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ESTIMATING FARM EFFICIENCY IN THE PRESENCE OF DOUBLE HETEROSCEDASTICITY USING PANEL DATA

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The accuracy of technical efficiency measures is important given the interest in such measures in policy discussions. In recent years the use of stochastic frontiers has become popular for estimating technical inefficiency, but estimated inefficiencies are sensitive to specification errors. One source of such errors is heteroscedasticity. This paper addresses this issue by extending the Hadri (1999) correction for heteroscedasticity to stochastic production frontiers and to panel data. It is argued that heteroscedasticity within an estimation can have a significant effect on results, and that correcting for heteroscedasticity yields more accurate measures of technical inefficiency. Using panel data on cereal farms, it is found that the usual technical efficiency measures used in stochastic production frontiers are significantly sensitive to the extended correction for heteroscedasticity.

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I. Introduction

In previous studies of inefficiency using stochastic frontier models, Caudill, Ford and Gropper (1995) noted that measures of inefficiency are based on residuals derived from the estimation of a stochastic frontier. They observed that residuals are sensitive to specification errors, particularly in stochastic frontier models, and that this sensitivity will be passed on to the inefficiency measures. To correct for this, they suggested that one should consider testing for and, if present, correcting for heteroscedasticity in the one-sided error term. Hadri (1999) argued that we might expect the two-sided error term to be affected by heteroscedasticity as well, and that if this likely eventuality is ignored, it will lead to inconsistent maximum likelihood (ML) estimators. Consequently, the usual tests will be no longer valid. Hence, in order to obtain correct estimators and conduct valid tests one must test for heteroscedasticity in both error terms and, if indicated, appropriate correction should be taken in the estimation procedure. In Hadri (1999), heteroscedastic frontier cost functions were estimated using cross-sectional data.

In this paper, we extend the Hadri (1999) correction for heteroscedasticity to stochastic production frontiers and to panel data, including unbalanced panel data. We consider one homoscedastic and three heteroscedastic specifications namely, heteroscedasticity in the one-sided term, heteroscedasticity in the symmetrical term and heteroscedasticity in both error terms. Using panel data on cereal farms, we find that the usual measures used in stochastic production frontiers are significantly sensitive to the extended correction for heteroscedasticity.

The paper is organised as follows. The theoretical models are presented in section 2. In section 3 the models are applied to a set of panel data on 102 mainly cereal farms in England for the harvest years 1982-1987. Section 4 concludes the paper.

II. Theoretical Models

Before introducing the heteroscedastic stochastic production frontier models, we briefly present the basic model used in the literature to describe a frontier production function. Greene (1993) provides a recent survey of this literature. The basic model can be written as follows:

$$y_{it} = X_{it}\ \beta + w_{it} - v_{it,} \tag{1}$$

where y_{it} denotes the logarithm of the production for the *i*th sample farm (i = 1, ... , N) in the *t*th time period (t = 1 , ... , T_i); X_{it} is a (1 x k) vector of the logarithm of the inputs associated with the *i*th sample farm in the *t*th time period (the first element would be one when an intercept term is included); β is a (k x 1) vector of unknown parameters to be estimated; w_{it} is a two-sided error term with $E\,[w_{it}] = 0$ for all i and t and $E\,[w_{it}\, w_{jl}] = 0$ for all i and j, $i \neq j$ and for all t and l; var $(w_{it}) = \sigma_w^2$; v_{it} is a non-negative one-sided error term with $E\,[v_{it}] > 0$, $E\,[v_{it}\, v_{jl}] = 0$ for all i and j, $i \neq j$ and for all t and l; and var$(v_{it}) = \sigma_v^2$. Furthermore, it is assumed that w and v are uncorrelated. The one-sided disturbance v reflects the fact that each firm's production must lie on or below its frontier. Such a term represents factors under the firm's control. The two-sided error term represents factors outside the firm's control.

Weinstein (1964) derived the density function of $w_{it} + v_{it}$ under the assumption that v_{it} is half-normal and w_{it} is normal. It is then easy to obtain the density function of their difference that takes the form:

$$f(\varepsilon_{it}) = (2/\sigma)\, f^*(\varepsilon_{it}/\sigma)(1 - F^*(\lambda\, \varepsilon_{it}/\sigma)), \qquad -\infty < \varepsilon_{it} < +\infty, \tag{2}$$

where $\varepsilon_{it} = w_{it} - v_{it}$, $\sigma^2 = \sigma_w^2 + \sigma_v^2$, $\lambda = \sigma_v / \sigma_w$ and f^* (.) and F^* (.) are respectively the standard normal density and distribution functions.

The advantage of stochastic frontier estimation is that it permits the estimation of firm-specific inefficiency. The most widely used measure of firm-specific inefficiency, suggested by Jondrow, Lovell, Materov and Schmidt (1982), is based on the conditional expected value of v_{it} given ε_{it}, and is given by:

$$E[v_{it} \mid \varepsilon_{it}] = \sigma_*[-\varepsilon_{it}\ \lambda/\sigma + f^*(\varepsilon_{it}\ \lambda/\sigma)/F^*(\varepsilon_{it}\ \lambda/\sigma)], \tag{3}$$

where $\sigma_* = \sigma_v \sigma_w / \sigma$.

In what follows, we derive the log-likelihood functions. The corresponding first partial derivatives for the three possible cases, heteroscedasticity in the one-sided, two-sided and both error terms, are given in the Appendix. These derivations are used among other things to evaluate log-likelihood ratios for testing purposes.

Following Hadri (1999) we assume the following multiplicative heteroscedasticity for the one-sided error term.

$$\sigma_{vit} = \exp(Z_{it}\ \alpha), \tag{4}$$

where Z_{it} is a vector of nonstochastic explanatory variables related to characteristics of firm management and α is a vector of unknown parameters. Z_{it} is assumed to include an intercept term. The standard deviation of the two-sided error term is also written in exponential form so that $\sigma_w = \exp(\gamma_o)$. The density function corresponding to model HV, where only the one-sided error term is assumed heteroscedastic, is given by:

$$f_{it}(\varepsilon_{it}) = (2/\sigma_{it}) f^*(\varepsilon_{it}/\sigma_{it})(1 - F^*(\lambda_{it}\ \varepsilon_{it}/\sigma_{it})),\ -\infty < \varepsilon_{it} < +\infty \tag{5}$$

where $\sigma_{it}^2 = \sigma_w^2 + \sigma_{vit}^2$, $\lambda_{it} = \sigma_{vit}/\sigma_w$ and $f^*(.)$ and $F^*(.)$ are as defined previously. The loglikelihood function is

$$\log\ L(\beta,\alpha,\gamma_0) = \sum_{i=1}^{N}\sum_{t=1}^{T_i} \log\ (f_{it}(\varepsilon_{it})). \tag{6}$$

As argued earlier, in the cross-section dimension the two-sided error is likely to be affected by size-related heteroscedasticity. The misspecification resulting from not incorporating heteroscedasticity in the ML estimation of our frontier can cause parameter estimators to be inconsistent as well as invalidating standard techniques of inference (White, 1982). In order to incorporate heteroscedasticity in the two-sided error term we specify $\sigma_{wit} = \exp\ (Y_{it}\ \gamma)$, where Y_{it} is a vector of nonstochastic explanatory variables related generally to characteristics of firm size and γ is a vector of unknown parameters. Y_{it} is assumed to include an intercept term. The standard deviation of the one-sided error term, assumed here to be homoscedastic, is now $\sigma_v = \exp(\alpha_0)$. The density function is still as in (5) but now $\sigma_{it}^2 = \sigma_{wit}^2 + \sigma_v^2$, and $\lambda_{it} = \sigma_v / \sigma_{wit}$. We call this model HW.

The most likely correct specification is the one where the two error terms are assumed to be concurrently heteroscedastic. This specification gives model

HVW. Equation (5) is still appropriate but now we have $\sigma_{it}^2 = \sigma_{wit}^2 + \sigma_{vit}^2$, and $\lambda_{it} = \sigma_{vit} / \sigma_{wit}$ where $\sigma_{wit} = \exp(Y_{it}\gamma)$ and $\sigma_{vit} = \exp(Z_{it}\alpha)$.

The first partial derivatives are needed when maximizing the likelihood function using the algorithm proposed by Berndt, Hall, Hall and Hausman (1974).

III. Empirical Applications

A set of panel data on 102 English farms, classified as 'mainly cereal' under the nationally organised Farm Business Survey, was used for the years 1982-1987 to estimate five stochastic frontier production functions. Data on output and input are collected only in value and cost terms, and are here deflated by the appropriate price index to proxy output and inputs. The characteristics of the data are summarised in Table 1. One feature of the sample is variability. In all variables, the standard deviation is large compared to the mean. Another feature is size dispersion; a farm that is one standard deviation above the mean is more than 9 times larger than a farm that is one standard deviation below the mean.

Table 1. Characteristics of the Sample Variables

	Mean	Std. deviation	Skewness	Kurtosis
Cereal output (C)	209.366	168.028	1.896	5.433
Cereal area (CA)	133.88	97.700	2.106	8.445
Crop protection (CP)	11.166	10.784	2.251	6.976
Seeds (CC)	7.014	6.415	2.917	13.869
Fertiliser (FC)	15.694	12.754	2.114	7.143
Labour (Lab)	22.133	17.047	2.184	6.827
Land (LPC)	21.425	16.923	1.821	4.281
Machinery, energy, & miscel. inputs (MEO)	35.967	28.827	2.447	9.314

Note: Cereal area in hectares; all other variables are in thousand Sterling Pounds at 1985 prices.

We estimated five stochastic frontier production functions using GQOPT/PC version 6.01 routines for the optimisation of our likelihood functions. Model H0 is the usual homoscedastic stochastic frontier production defined by

$$C_{it} = \beta_o + \beta_1 Lab_{it} + \beta_2 MEO_{it} + \beta_3 CC_{it} + \beta_4 CA_{it} + \beta_5 LPC_{it} + \beta_6 FC_{it} + \beta_7 CP_{it} + \beta_8 t + w_{it} - v_{it} \quad (7)$$

All the variables are in logarithms. C represents the total value of cereals output; Lab represents the total cost of labour; MEO represents the total cost of machinery, energy and miscellaneous items; CC represents the total cost of seeds; CA represents area under cereals; LPC represents land and property charges; FC represents total cost of fertilizer; CP represents total cost of crop protection products; *t* indicates the year of observation; and *w* and *v* are the random variables whose distributional properties are defined in the previous sections.

The value of output and inputs were deflated by the appropriate price index. The year of observation is included in the model to account for technological change (Hicksian neutral) even though the time period considered is short.

Model HO, defined by equation (7), $\sigma_w = \exp(\gamma_0)$ and $\sigma_v = \exp(\alpha_0)$, contains nine β parameters and two additional parameters associated with the distributions of the *w* and *v* random variables. The two error terms are clearly assumed to be homoscedastic.

In model HV we assume that *v* is heteroscedastic and *w* homoscedastic. The model is defined by equation (7), $\sigma_w = \exp(\gamma_0)$, and

$$\sigma_{vit} = \exp(\alpha_0 + \alpha_1 Lab_{it} + \alpha_2 MEO_{it} + \alpha_3 CA_{it} + \alpha_4 FC_{it} + \alpha_5 t) \quad (8)$$

For Model HW we assume that *w* is heteroscedastic and *v* homoscedastic. Model HW is defined by equation (7), $\sigma_v = \exp(\alpha_0)$ and

$$\sigma_{wit} = \exp(\gamma_0 + \gamma_1 Lab_{it} + \gamma_2 MEO_{it} + \gamma_3 CA_{it} + \gamma_4 FC_{it} + \gamma_5 t) \quad (9)$$

Finally, in model HVW we assume that both disturbance terms are heteroscedastic. The model is defined by equations (7), (8) and (9). The maximum-likelihood estimates of each model are reported in Table 2.

Table 2. Estimation Results

	Model HO	Model HV	Model HW	Model HVW	Model HVWR
Constant	3.736	3.730	3.770	3.791	3.999
	(19.28)	(13.08)	(18.78)	(11.85)	(20.23)
Lab	0.002	0.027	0.003	0.029	0.030
	(0.323)	(2.888)	(0.711)	(3.769)	(4.167)
MEO	0.250	0.272	0.237	0.283	0.256
	(9.083)	(6.248)	(8.509)	(5.295)	(6.516)
CC	0.125	0.120	0.099	0.116	0.116
	(5.128)	(4.727)	(4.064)	(4.913)	(4.948)
CA	0.277	0.128	0.283	0.149	0.182
	(5.921)	(1.733)	(5.706)	(2.184)	(3.226)
LPC	0.076	0.116	0.092	0.125	0.127
	(2.268)	(3.220)	(2.659)	(3.597)	(3.774)
FC	0.135	0.096	0.123	0.063	0.052
	(3.990)	(1.883)	(3.629)	(1.232)	(1.111)
CP	0.172	0.193	0.198	0.193	0.192
	(10.80)	(11.07)	(10.09)	(13.71)	(14.19)
T	-0.033	-0.027	-0.032	-0.025	-0.024
	(-6.926)	(-2.947)	(-3.087)	(-3.087)	(-3.373)
σ_v					
Constant	-1.472	-1.779	-1.516	-1.771	
	(-15.24)	(-0.885)	(-15.58)	(-0.890)	
Lab		0.297		0.280	0.258
		(1.853)		(2.013)	(2.235)
MEO		0.315		0.425	0.218
		(0.836)		(1.261)	(1.037)

Table 2. (Continued) Estimation Results

	Model HO	Model HV	Model HW	Model HVW	Model HVWR
CA		-1.099		-0.971	-0.611
		(-1.302)		(-1.899)	(-2.253)
FC		-0.148		-0.299	-0.415
		(-0.491)		(-1.002)	(-1.651)
T		0.071		0.066	0.074
		(1.323)		(1.184)	(1.739)
σ_w					
Constant	-2.050	-1.878	-2.529	-2.379	-2.843
	(-21.97)	(-15.06)	(-1.869)	(-1.873)	(-3.345)
Lab			0.245	0.065	0.065
			(1.802)	(1.500)	(1.431)
MEO			0.026	-0.365	-0.319
			(0.154)	(-1.788)	(-1.847)
CA			-0.602	0.029	
			(-1.996)	(0.111)	
FC			0.052	0.355	0.372
			(0.237)	(1.416)	(2.103)
T			0.036	0.015	
			(1.909)	(0.461)	
Log likelihood	297.58	308.89	308.27	314.85	314.21
LR value	33.54	11.92	13.16		1.28

Note: t-values in parentheses.

Likelihood ratio statistics were used to test hypotheses. All the tests were carried out using the 5% significance level. Model HVW nests all the other models. Using general to specific methodology (see Abadir et al (1999) and Abadir and Hadri (2000) on the importance of general to specific methodology), we started by testing the hypothesis of a homoscedastic *v* (H_0: $\alpha_1 = \alpha_2 = \alpha_3 = \alpha_4 = \alpha_5 = 0$, against the alternative that at least one parameter is different from zero). We obtained a likelihood ratio of 13.16 indicating the rejection of the null hypothesis. We then tested the hypothesis of a homoscedastic *w* (H_0: $\gamma_1 = \gamma_2 = \gamma_3 = \gamma_4 = \gamma_5 = 0$, against the alternative that at least one parameter is different from zero). The likelihood ratio reached a value of 11.92 indicating the rejection of the null hypothesis. Next, we tested the joint hypothesis that *v* and *w* are homoscedastic. This hypothesis is also rejected on the basis of a likelihood ratio of 33.54. Therefore, HVW is statistically the preferred model as far as testing for heteroscedasticity is concerned.

This result shows the necessity of testing for heteroscedasticity in both error terms and making the appropriate corrections. By allowing both error terms to be heteroscedastic, model HVW is correcting for the corresponding double heteroscedasticity. Now, model HVW can be reduced further by noticing that all the parameters in the production function are significant, while three parameters associated with the error terms appear to be insignificant, namely α_0, γ_3 and γ_5. To test the joint hypothesis $H_0 : \alpha_0 = \gamma_3 = \gamma_5 = 0$, we estimated a restricted model called model HVWR. Its parameter estimates are shown in Table 2. We obtained a likelihood ratio of 1.28 leading to the acceptance of the restrictions.

Table 3 shows some descriptive statistics of efficiencies estimated from the five models. While the maxima are similar, there is a clear difference between the minima of the two doubly heteroscedastic models (HVW and HVWR) and the other three specifications. The means of models HVW and HVWR are equal, and the standard deviations and skewness are very close. We notice that model HV is the closest to model HVW and dissimilar from model HO and model HW. This suggests that heteroscedasticity is stronger in the one-sided term.

Table 4 confirms this last result where we find a very high correlation between model HV and model HVW efficiencies. Table 4 displays correlations and rank correlations between efficiencies estimated from the five models. The ranking is clearly affected by the specification used. Hence, accounting correctly for heteroscedasticity has a significant effect not only on estimation and on testing but on ranking farm efficiencies as well.

Table 3. Summary Statistics for Efficiencies

Model	Minimum	Maximum	Mean	St. dev	Skewness
HO	0.49	0.96	0.83	0.081	-1.06
HV	0.47	0.98	0.88	0.082	-2.02
HW	0.49	0.97	0.84	0.077	-1.01
HVW	0.41	0.98	0.86	0.090	-1.91
HVWR	0.42	0.98	0.86	0.095	-1.81

Table 4. Correlation between Efficiencies

	Correlation coefficient				Pearson and Spearman rank correlation coefficient			
Model	HV	HW	HVW	HVWR	HV	HW	HVW	HVWR
HO	0.68	0.97	0.72	0.74	0.43	0.88	0.51	0.53
HV		0.55	0.98	0.98		0.44	0.90	0.85
HW			0.60	0.62			0.52	0.55
HVW				0.99				0.92

In our selected model HVWR, it is clear that neither size (CA) nor time have any effect on the variance of the double-sided error term (*w*). For the inefficiency term *v*, the parameters associated with Labour and MEO are positive (0.258 and 0.218 respectively), suggesting that larger farms in terms of labour and machinery cost tend to have more variability in efficiency. Typically, an increase by 100% in labour tends to increase the variance of the inefficiency error term by around 5%, and an increase of 100% in machinery, energy and other costs (MEO) tends to increase the variance by around 4%. We can deduce from this that farms with higher expenditure on labour, machinery, energy, and other costs tend to be different in terms of efficiency than farms with lower expenditure on these items. Similarly, farms with lower levels of expenditure on labour and MEO tend to have a smaller variance, which means that they are similar to each other in terms of efficiency than farms with higher levels of expenditure.

By contrast, land area (CA) and fertilizer cost (FC) have negative parameters in the variance of the inefficiency term. This means that these two variables tend to dampen variability in efficiency. The time parameter is small but significant, indicating that time has a slight positive effect on the inefficiency variance.

The parameter estimates for model HVWR have the expected sign and are all positive except for the time variable. Although the parameter associated with time is very small (-0.024), it is nevertheless significant. The elasticity for MEO, cost of seeds, cereal area, land and property charges, and crop protection costs are relatively important, with values of 0.25, 0.11, 0.18, 0.12, and 0.19 respectively. The elasticities for labour and fertilizer costs are less important with values of 0.03 and 0.05 respectively. The return to scale parameter is 0.931, which indicates roughly constant returns to scale. The estimated technical efficiencies for the 102 farms are available from the authors.

IV. Conclusion

This paper extends the Hadri (1999) correction for heteroscedasticy to stochastic production frontiers and to panel data. It demonstrates that heteroscedasticity within an estimation can have a significant effect on results. The models developed in this paper demonstrate that the correction for heteroscedasticity is essential in order to obtain valid estimates, tests and correct measures of efficiency.

Appendix

The first partial derivatives of the log-likelihood function where only the one-sided term is assumed heteroscedastic:

$$\frac{\partial \log L}{\partial \beta} = \sum\sum\left[\frac{(y_{it} - X_{it}\beta)}{\sigma_{it}^2} + \frac{\lambda_{it} f_{it}^*}{\sigma_{it}(1 - F_{it}^*)}\right] X_{it}^{'}, \tag{1}$$

$$\frac{\partial \log L}{\partial \alpha} = \sum\sum\left[-\frac{\sigma_{vit}^2}{\sigma_{it}^2} + \frac{\sigma_{vit}^2}{\sigma_{it}^2}\left(\frac{(y_{it} - X_{it}\beta)}{\sigma_{it}}\right)^2 - \right. \tag{2}$$

$$\left. - \frac{\lambda_{it} f_{it}^*}{(1-F_{it}^*)}\left(\frac{(y_{it} - X_{it}\beta)}{\sigma_{it}}\right) \times \left(1 - \frac{\sigma_{vit}^2}{\sigma_{it}^2}\right)\right] Z_{it}^{'},$$

$$\frac{\partial \log L}{\partial \gamma_0} = \sum\sum\left[-\frac{\sigma_{w}^2}{\sigma_{it}^2} + \frac{\sigma_{w}^2}{\sigma_{it}^2}\left(\frac{(y_{it} - X_{it}\beta)}{\sigma_{it}}\right)^2 + \right. \tag{3}$$

$$\left. + \frac{\lambda_{it} f_{it}^*}{(1-F_{it}^*)}\left(\frac{(y_{it} - X_{it}\beta)}{\sigma_{it}}\right) \times \left(1 + \frac{\sigma_{w}^2}{\sigma_{it}^2}\right)\right].$$

The first partial derivatives of the log-likelihood function where only the two-sided term is assumed heteroscedastic:

$$\frac{\partial \log L}{\partial \beta} = \sum\sum\left[\frac{(y_{it} - X_{it}\beta)}{\sigma_{it}^2} + \frac{\lambda_{it} f_{it}^*}{\sigma_{it}(1-F_{it}^*)}\right] X_{it}^{'}, \tag{4}$$

$$\frac{\partial \log L}{\partial \alpha_0} = \sum\sum\left[-\frac{\sigma_{v}^2}{\sigma_{it}^2} + \frac{\sigma_{v}^2}{\sigma_{it}^2}\left(\frac{(y_{it} - X_{it}\beta)}{\sigma_{it}}\right)^2 - \right. \tag{5}$$

$$\left. - \frac{\lambda_{it} - f_{it}^*}{(1-F_{it}^*)}\left(\frac{(y_{it} - X_{it}\beta}{\sigma_{it}}\right) \times \left(1 - \frac{\sigma_{v}^2}{\sigma_{it}^2}\right)\right],$$

$$\frac{\partial \log L}{\partial \gamma} = \sum\sum\left[-\frac{\sigma_{wit}^2}{\sigma_{it}^2} + \frac{\sigma_{wit}^2}{\sigma_{it}^2}\left(\frac{(y_{it} - X_{it}\beta)}{\sigma_{it}}\right)^2 - \right. \tag{6}$$

$$\left. - \frac{\lambda_{it} - f_{it}^*}{(1-F_{it}^*)}\left(\frac{(y_{it} - X_{it}\beta)}{\sigma_{it}}\right) \times \left(1 - \frac{\sigma_{wit}^2}{\sigma_{it}^2}\right)\right] Y_{it}^{'}.$$

The first partial derivatives of the log-likelihood function where both disturbance terms are assumed heteroscedastic:

$$\frac{\partial \log L}{\partial \beta} = \sum\sum\left[\frac{(y_{it} - X_{it}\beta)}{\sigma_{it}^2} + \frac{\lambda_{it} f_{it}^*}{\sigma_{it}\left(1-F_{it}^*\right)}\right] X_{it}^{'}, \tag{7}$$

$$\frac{\partial \log L}{\partial \alpha} = \sum\sum\left[-\frac{\sigma_{vit}^2}{\sigma_{it}^2} + \frac{\sigma_{vit}^2}{\sigma_{it}^2}\left(\frac{(y_{it} - X_{it}\beta)}{\sigma_{it}}\right)^2 - \right. \tag{8}$$

$$\left. - \frac{\lambda_{it} f_{it}^*}{\left(1 - F_{it}^*\right)}\left(\frac{(y_{it} - X_{it}\beta)}{\sigma_{it}}\right) \times \left(1 - \frac{\sigma_{vit}^2}{\sigma_{it}^2}\right)\right] Z_{it}^{'},$$

$$\frac{\partial \log L}{\partial \gamma} = \sum\sum\left[-\frac{\sigma_{wit}^2}{\sigma_{it}^2} + \frac{\sigma_{wit}^2}{\sigma_{it}^2}\left(\frac{(y_{it} - X_{it}\beta)}{\sigma_{it}}\right)^2 + \right. \tag{9}$$

$$\left. + \frac{\lambda_{it} f_{it}^*}{\left(1 - F_{it}^*\right)}\left(\frac{(y_{it} - X_{it}\beta)}{\sigma_{it}}\right) \times \left(1 + \frac{\sigma_{vit}^2}{\sigma_{it}^2}\right)\right]$$

where $\sum\sum \equiv \sum_{i=1}^{N} \sum_{t=1}^{T_i} \cdot T_i$ is used here instead of T in order to allow for the possibility of unbalanced data.

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