Numerical Example of Creating a Zero Risk Portfolio with Two Risky Securities Whose Returns are Perfectly Negatively Correlated

William L. Silber

	Possible Outcomes	
	Good Year (Pr=.5)	Bad Year (Pr. = .5)
Security A	16%	2%
B	-2%	12%

Notes:

1) The data in the above table show 2 possible returns for each security, with each outcome having a probability of .5

2) The expected return on security A is 9% (=.5*16%+.5*2%), with substantial variance around that mean. Security B has an expected return of 5%, also with substantial variance around the mean.

3) Since there are only two outcomes for each security that vary in opposite directions, the correlation between the 2 securities is –1. Thus it is possible to combine the 2 securities to produce a riskless portfolio (no variance around the mean) with a return that lies between the returns on the 2 securities.

4) An allocation of X1=.5 and X2=.5 will do the job. These weights are derived by solving the following equation for X1 (the equation sets the return in the good year equal to the return in the bad year) and recalling that X1+X2=1:

$$X1*(16\%)+ (1-X1)*(-2\%)=X1*(2\%)+ (1-X1)*(12\%)$$

5) Here is the numerical example of zero risk (no variability in return between good and bad years). Suppose your wealth equaled $200 and you put $100 in security A and $100 in security B. Then you will earn the following:

Good Year: .16*$100 + (-.02)*$100 = $14
Bad Year: .02*$100 + .12* $100=$14

6) No matter what happens you earn $14/$200 = .07 or 7%. The reason is that the outcomes of the two securities move in opposite directions and you have allocated the funds in your portfolio so that the dollar amounts offset each other.