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Estimation of growth convergence using a stochastic production frontier approach

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Abstract

In this paper we estimate stochastic world production frontiers econometrically taking country heterogeneity into account. From the estimated frontier we obtain estimates of technical change, technological catch-up (efficiency improvement/convergence), and scale related components of total factor productivity (TFP) growth. © 2005 Elsevier B.V. All rights reserved.

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1. Introduction

In a seminal paper [Kumar and Russell \(2002\)](#) show that economic growth convergence can be viewed as countries' movements toward the world production frontier. Given its resources (input levels in the context of a production function) a country might fall short of producing the maximum possible (frontier) output—a phenomenon popularly known as technical inefficiency. Such inefficiency may arise because of factors such as inadequate financial institutions and inapposite regulatory intervention, etc. Economic convergence is observed if countries operating below the frontier move toward the world

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production frontier (efficiency improvements) over time. This phenomenon is often labeled as technological catch-up. Kumar and Russell (2002) use a non-parametric approach to estimate the world production frontier. In this study we use a parametric approach, in which a stochastic production frontier is econometrically estimated. From the estimated frontier we obtain estimates of technical change, technological catch-up (convergence), and scale related components of total factor productivity (TFP) growth.

Kneller and Stevens (2003) also estimate a world production frontier model that allows for the technical inefficiency. The inclusion of the inefficiency in their model, however, is for the sake of completeness of production function specifications. Our model differs from theirs in several accounts. For example, technical inefficiency is interpreted in the context of growth convergence in our model, and we adopt an inefficiency parameterization from which the rate of convergence can be easily computed and tested. Efficiency improvements are also explicitly related to TFP growth (convergence) in our model. In addition, our model recognizes country heterogeneity and includes country-specific fixed effects in estimating the world frontier. As argued by Greene (2002), subject-specific effects are important sources of heterogeneity that have been largely ignored in the production efficiency literature.

2. The model

The frontier-based growth model for cross-country panel data is specified as follows.

$$y_{it} = \beta_i + \beta_1 k_{it} + \beta_2 l_{it} + \beta_3 k_{it}^2 + \beta_4 l_{it}^2 + \beta_5 k_{it} l_{it} + \beta_6 t + v_{it} - u_{it}, \quad (1)$$

$$v_{it} \sim N(0, \sigma_v^2), \quad (2)$$

$$u_{it} = G_t u_i = \exp[\gamma(t - \underline{t})] u_i, \quad (3)$$

$$u_i \sim N^+(u_i, \sigma^2), \quad (4)$$

$$u_i = \delta_0 + \delta_1 (k_{i\underline{t}} - l_{i\underline{t}}), \quad (5)$$

$$\sigma_v^2 = \exp(c_v), \quad \sigma^2 = \exp(c_u). \quad (6)$$

In the model specified above, the subscripts i and t refer to country and year, respectively; y_{it} , k_{it} , and l_{it} are the log of output, capital, and human capital-adjusted labor supply defined as the labor force weighted by the years of schooling. The production frontier function has a translog specification in terms of k and l , and it augments by a time trend variable (t) (same as in Kneller and Stevens, 2003). The model allows the frontier function to have country-specific intercepts (β_i). That is, instead of assuming a single world frontier function for all the countries, we allow heterogeneity via country-specific intercepts in the frontier production function. The stochastic noise component v_{it} is added to allow for unobserved random errors affecting the frontier function. The inefficiency term $u_{it} \geq 0$ measures the distance from the frontier for a country i at time t , and growth convergence implies a shrinkage of u_{it} over time. The term u_{it} is specified as a product of two components, viz., a deterministic function of time (G_t) and a

stochastic country-specific, non-negative variable (u_i). The notation $N^+(\cdot)$ indicates a non-negative truncation of the underlying normal distribution.

Eqs. (3)–(5) deserve further explanations. In these equations, \underline{t} denotes the starting year of the data (initial time period), and thus $u_{it}=u_i$ when $t=\underline{t}$. Therefore, the initial inefficiency (distance from the frontier) is assumed to follow a $N(\mu_i, \sigma^2)$ distribution truncated at zero from below. The mean of the pre-truncated distribution is related to (log of) the initial capital to labor ratio ($k_{i\underline{t}}-l_{i\underline{t}}$) which is country-specific. The inefficiency evolves over time according to $\exp[\gamma(t-\underline{t})]$. Note that when $t\rightarrow\infty$, $u_{it}\rightarrow 0$ if $\gamma<0$. Therefore, the convergence hypothesis is tested by $H_0:\gamma\leq 0$ against $H_1:\gamma>0$.

The coefficient γ in the G_t function above has a straightforward interpretation. Since $\gamma=\partial\ln u_{it}/\partial t$, one can interpret it as the *percentage change in inefficiency* over time. Thus, if γ is negative, then technological catch-up (movement towards the frontier) is observed, and we may define $-\gamma$ as the technological catch-up rate. This coefficient is also related to the rate of convergence (*change in inefficiency*) which can be computed from

$$\rho_{it} = - \left[\frac{\partial u_{it}}{\partial t} \right] = - \gamma \exp \left[\gamma (t - \underline{t}) \right] u_i. \quad (7)$$

Since $u_i \geq 0$, a negative value of γ will indicate convergence. The rate of convergence in the above formulation depends on the initial level of technical inefficiency, u_i , which is country-specific. To estimate ρ_{it} we need to replace u_i by its estimate (see Kumbhakar and Lovell, 2000, p. 111) since u_i is not observed.

The model used by Kneller and Stevens (2003) does not allow country-specific intercepts in the production frontier. It instead allows the pre-truncated mean of inefficiency to vary over time. That is, u_{it} is modeled as

$$u_{it} \sim N^+(\delta_0 + \delta_1 t, \sigma_u^2). \quad (8)$$

The above specification assumes that at each point in time the technical inefficiency for each country is drawn from a truncated normal distribution that is independently and identically distributed among countries. The temporal pattern of inefficiency is captured in the mean of the distribution. Although a negative value of δ_1 would imply shrinking inefficiency over time (meaning convergence), a straightforward interpretation of δ_1 is difficult because of the nonlinearity between $E(u_{it})$ and t . In addition, the convergence hypothesis implies that $\mu_{it} \rightarrow -\infty$ as $t \rightarrow \infty$, which is not a testable hypothesis in Eq. (8).

3. Data and estimation results

The data we use here are the same as that in Duffy and Papageorgiou (2000) and Kneller and Stevens (2003), and the World Bank STARS database is the data source. The output and capital variable are measures of GDP and the aggregate physical capital stock (converted into constant, end-of-period 1987 U.S. dollars) for all 82 countries over the period 1960–1987. Labor is the number of individuals in the workforce between the ages of 15 and 64. Human capital-adjusted labor is obtained by weighting labor force by the mean years of schooling of the workforce. Further details concerning the construction of this data are provided in the appendix of Duffy and Papageorgiou (2000).

The model outlined in Eqs. (1)–(6) needs to be estimated using the maximum likelihood (ML) method. The likelihood function of the model can be derived from the distributional assumptions on v_{it} and u_i . The function can be found in Kumbhakar and Lovell (2000, p. 111). We need to substitute the country dummies, the appropriate expressions of G_t , from Eq. (3) and μ_i from Eq. (5) into it to obtain the appropriate likelihood function for our model. Once the parameters are estimated, u_i can be estimated from $E(u_i|\epsilon_i)$ where $\epsilon_i=(\epsilon_{i1}, \dots, \epsilon_{iT})=(v_{i1}-u_{i1}, \dots, v_{iT}-u_{iT})$ (see Kumbhakar and Lovell, 2000, p. 111). Technical inefficiency is then estimated from $\hat{u}_{it}=\hat{G}_t \cdot E(u_i|\epsilon_i)$.

Table 1 reports the estimation results of the model of Eqs. (1)–(6), with (Model 2) and without (Model 1) country-specific intercepts in the frontier functions. For comparison purposes, we also estimate a Cobb–Douglas specification of the model which imposes the constraints $\beta_3=0, \beta_4=0,$ and $\beta_5=0$ on Eq. (1). The estimation results with (Model 4) and without (Model 3) country-specific intercepts are also in Table 1. Since the translog parameters cannot be directly interpreted, the table also reports the output elasticities with respect to capital (θ_k) and labor (θ_l) evaluated at the sample mean. These elasticities are

$$\theta_k = \beta_1 + 2\beta_3k + \beta_5l, \tag{9}$$

$$\theta_l = \beta_2 + 2\beta_4l + \beta_5k. \tag{10}$$

The standard errors of the two statistics are obtained using the delta method on the above formulae.

Table 1
Estimation results

	TL specification				CD specification			
	Model 1		Model 2		Model 3		Model 4	
	Coefficient	Standard error	Coefficient	Standard error	Coefficient	Standard error	Coefficient	Standard error
Frontier function								
β_1	0.501***	(0.023)	0.501***	(0.034)	0.853***	(0.008)	0.498***	(0.018)
β_2	-0.031	(0.020)	0.010	(0.043)	0.111***	(0.011)	0.039	(0.029)
β_3	0.018***	(0.005)	-0.023***	(0.007)				
β_4	0.011*	(0.006)	-0.021**	(0.098)				
β_5	0.007	(0.012)	0.045***	(0.015)				
β_6	-0.087***	(0.012)	-0.030***	(0.002)	-0.041***	(0.004)	-0.031***	(0.002)
u_{it} equation								
γ	-0.012***	(0.001)	-0.024***	(0.002)	-0.024***	(0.001)	-0.024***	(0.001)
δ_0	8.567***	(1.950)	2.355***	(0.202)	1.551***	(0.282)	2.336***	(0.195)
δ_1	-0.107*	(0.065)	0.029	(0.071)	0.036	(0.048)	0.003	(0.064)
c_u	-0.900***	(0.185)	-0.933***	(0.202)	-1.535***	(0.162)	-0.924***	(0.186)
v_{it} equation								
c_v	-4.294***	(0.030)	-5.192***	(0.031)	-4.117***	(0.030)	-5.188***	(0.030)
θ_k	0.651***	(0.018)	0.474***	(0.019)	0.853***	(0.008)	0.498***	(0.018)
θ_l	0.066***	(0.013)	0.036	(0.031)	0.111***	(0.011)	0.039	(0.029)
Log-likelihood	1408.630		2415.379		1249.913		2409.812	

Models 2 and 4 include country-specific intercepts in the production function while Models 1 and 3 do not. Estimates of the intercepts are not reported in the table to save space. Significance: ***: 1% level; **: 5% level; *: 10% level.

The estimates of the frontier functions are noticeably different between models with and without country-specific effects. For example, the output elasticities of capital and labor in Model 2 are about 60% to 70% smaller compared to those from Model 1. Similar observations are found between Models 4 and 3. This may not be surprising since the capital and labor variables in Models 1 and 3 are likely to pick up some of the omitted heterogeneity effects, which are captured in the alternative models (i.e., Models 2 and 4) by the country dummy variables. Since $\partial y/\partial t < 0$, all the models predict technical regress, and the effects are between 8.7% and 3.0%. Technical regress is also found in [Duffy and Papageorgiou \(2000\)](#) and [Kneller and Stevens \(2003\)](#). The estimates of γ are -0.024 except for Model 1,¹ and all the estimates are statistically significant at the 1% level. The estimate of -0.024 implies that the countries are converging to the frontier at a rate of 2.4% per year (meaning that technical inefficiency is shrinking at a rate of 2.4% per year). Although [Kneller and Stevens \(2003\)](#) obtain $\delta_1 = -0.057$ in Eq. (8), the estimate does not imply convergence at a rate of 5.7%.

Since Models 1 and 3 are nested in Models 2 and 4, respectively, we conduct likelihood ratio (LR) tests on the joint significance of the country-specific intercepts. The test statistic for the translog model is $\chi_{81}^2 = 2013.5$, and that for the Cobb–Douglas model is $\chi_{81}^2 = 2319.8$; both of them are statistically significant at the 1% level. Therefore, the data provides evidence of heterogeneity in the production functions, for both the translog and Cobb–Douglas specifications, across countries.

We also use the results to test the assumption of a Cobb–Douglas production technology which is widely adopted in the growth literature. The assumption has been rejected by [Duffy and Papageorgiou \(2000\)](#) against a general CES production technology, and by [Kneller and Stevens \(2003\)](#) against a translog technology. None of the studies, however, takes into account country heterogeneities which are found to be important in this study. By taking the heterogeneity into account, the test of a Cobb–Douglas assumption against the translog assumption is a test of Model 4 against Model 2. The test yields a statistic of $\chi_3^2 = 11.134$, and the Cobb–Douglas assumption is rejected. The results reinforce the findings of the aforementioned studies.

Technical regress predicted by the model used in [Kneller and Stevens \(2003\)](#) (2.8%) is close to the values predicted by Model 2 (3.0%) and Model 4 (3.1%). However, the model used by [Kneller and Stevens \(2003\)](#) is not nested in neither of the models estimated in this paper, and therefore it is not possible to conduct LR tests to for the models.

As mentioned in [Kumar and Russell \(2002\)](#), productivity change can take place due to (i) a shift in the production frontier, (ii) change in efficiency, and (iii) scale economies (dis-economies). We now report TFP growth and decompose it into the above three components (see [Kumbhakar and Lovell, 2000](#), p. 284), viz.,

$$\text{TFP} = \text{TC} + \text{TE}\Delta + \text{Scale}, \quad (11)$$

where $\text{TC} = \partial y/\partial t = \beta_6$, $\text{TE}\Delta = -\partial u_{it}/\partial t$, and $\text{Scale} = (\theta - 1)\{(\theta_k \dot{K} + \theta_l \dot{L})\}/\theta$. Finally, $\theta = \theta_k + \theta_l$ is a measure of returns to scale.

¹ We have checked and confirmed the robustness of the result using different initial values for the ML estimations.

Table 2
Model statistics

	Model 1		Model 2		Model 3		Model 4	
	Mean	Standard deviation						
ρ_{it}	0.088	0.011	0.042	0.014	0.028	0.010	0.042	0.014
TC	-0.087	–	-0.029	–	-0.041	–	-0.031	–
TE Δ	0.088	0.011	0.042	0.014	0.028	0.010	0.042	0.014
Scale	-0.015	0.013	-0.026	0.017	-0.002	0.001	-0.025	0.015
TFP	-0.015	0.014	-0.014	0.020	-0.016	0.010	-0.015	0.019
TFP ₁	0.001	0.011	0.012	0.014	-0.013	0.010	0.011	0.014

TFP is the sum of TC, Scale and TE Δ , while TFP₁ is the sum of TC and TE Δ .

An alternative way to measure productivity change is to examine the rate at which output changes while holding inputs unchanged. This is equivalent to ignoring the scale component (changes in input levels) above. Consequently, the TFP growth formula is:

$$\dot{\text{TFP}}_1 = \text{TC} + \text{TE}\Delta, \quad (12)$$

where the TC and TE Δ components are as previously defined.

The TFP growth and its components are reported in Table 2. Although the individual components differ between the two models, the TFP growth results are comparable. All the models show a decline in TFP growth at a rate between 1.4% and 1.6%. If TFP growth is measured holding the input levels unchanged, then three of the models show positive TFP growth, indicating technological catch-up. The only exception is the Cobb–Douglas model without country heterogeneity, which is found to be misspecified based on the LR tests.

4. Conclusion

In this paper we have considered technical inefficiency in measuring and decomposing TFP growth into technical change (shift in the frontier) and technological catch-up (movement towards or away from the frontier). In doing so, we specify a model that can accommodate heterogeneity in the world production function. We find evidence in favor of country heterogeneity. Ignoring heterogeneity tends to underestimate the catch-up rate and overestimate the technical regress, although the joint effects of the catch-up rate and technical change are quite similar in those models with and without heterogeneity.

References

- Duffy, J., Papageorgiou, C., 2000. A cross-country empirical investigation of the aggregate production function specification. *Journal of Economic Growth* 5, 87–120.
- Greene, W., 2002. Fixed and Random Effects in Stochastic Frontier Models. Stern School of Business, New York University.
- Kneller, R., Stevens, A., 2003. The specification of the aggregate production function in the presence of inefficiency. *Economics Letters* 81, 223–226.
- Kumar, S., Russell, R.R., 2002. Technological change, technological catch-up, and capital deepening: relative contributions to growth and convergence. *American Economic Review* 92, 527–548.
- Kumbhakar, S., Lovell, K.C.A., 2000. *Stochastic Frontier Analysis*. Cambridge University Press, New York.