  $\beta_2^{Indirect}$ , and  $\beta_1^{Total}$  and  $\beta_2^{Total}$ ,



respectively) using (20) we calculate direct, indirect and total returns to scale at the sample mean, which we denote  $RTS^{Direct}$ ,  $RTS^{Indirect}$  and  $RTS^{Total}$ .

$RTS^{Direct}$  from the Cliff-Ord type frontier models are also own returns to scale. We distinguish between own returns to scale from the non-spatial frontier model,  $RTS$ , and own returns to scale from a Cliff-Ord type frontier model,  $RTS^{Direct}$ , because the latter takes into account feedback returns to scale to the  $i$ -th unit (i.e. the effect of changing the scale of inputs in the  $i$ -th unit which affects a neighboring unit's output which in turn affects the  $i$ -th unit's output), whereas  $RTS$  ignores these feedback effects.  $RTS^{Indirect}$  denotes the spillover returns to scale from a Cliff-Ord type frontier model which in this application refers to the effect on the  $i$ -th unit's output following a change in the scale of inputs in the other  $J$  units.<sup>12</sup>  $RTS^{Total}$  denotes the sum of the direct and indirect returns to scale from a Cliff-Ord type frontier model and therefore refers to the effect on the output of the  $i$ -th unit following a change in the scale of inputs in all the units in the sample.

For the sample average country the estimate of  $RTS$  is 0.949 and the estimates of  $RTS^{Direct}$  range from 0.906 – 0.970. We perform one-sided  $t$ -tests of own, spillover and total constant returns to scale.<sup>13</sup> The null of the tests for constant own returns to scale is  $RTS = 1$  or  $RTS^{Direct} = 1$  and the alternative hypothesis since all our estimates indicate decreasing own returns to scale is  $RTS < 1$  or  $RTS^{Direct} < 1$ . The  $t$ -test results indicate that at the 5% level  $RTS$  and  $RTS^{Direct}$  are not significantly different from 1.

When the specification of  $W$  in the Cliff-Ord type frontier models is  $W_{3Near} - W_{7Near}$  and  $W_{3Big} - W_{7Big}$  the estimates of  $RTS^{Indirect}$  for the sample average country range from 0.131 – 0.369 and 0.788 – 1.213, respectively. The  $t$ -tests of constant spillover returns to scale indicate that production in the sample average country is characterized by decreasing spillover returns when the specification of  $W$  in the Cliff-Ord type frontier models is  $W_{3Near} - W_{7Near}$ . The estimates of  $RTS^{Indirect}$  from the  $W_{3Near} - W_{7Near}$  models with the specification of spatial interaction in parentheses are: 0.131 ( $W_{3Near}$ ); 0.252 ( $W_{4Near}$ ); 0.321 ( $W_{5Near}$ ); 0.325 ( $W_{6Near}$ ); and 0.369 ( $W_{7Near}$ ). In contrast, the  $t$ -test results when the specification of  $W$  is  $W_{3Big} - W_{7Big}$  all suggest that production in the sample average country is characterized by constant spillover returns to scale.

The estimates of  $RTS^{Total}$  from the Cliff-Ord type frontier models for the sample average country range from 1.090 – 1.316 when the specification of  $W$  is  $W_{3Near} - W_{7Near}$  and are of the order 1.702 – 2.135 when the spatial interaction is  $W_{3Big} - W_{7Big}$ . A  $t$ -test of constant total returns reveals that  $RTS^{Total}$  from the  $W_{3Near}$  model is not significantly different from 1. For all the other Cliff-Ord type frontier models the  $t$ -tests indicate that production in the sample average country is characterized by increasing total returns. The

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<sup>12</sup>Our interpretation of  $RTS^{Indirect}$  and hence  $RTS^{Total}$  follows from what we referred to as the second interpretation of the spillover elasticity from a Cliff-Ord type frontier model in the discussion of the estimation results.

<sup>13</sup>Our test statistics are available from the corresponding author upon request.

estimates of  $RTS^{Total}$  which are significantly greater than 1 are: 1.222 ( $W_{4Near}$ ); 1.277 ( $W_{5Near}$ ); 1.264 ( $W_{6Near}$ ); 1.316 ( $W_{7Near}$ ); 1.784 ( $W_{3Big}$ ); 1.702 ( $W_{4Big}$ ); 2.135 ( $W_{5Big}$ ); 2.071 ( $W_{6Big}$ ); and 1.914 ( $W_{7Big}$ ).

Summarizing our findings from the  $t$ -tests of own, spillover and total constant returns to scale from the Cliff-Ord type frontier models for the sample average country. We find robust evidence of constant own returns to scale. We also find that when the specification of the spatial interaction is based on different measures of economic distance we consistently observe constant spillover returns but when the spatial interaction is specified according to various composite measures of geographical distance and economic distance there is conclusive evidence of decreasing returns. From all but one of the ten Cliff-Ord type frontier models we find evidence of increasing total returns. Where we find constant spillover returns it follows that the estimates of increasing total returns are larger than when we observe decreasing spillover returns.

In Appendix A.1 for each country we present various estimates of average returns to scale outside the sample mean. For 26 of the 40 countries, the estimate of average own returns to scale from the non-spatial frontier model is below unity indicating decreasing returns. Average direct returns across the  $W_{3Near} - W_{7Near}$  models and across the  $W_{3Big} - W_{7Big}$  models both suggest that there are 26 countries where production is characterized by decreasing own returns. Considering jointly the average estimates of own and spillover returns by turning our attention to the average total returns. Average total returns across the  $W_{3Near} - W_{7Near}$  models indicate increasing total returns for 33 countries. The average total returns across the  $W_{3Near} - W_{7Near}$  models and across the  $W_{3Big} - W_{7Big}$  models suggest that specifying  $W$  according to economic distance as opposed to a combination of geographical distance and economic distance leads to a marked increase in average total returns. In particular, average total returns across the  $W_{3Big} - W_{7Big}$  models indicate increasing total returns for all 40 countries.

As a final point on the estimates of the average own, spillover and total returns to scale outside the sample mean we discuss the findings for individual countries. The rankings of the average returns in Appendix A.1 are extremely robust. This is evident because there is perfect positive rank correlation for each pair of average returns in Appendix A.1. The five countries at the top of the rankings for average  $RTS^{Direct}$ , average  $RTS^{Indirect}$  and average  $RTS^{Total}$  are: 1. Malta; 2. Iceland; 3. Luxembourg; 4. Cyprus; 5. Estonia. And the five countries at the bottom of the three rankings are: 40. Russia; 39. Germany; 38. UK; 37. France; 36. Italy. We also note that the correlation between mean real GDP for a country over the sample period and average  $RTS^{Direct}$ , average  $RTS^{Indirect}$  or average  $RTS^{Total}$  is  $-0.76$ . What may be reason for this finding? One possible explanation is a negative relationship between the size of an economy and the magnitude of the dynamic gains from trade (Alesina and Wacziarg, 1998, and Alesina *et al.*, 2000).

## 6.4 Efficiency Results

An efficiency score of 100% would indicate that a country's output is as high as possible given its inputs, relative to the other countries in the sample. In Appendix A.2 we present various average efficiency scores and average efficiency rankings for each country over the sample period. For each country we also present the minimum and maximum efficiency score and efficiency ranking across the  $W_{3Near} - W_{7Near}$  models and across the  $W_{3Big} - W_{7Big}$  models. Selected average annual efficiency scores are plotted in Figure 2 and selected efficiency distributions are presented in Figure 3.<sup>14</sup>

[Insert Figures 2 and 3]

To facilitate the discussion the efficiencies from the Cliff-Ord type frontier models are compared to the base set of efficiencies from the non-spatial frontier model (Model 1 in Table 2). The mean efficiency score from the non-spatial frontier model over the sample period and across the 40 countries is 49%. The corresponding average efficiency scores from the Cliff-Ord type frontier models when the spatial interaction is  $W_{3Near} - W_{7Near}$  and  $W_{3Big} - W_{7Big}$  are 47% and 40%, respectively. From Figure 2 we can see that the average annual efficiency from the non-spatial frontier model is, in general, larger than the average annual efficiencies across the  $W_{3Near} - W_{7Near}$  models and across the  $W_{3Big} - W_{7Big}$  models. We can therefore conclude that, in general, the average annual efficiency score is upwardly biased when spatial spillovers are not taken into account in the production frontier modeling. In addition, we can see from Figure 2 that over the entire sample period the average annual efficiency across the  $W_{3Big} - W_{7Big}$  models is lower than the average annual efficiency across the  $W_{3Near} - W_{7Near}$  models. This suggests that mean annual efficiency is lower when  $W$  is weighted by economic distance as opposed to a combination of geographical distance and economic distance.

The correlation between any pair of the following efficiencies is at least 0.90: efficiencies from Model 1, the average efficiencies across the  $W_{3Near} - W_{7Near}$  models and the average efficiencies across the  $W_{3Big} - W_{7Big}$  models. However, it is apparent from Figure 3 that the distributions of these three sets of efficiencies differ. The distributions of the average efficiencies across the  $W_{3Near} - W_{7Near}$  models and across the  $W_{3Big} - W_{7Big}$  models are characterized by moderate multi-modality. In contrast, the distribution of the efficiency scores from Model 1 is more typical of unimodality.

We conducted ten Kruskal-Wallis tests of the null that the base efficiencies from Model 1 do not differ from the efficiencies from a Cliff-Ord type frontier model. The null is rejected at the 1% level when the efficiencies from Model 1 are tested against any of

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<sup>14</sup>To plot the kernel densities in Figure 3 we use the Gaussian density and obtain the bandwidth  $h$  using the Sheather and Jones (1991) solve-the-equation plug-in-approach. To avoid bias problems near the boundary when estimating the kernel densities the reflection method is used (see Silverman, 1986, and Scott, 1992).

the five sets of efficiencies when the spatial interaction is  $W_{3Big} - W_{7Big}$ . In general, the null is accepted at a reasonable level of significance when the efficiencies from Model 1 are tested against a set of efficiencies from the  $W_{3Near} - W_{7Near}$  models. The only case where the null is rejected at a reasonable level of significance is when the efficiencies from Model 1 are tested against the efficiencies from the  $W_{6Near}$  model.

The null of the Kruskal-Wallis test is accepted at a reasonable level of significance when the efficiencies from the  $W_{3Big} - W_{7Big}$  models are tested against one another. This is also the case for a Kruskal-Wallis test of any pairwise combination of efficiencies from the  $W_{3Near} - W_{7Near}$  models. However, the null of the Kruskal-Wallis test is rejected at a reasonable level of significance when any of the five sets of efficiencies from the  $W_{3Big} - W_{7Big}$  models is tested against the efficiencies from one of the  $W_{3Near} - W_{7Near}$  models.

Univariate efficiency distributions such as those in Figure 3 do not provide any information about the relative performance of the countries. To shed some light on the relative performance of the countries we use the contour plots in Figure 4 which relate to pairwise combinations of: the normalized efficiencies from Model 1; the normalized average efficiencies across the  $W_{3Near} - W_{7Near}$  models; and the normalized average efficiencies across the  $W_{3Big} - W_{7Big}$  models.<sup>15</sup> In all three contour plots the countries are highly concentrated on the diagonal line. This indicates that each set of normalized efficiency scores is no better or worse than either of the other two sets. We can therefore conclude that whether/how spatial spillovers are taken into account in the production frontier modeling has no implications for the relative performance of the countries. We documented above, however, the implications for the absolute performance of the countries when spatial dependence is ignored (see the above discussion of Figures 2 and 3).

[Insert Figure 4]

Turning to the average efficiency rankings over the sample period for individual countries. The non-spatial frontier model and the Cliff-Ord type frontier models yield a robust set of average efficiency rankings. For instance, just seven countries are in the bottom five of the average efficiency rankings (Albania; Armenia; Azerbaijan; Belarus; Moldova; Romania; Ukraine). There is slightly less agreement across the models with regard to the most efficient countries with ten countries featuring in the top five of the average efficiency rankings (Austria; France; Germany; Greece; Iceland; Italy; Luxembourg; Norway; Sweden; UK). Finding a lot of similarity between the average efficiency rankings

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<sup>15</sup>The contour plots are configured using bivariate Gaussian kernels where the bandwidths are calculated using the solve-the-equation plug-in approach for a bivariate Gaussian kernel à la Wand and Jones (1994). Prior to the configuration of the contour plots, the efficiency scores are normalized relative to the mean. The contour plots of the efficiency scores from Model 1 against the scores from the individual Cliff-Ord type frontier models are very similar to one another. Hence why Figure 4 relates to two sets of average efficiencies from the Cliff-Ord type frontier models.

from the non-spatial frontier model and the Cliff-Ord type frontier models is consistent with the above discussion of Figure 4 (i.e. ignoring spatial dependence has very little impact on the relative performance of the countries). Finally, we note that the average efficiency rankings from the non-spatial frontier model and the Cliff-Ord type frontier models are to a large extent in line with what we expected. In particular, we would expect the majority of the above countries to be at the top and bottom of the average efficiency rankings because of their geographical location and their tendency to have mean real income per capita in the top and bottom thirds of the sample.

## 7 Concluding Remarks and Further Work

In this paper we have blended seminal early work in spatial econometrics with a notable contribution on parametric efficiency estimation using panel data. Specifically, we introduce a spatial autoregressive frontier model by combining the spatial autoregressive model (Cliff and Ord, 1973; 1981) and the CSS approach to estimate time-varying efficiency from the unit specific effects. For some time Druska and Horrace's (2004) analysis of Indonesian rice farm production using a GMM spatial error frontier model has been the only major contribution to the literature on spatial frontier modeling. Our study represents a further contribution to this literature.

We applied our model to a classic case of aggregate country production in Europe for the period 1995 – 2008. The efficiency rankings from our fitted Cliff-Ord type frontier models are plausible because they are broadly in line with our expectations, with the best performing countries tending to be from Northern and Western Europe and the worst performing countries are predominantly Eastern European.

The fitted Cliff-Ord type frontier models were used in a Bayesian MCMC experiment to simulate the average own, spillover and total marginal effects, and the associated  $t$ -statistics. Using these average marginal effects we explored a new line of enquiry for productivity analysis. In particular, we calculate three new measures of returns to scale (own, spillover and total returns). There are other new lines of enquiry for productivity analysis using the own, spillover and total marginal effects. For instance, a logical piece of further work would be to extend the parametric Malmquist TFP growth decomposition to include own and spillover components (e.g. own and spillover scale effects).

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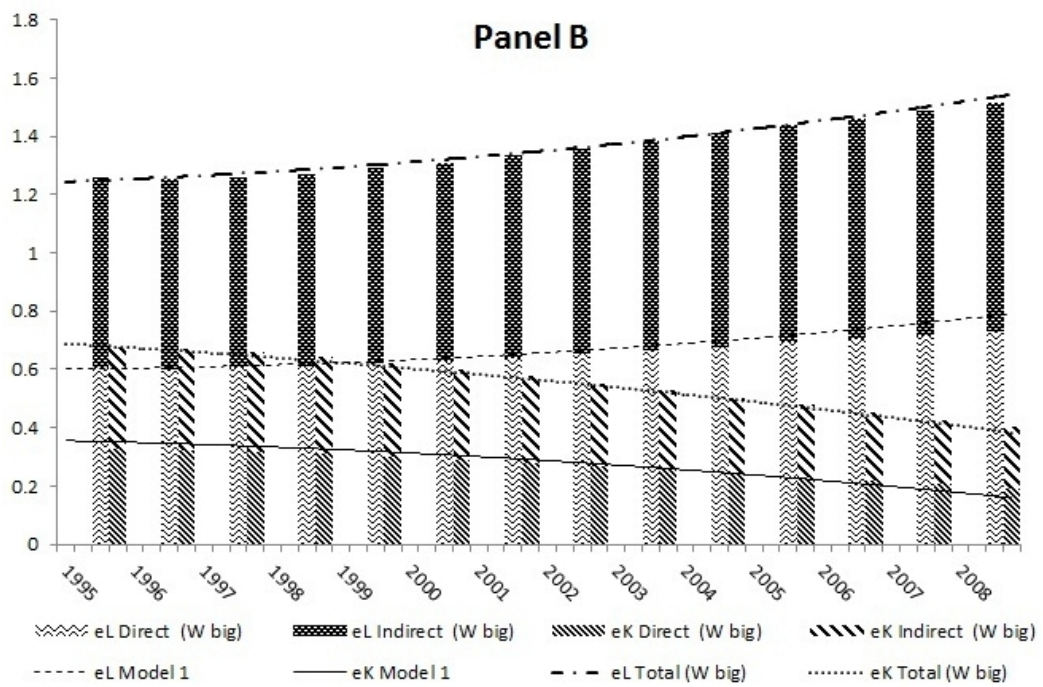
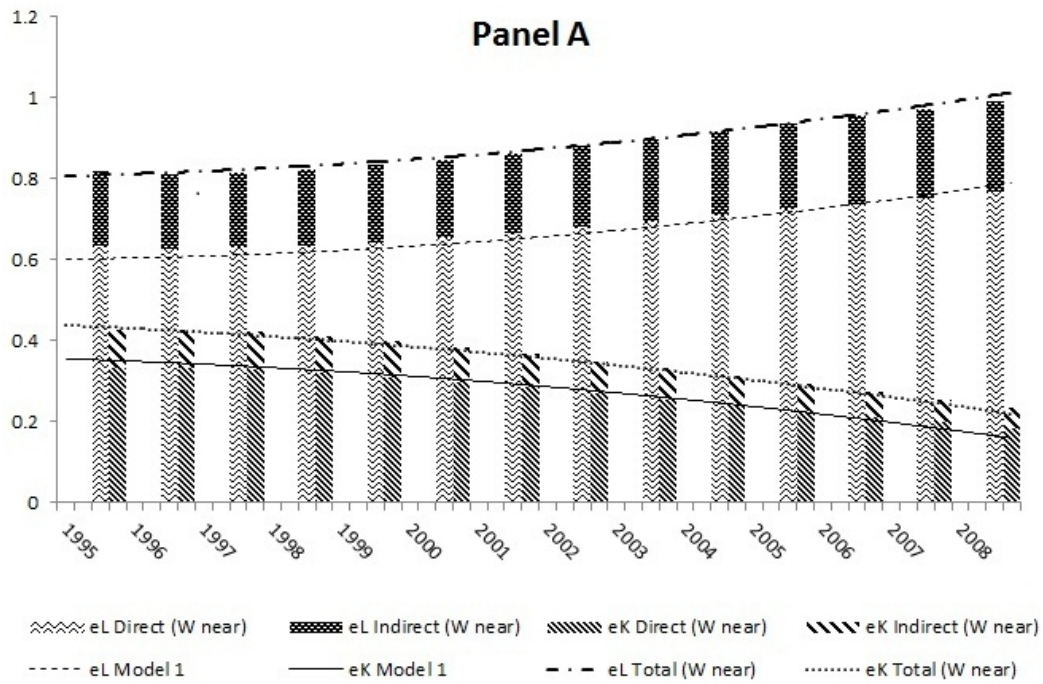


Figure 1: Average capital and labor elasticities (eK and eL) outside the sample mean



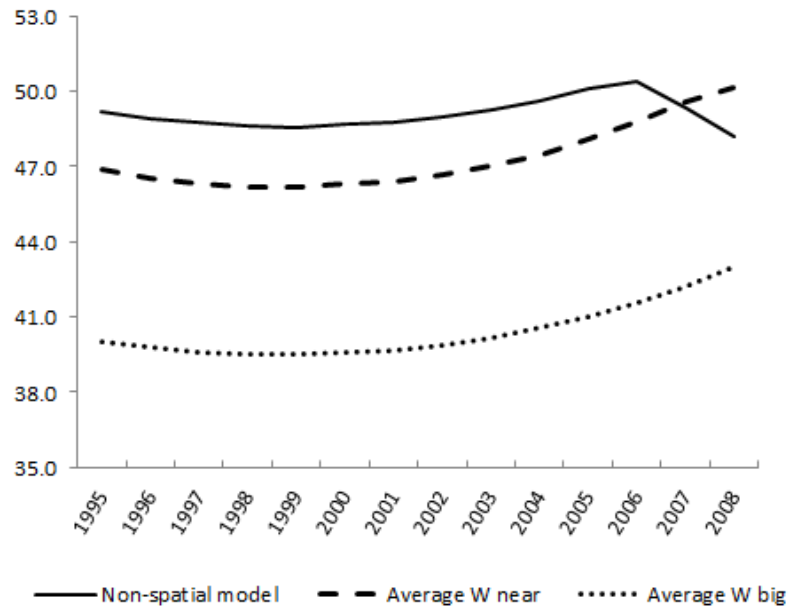


Figure 2: Average annual technical efficiency scores

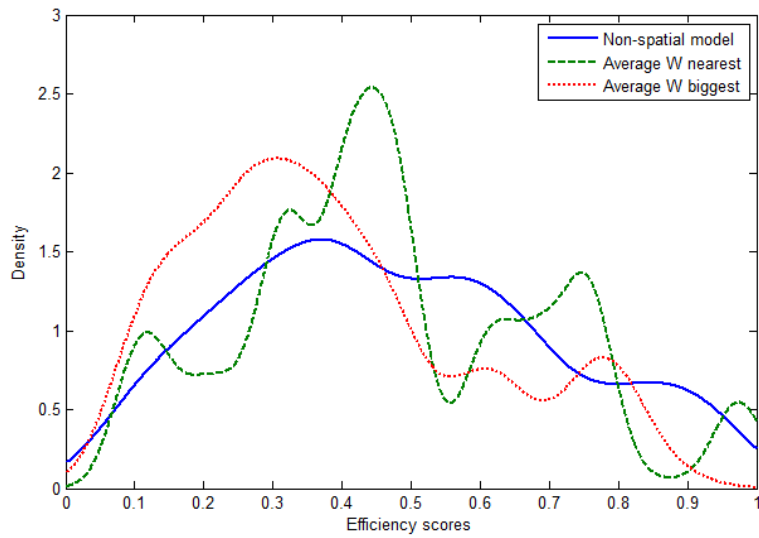


Figure 3: Kernel densities for three sets of efficiency scores

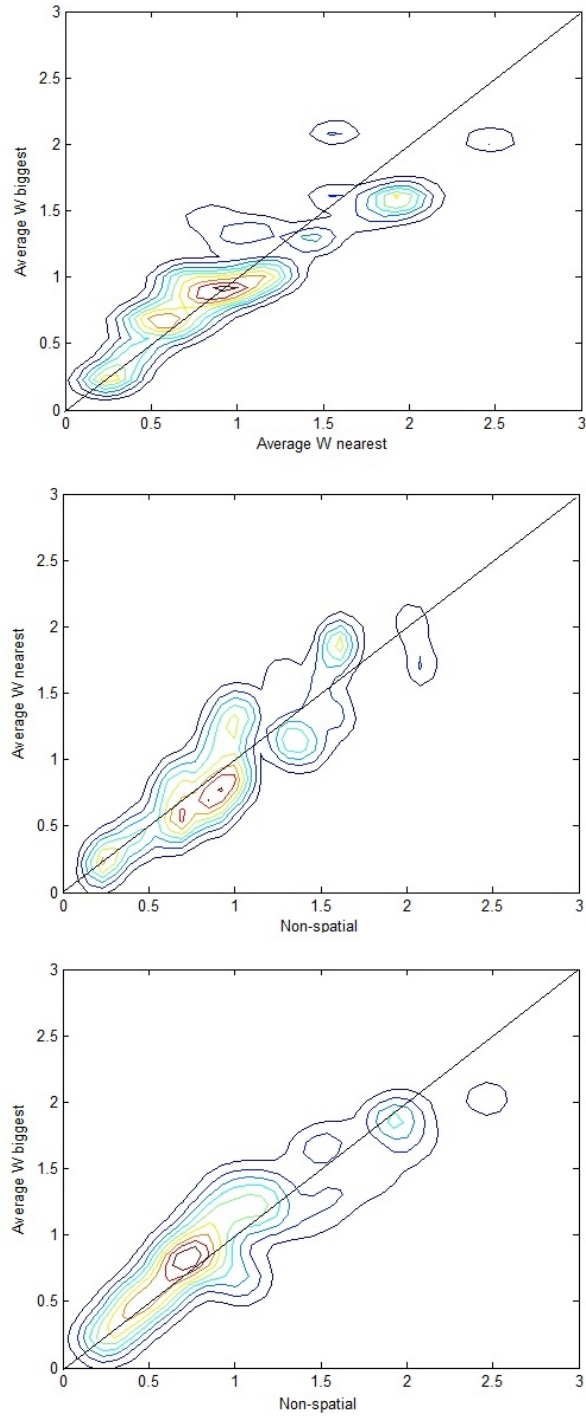


Figure 4: Contour plots of normalized efficiency scores

Table 1: Summary statistics

	Variable	Mean	St.Dev.	Min	Max
Real GDP (millions)	$y$	391,000	618,000	5,877	2,800,000
Labor (000s)	$x_1$	9,338	14,212	139	75,730
Real capital stock (billions) Dep Rate=6%	$x_2$	48,100	85,000	179	513,000
Sum of exports and imports as a share of GDP i.e. trade openness	$z_1$	0.92	0.44	0.29	3.24
Government spending as a share of GDP	$z_2$	0.09	0.03	0.05	0.16

Table 2: Base Non-Spatial Frontier Model and Selected W Near Frontier Models

Variable	Parameter	Model 1— No Spatial Dependence		Model 2 $W_{3Near}$		Model 3 $W_{5Near}$		Model 4 $W_{7Near}$	
		Coef	t-stat	Coef	t-stat	Coef	t-stat	Coef	t-stat
$\ln x_1$	$\beta_1$	0.669***	9.10						
	$\beta_1^{Direct}$			0.686***	9.04	0.685***	9.71	0.680***	9.69
	$\beta_1^{Indirect}$			0.094**	2.74	0.230***	4.29	0.266***	4.13
	$\beta_1^{Total}$			0.781***	8.15	0.916***	8.50	0.945***	8.03
$\ln x_2$	$\beta_2$	0.280***	11.81						
	$\beta_2^{Direct}$			0.272***	11.21	0.271***	11.13	0.267***	11.46
	$\beta_2^{Indirect}$			0.037**	3.12	0.091***	4.83	0.103***	5.05
	$\beta_2^{Total}$			0.309***	11.54	0.361***	10.68	0.370***	11.34
$(\ln x_1)^2$	$\beta_3$	-0.003	-0.11						
	$\beta_3^{Direct}$			-0.001	-0.02	-0.010	-0.45	-0.011	-0.47
	$\beta_3^{Indirect}$			0.000	0.00	-0.004	-0.44	-0.004	-0.45
	$\beta_3^{Total}$			-0.001	-0.02	-0.014	-0.45	-0.015	-0.47
$(\ln x_2)^2$	$\beta_4$	0.035***	4.13						
	$\beta_4^{Direct}$			0.030***	3.50	0.026**	2.82	0.022*	2.51
	$\beta_4^{Indirect}$			0.004*	2.38	0.009*	2.57	0.008*	2.35
	$\beta_4^{Total}$			0.034***	3.52	0.034**	2.85	0.030*	2.54
$\ln x_1 \ln x_2$	$\beta_5$	-0.094***	-4.70						
	$\beta_5^{Direct}$			-0.079***	-3.90	-0.067**	-3.25	-0.060**	-2.98
	$\beta_5^{Indirect}$			-0.011*	-2.50	-0.022**	-2.85	-0.023**	-2.71
	$\beta_5^{Total}$			-0.090***	-3.93	-0.090**	-3.27	-0.083**	-3.02
$t$	$\beta_6$	-0.008*	-2.21						
	$\beta_6^{Direct}$			-0.011**	-3.27	-0.015***	-4.22	-0.015***	-4.47
	$\beta_6^{Indirect}$			-0.001*	-2.09	-0.005**	-2.90	-0.006**	-3.06
	$\beta_6^{Total}$			-0.012**	-3.20	-0.020***	-3.96	-0.020***	-4.19
$t^2$	$\beta_7$	0.006***	16.44						
	$\beta_7^{Direct}$			0.006***	14.19	0.006***	13.47	0.005***	12.56
	$\beta_7^{Indirect}$			0.001**	3.19	0.002***	5.33	0.002***	5.51
	$\beta_7^{Total}$			0.007***	15.22	0.007***	14.08	0.007***	13.95
$\ln x_1 t$	$\beta_8$	0.027***	8.63						
	$\beta_8^{Direct}$			0.024***	7.37	0.021***	6.60	0.020***	6.08
	$\beta_8^{Indirect}$			0.003**	3.02	0.007***	4.44	0.008***	4.39
	$\beta_8^{Total}$			0.027***	7.56	0.028***	6.75	0.027***	6.31
$\ln x_2 t$	$\beta_9$	-0.024***	-8.22						
	$\beta_9^{Direct}$			-0.021***	-7.34	-0.019***	-6.44	-0.017***	-5.90
	$\beta_9^{Indirect}$			-0.003**	-2.96	-0.006***	-4.40	-0.007***	-4.25
	$\beta_9^{Total}$			-0.024***	-7.45	-0.026***	-6.59	-0.024***	-6.06
$z_1$	$\phi_1$	-0.018	-0.66						
	$\phi_1^{Direct}$			-0.008	-0.28	-0.011	-0.42	-0.011	-0.43
	$\phi_1^{Indirect}$			-0.001	-0.22	-0.003	-0.39	-0.004	-0.41
	$\phi_1^{Total}$			-0.008	-0.28	-0.014	-0.42	-0.015	-0.43
$z_2$	$\phi_2$	-1.644***	-4.25						
	$\phi_2^{Direct}$			-1.479***	-3.93	-1.359***	-3.67	-1.389***	-3.64
	$\phi_2^{Indirect}$			-0.202*	-2.29	-0.455**	-2.96	-0.541**	-2.83
	$\phi_2^{Total}$			-1.681***	-3.86	-1.814***	-3.63	-1.930***	-3.54
$z_3$	$\phi_3$	0.021	1.63						
	$\phi_3^{Direct}$			0.021	1.66	0.023	1.87	0.026*	2.09
	$\phi_3^{Indirect}$			0.003	1.41	0.008	1.67	0.010	1.84
	$\phi_3^{Total}$			0.024	1.65	0.031	1.85	0.036*	2.05
$\sum_{j=1}^N w_{ij} y_{jt}$	$\lambda$	-	-	0.121**	3.27	0.258***	6.33	0.287***	6.78
Log-likelihood				814.87		822.55		824.13	

Note: \*, \*\*, \*\*\* denote statistical significance at the 5%, 1% and 0.1% levels, respectively.

Table 3: Selected W Big Frontier Models

Variable	Parameter	Model 5 $W_{3Big}$		Model 6 $W_{5Big}$		Model 7 $W_{7Big}$	
		Coef	t-stat	Coef	t-stat	Coef	t-stat
$\ln x_1$	$\beta_1^{Direct}$	0.630***	9.08	0.658***	8.85	0.673***	9.42
	$\beta_1^{Indirect}$	0.612***	5.43	0.867***	5.44	0.698***	5.10
	$\beta_1^{Total}$	1.241***	7.42	1.524***	7.05	1.371***	7.29
$\ln x_2$	$\beta_2^{Direct}$	0.276***	11.69	0.264***	11.54	0.267***	11.30
	$\beta_2^{Indirect}$	0.267***	7.19	0.346***	7.33	0.276***	6.70
	$\beta_2^{Total}$	0.543***	10.79	0.611***	10.40	0.543***	10.46
$(\ln x_1)^2$	$\beta_3^{Direct}$	-0.038	-1.60	-0.032	-1.41	-0.029	-1.31
	$\beta_3^{Indirect}$	-0.037	-1.55	-0.043	-1.38	-0.030	-1.26
	$\beta_3^{Total}$	-0.074	-1.59	-0.075	-1.40	-0.060	-1.30
$(\ln x_2)^2$	$\beta_4^{Direct}$	0.012	1.48	0.015	1.75	0.018*	2.11
	$\beta_4^{Indirect}$	0.012	1.48	0.019	1.74	0.018*	2.03
	$\beta_4^{Total}$	0.024	1.49	0.033	1.76	0.036*	2.10
$\ln x_1 \ln x_2$	$\beta_5^{Direct}$	-0.048*	-2.48	-0.049*	-2.52	-0.057**	-2.94
	$\beta_5^{Indirect}$	-0.046*	-2.44	-0.064*	-2.46	-0.059**	-2.71
	$\beta_5^{Total}$	-0.094*	-2.50	-0.113*	-2.53	-0.116**	-2.90
$t$	$\beta_6^{Direct}$	-0.019***	-6.00	-0.020***	-6.38	-0.019***	-6.09
	$\beta_6^{Indirect}$	-0.019***	-4.41	-0.026***	-4.70	-0.020***	-4.34
	$\beta_6^{Total}$	-0.038***	-5.38	-0.047***	-5.62	-0.039***	-5.43
$t^2$	$\beta_7^{Direct}$	0.005***	12.13	0.005***	12.23	0.005***	12.35
	$\beta_7^{Indirect}$	0.005***	8.07	0.006***	8.09	0.005***	7.22
	$\beta_7^{Total}$	0.009***	12.50	0.011***	11.91	0.010***	12.01
$\ln x_1 t$	$\beta_8^{Direct}$	0.017***	5.70	0.018***	5.86	0.020***	6.46
	$\beta_8^{Indirect}$	0.017***	4.89	0.023***	4.97	0.020***	4.78
	$\beta_8^{Total}$	0.034***	5.64	0.041***	5.69	0.040***	6.01
$\ln x_2 t$	$\beta_9^{Direct}$	-0.013***	-4.63	-0.014***	-5.15	-0.016***	-5.83
	$\beta_9^{Indirect}$	-0.012***	-4.28	-0.019***	-4.56	-0.017***	-4.55
	$\beta_9^{Total}$	-0.025***	-4.68	-0.033***	-5.07	-0.033***	-5.53
$z_1$	$\phi_1^{Direct}$	-0.020	-0.78	-0.008	-0.33	-0.008	-0.32
	$\phi_1^{Indirect}$	-0.019	-0.77	-0.011	-0.32	-0.008	-0.31
	$\phi_1^{Total}$	-0.039	-0.78	-0.019	-0.32	-0.016	-0.31
$z_2$	$\phi_2^{Direct}$	-1.532***	-4.23	-1.299***	-3.44	-1.268***	-3.41
	$\phi_2^{Indirect}$	-1.491***	-3.50	-1.710**	-3.08	-1.314**	-2.98
	$\phi_2^{Total}$	-3.023***	-3.97	-3.009***	-3.31	-2.582**	-3.28
$z_3$	$\phi_3^{Direct}$	0.029*	2.42	0.025*	2.03	0.024	1.97
	$\phi_3^{Indirect}$	0.028*	2.23	0.034	1.90	0.025	1.81
	$\phi_3^{Total}$	0.057*	2.36	0.059	1.98	0.049	1.91
$\sum_{j=1}^N w_{ij} y_{jt}$	$\lambda$	0.501***	14.01	0.579***	17.25	0.520***	13.15
Log-likelihood		837.47		841.59		831.72	

Note: \*, \*\*, \*\*\* denote statistical significance at the 5%, 1% and 0.1% levels, respectively.

# A Appendix

## A.1 Average Returns to Scale (RTS): 1995-2008

	Model 1– No Spatial Dependence	Average: $W_{3Near} - W_{7Near}$			Average: $W_{3Big} - W_{7Big}$		
Country	Average <i>RTS</i>	Average <i>RTS</i>			Average <i>RTS</i>		
		<i>Total</i>	<i>Direct</i>	<i>Indirect</i>	<i>Total</i>	<i>Direct</i>	<i>Indirect</i>
Albania	1.098	1.393	1.078	0.315	2.254	1.085	1.170
Armenia	1.106	1.399	1.082	0.316	2.265	1.090	1.175
Austria	0.916	1.203	0.931	0.272	1.855	0.892	0.963
Azerbaijan	0.993	1.277	0.988	0.289	2.006	0.965	1.041
Belarus	0.938	1.220	0.944	0.276	1.886	0.907	0.979
Belgium	0.899	1.185	0.917	0.268	1.815	0.873	0.942
Bulgaria	0.983	1.269	0.982	0.287	1.989	0.957	1.032
Croatia	1.026	1.318	1.020	0.298	2.098	1.009	1.089
Cyprus	1.200	1.507	1.166	0.341	2.503	1.204	1.298
Czech Republic	0.899	1.183	0.915	0.268	1.809	0.870	0.939
Denmark	0.960	1.250	0.967	0.283	1.954	0.940	1.014
Estonia	1.159	1.462	1.131	0.331	2.405	1.157	1.248
Finland	0.976	1.267	0.980	0.287	1.990	0.957	1.033
France	0.683	0.950	0.735	0.215	1.313	0.630	0.682
Germany	0.637	0.901	0.697	0.204	1.208	0.580	0.628
Greece	0.910	1.195	0.925	0.271	1.837	0.883	0.954
Hungary	0.932	1.218	0.942	0.276	1.883	0.906	0.978
Iceland	1.312	1.630	1.261	0.369	2.767	1.332	1.435
Ireland	1.015	1.309	1.013	0.296	2.081	1.001	1.080
Italy	0.695	0.964	0.746	0.218	1.343	0.645	0.698
Latvia	1.106	1.403	1.086	0.317	2.278	1.096	1.182
Lithuania	1.058	1.351	1.045	0.306	2.165	1.042	1.124
Luxembourg	1.233	1.546	1.196	0.350	2.588	1.246	1.342
Macedonia	1.145	1.445	1.118	0.327	2.366	1.139	1.228
Malta	1.319	1.637	1.267	0.370	2.781	1.339	1.442
Moldova	1.077	1.367	1.058	0.309	2.198	1.057	1.140
Netherlands	0.833	1.112	0.860	0.252	1.659	0.798	0.862
Norway	0.977	1.269	0.982	0.287	1.996	0.960	1.036
Poland	0.765	1.036	0.801	0.234	1.493	0.718	0.776
Portugal	0.891	1.174	0.908	0.266	1.791	0.861	0.930
Romania	0.832	1.106	0.856	0.250	1.643	0.790	0.854
Russian Fed	0.597	0.853	0.660	0.193	1.103	0.529	0.574
Slovakia	0.990	1.280	0.990	0.290	2.016	0.970	1.046
Slovenia	1.096	1.396	1.080	0.316	2.266	1.090	1.176
Spain	0.728	1.000	0.773	0.226	1.419	0.682	0.737
Sweden	0.910	1.195	0.925	0.270	1.836	0.883	0.953
Switzerland	0.904	1.191	0.921	0.269	1.828	0.879	0.949
Turkey	0.722	0.989	0.765	0.224	1.393	0.669	0.724
Ukraine	0.754	1.020	0.789	0.231	1.457	0.700	0.757
UK	0.682	0.949	0.734	0.215	1.310	0.629	0.681

## A.2 Average Efficiency Scores: 1995-2008

Country	Model 1— No Spatial Dependence		Average: $W_{3Near} - W_{7Near}$						Average: $W_{3Big} - W_{7Big}$					
	Av. Eff. Score (%)	Rank	Av. Eff. Score (%)			Mean	Rank		Av. Eff. Score (%)			Mean	Rank	
			Mean	Min.	Max.		Min.	Max.	Mean	Min.	Max.			
Albania	14.9	37	14.2	11.7	18.2	37	37	38	13.0	12.4	14.1	37	37	37
Armenia	11.8	39	12.0	11.1	13.8	38	37	39	10.3	9.9	11.0	39	38	40
Austria	59.9	13	61.1	39.7	79.2	13	4	21	39.3	37.6	40.8	17	17	18
Azerbaijan	13.4	38	11.8	10.2	12.8	38	37	39	10.5	9.6	11.1	38	38	39
Belarus	23.1	35	21.9	15.9	26.3	35	34	35	16.7	16.0	17.6	35	34	36
Belgium	63.3	12	49.0	42.8	55.6	16	13	18	49.3	45.2	52.4	11	11	12
Bulgaria	27.7	32	32.5	31.0	34.9	29	27	31	19.5	17.7	21.7	33	33	33
Croatia	30.4	30	34.2	26.9	40.9	30	27	33	23.4	18.4	25.9	31	30	32
Cyprus	46.2	20	47.9	44.7	51.4	16	14	19	36.9	35.4	42.3	19	15	21
Czech Republic	43.4	21	33.0	30.1	36.0	29	28	30	32.6	31.3	33.8	24	21	25
Denmark	56.3	15	46.7	43.0	49.9	18	17	20	47.8	44.2	51.6	13	12	14
Estonia	35.2	28	42.7	37.8	45.7	22	19	26	42.2	34.4	50.6	16	11	20
Finland	55.1	16	64.5	59.3	68.6	10	9	12	45.8	44.6	48.4	14	12	15
France	90.9	4	76.0	67.5	79.8	6	3	10	77.7	74.5	82.4	4	2	5
Germany	99.1	1	96.1	90.3	100.0	2	1	2	100.0	100.0	100.0	1	1	1
Greece	53.2	18	68.9	61.1	80.1	9	5	12	33.9	32.5	35.1	22	21	23
Hungary	41.4	23	46.6	43.3	49.8	18	16	21	27.3	26.7	28.4	27	27	28
Iceland	81.5	6	72.4	66.6	77.2	7	4	10	82.4	79.3	84.6	3	2	5
Ireland	58.4	14	42.2	37.5	48.4	24	18	26	36.9	35.9	37.3	19	18	20
Italy	83.3	5	98.7	96.8	100.0	1	1	2	63.3	60.8	67.5	7	7	8
Latvia	30.1	31	42.4	37.6	47.4	22	20	27	42.4	33.9	47.0	16	14	20
Lithuania	31.9	29	39.2	34.2	42.0	26	22	30	24.4	22.2	25.2	30	29	31
Luxembourg	93.9	2	77.5	64.2	90.8	5	3	8	78.7	76.0	80.1	4	3	4
Macedonia	24.1	33	29.6	27.7	32.8	32	28	34	22.6	18.9	25.3	31	29	32
Malta	68.2	9	44.5	38.5	51.7	21	15	25	39.4	37.8	41.1	17	16	18
Moldova	7.6	40	8.9	8.1	9.5	40	40	40	9.4	8.7	11.4	40	38	40
Netherlands	80.0	7	60.3	53.4	70.8	11	8	13	58.5	56.5	63.2	9	9	10
Norway	69.9	8	69.8	61.4	83.5	8	4	11	73.0	68.0	79.7	6	4	6
Poland	41.4	22	31.8	27.8	43.2	30	22	33	27.3	26.1	28.4	27	26	28
Portugal	35.3	27	27.0	25.0	29.6	34	32	35	25.3	24.6	26.0	30	29	31
Romania	23.5	34	31.9	29.4	38.3	31	29	33	16.6	15.0	17.6	35	34	36
Russian Fed	54.7	17	62.6	51.6	69.8	11	6	16	59.4	56.0	61.8	9	8	10
Slovakia	37.7	26	39.5	32.6	43.9	25	21	29	34.5	32.3	37.7	21	19	23
Slovenia	40.7	24	43.1	34.6	51.0	21	15	27	31.2	27.5	34.0	24	23	26
Spain	67.6	10	50.3	47.1	54.7	15	14	19	47.1	44.4	49.2	13	12	15
Sweden	64.2	11	76.3	69.1	80.6	6	3	9	62.9	57.0	67.7	8	7	10
Switzerland	50.2	19	40.5	38.3	43.2	25	23	28	32.1	30.8	33.9	24	22	25
Turkey	40.0	25	48.3	39.9	54.6	17	13	24	28.1	27.4	29.8	26	25	27
Ukraine	19.9	36	20.0	15.1	23.2	36	36	36	16.3	15.5	17.2	35	35	36
UK	92.5	3	76.4	69.4	81.2	5	4	7	79.5	74.9	85.8	4	2	6
<i>Av Eff Sc</i>	49.0		47.3						40.4					
<i>St Dev Eff Sc</i>	24.7		22.4						22.8					