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Foundations of Finance (B01.2311)

OPTIONS PRICING EXERCISE

Minnesota Mining and Manufacturing (3M) is awarding a year-end bonus to its Senior Vice-President of Marketing in the form of call options on 3M stock. Each option gives the VP the right to purchase one share of 3M stock three years hence at a price of $100. In addition to the options, the VP has a position in 3M stock. The VP believes that she should be flattered by the bonus, but she doesn't really know how much the options are worth since they are not traded on an exchange. The stock (MMM-NYSE) is trading at $100 per share and the three year continuously compounded risk-free rate is 6% .

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a) What should the minimum value of this option be? (Assume that the VP can borrow and lend at the risk-free rate).

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b) Given that the annual volatility of 3M stock is 30%, what is the fair value of the option according to the Black-Scholes pricing model?

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c) Given the fair value of this call, what should the fair value of the put be?

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d) Now, suppose that the VP believes that the fair value of the call is too high and is willing to sell it to you for $23.00. Assume that both the puts and calls can be traded and that the puts are priced at their fair value. What would you do? Is there an arbitrage opportunity?

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e) Now, suppose that the VP believes that the fair value of the call is too low and is willing to purchase it from you for $32.00. What would you do? Is there an arbitrage opportunity?

OPTIONS PRICING EXERCISE (SOLUTION)

Minnesota Mining and Manufacturing (3M) is awarding a year-end bonus to its Senior Vice-President of Marketing in the form of call options on 3M stock. Each option gives the VP the right to purchase one share of 3M stock three years hence at a price of $100. In addition to the options, the VP has a position in 3M stock. The VP believes that she should be flattered by the bonus, but she doesn't really know how much the options are worth since they are not traded on an exchange. The stock (MMM-NYSE) is trading at $100 per share and the three year continuously compounded risk-free rate is 6%.

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a) What should the minimum value of this option be? (Assume that the VP can borrow and lend at the risk-free rate).

The VP can sell her 3M stock now, invest the proceeds for three years, and then buy the stock back at $100 by exercising the option at expiration.

Sell stock: receive $100.00
Invest the proceeds for three years: 100.00*e^(.06)(3) = $119.72
Risk-free interest received in three years: 119.72 - 100.00 = $19.72

Minimum value of the option = the present value of the risk-free
interest received: 19.72*e^-(.06)(3) = $16.47

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b) Given that the annual volatility of 3M stock is 30%, what is the fair value of the option according to the Black-Scholes pricing model?

C = S*N(d1) - K*e^-(r)(t)*N(d2)

Calculate the values of d1 and d2:

d1 = [ln(S/K) + (r+s^2/2)t1/s*t^(1/2)
d1 = [ln(100/100) + (.06+(.3)^2/2)*3]1/.3*3^(1/2)
d1 = 0.60621778

d2 = d1 - s*t^(1/2)
d2 = 0.60621778 - .3*3^(1/2)
d2 = 0.08660254

Using table 15.2, find the values of N(d1) and N(d2) by interpolation:

d	N(d)
.60	.7258
.62	.7324

Interpolating:

[(.60621778 -.60)/(.62 - .60)]*(.7324 - .7258) = 0.002052
Therefore:

N(d1) = .7258 + .002052 = .727852

d	N(d)
.08	.5319
.10	.5398

Interpolating:

[(.08660254 - .08)/(.10 - .08)]*(.5398 - .5319) = 0.002608

Therefore:

N(d2) = .5319 + .002608 = .534508

Price the option:

C = S*N(d1) - [K*e^ - (r)(t)]*N(d2)
C = 100*(.727852) - [100*e^ - (.06)(3)]*(.534508)
C = $28.14

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c) Given the fair value of this call, what should the fair value of the put be?

Price the put using put-call parity:

C - P = S - PV(K
P = C - S + PV(K)
P = 28.14 - 100 + 100*e^ - (.06)(3)
P = 28.14 - 100 + 83.53
P = $11.67

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d) Now, suppose that the VP believes that the fair value of the call is too high and is willing to sell it to you for $23.00. Assume that both the puts and calls can be traded and that the puts are priced at their fair value. What would you do? Is there an arbitrage opportunity?

Check put-call parity:

C - P = *S* - PV(K)
23.00 - 11.67 = (?) 100 - l00e^ - (.06)(3)
11.33 < 16.47

Strategy: Buy low (C-P @11.33), Sell high (S-PV(K) @16.47)

		cash outlay
Long:	buy call	- 23.00
	sell put	+ 11.67
Short:	sell stock	+100.00
	lend difference	+88.7

$88.67 lent for three years will grow to: 88.67*e^ (.06)(3) = $106.16

Examine the three possible outcomes at expiration of the option:

S>E	cash outlay
long C : exercise, buy stock	-100.00
short P: expires worthless	---
short S: deliver stock from c	---

net profit = 106.16 - 100.00 = $6.16

S=E	cash outlay
long C : expires worthless	
short P: expires worthless	---
short S: buy stock @100 & deliver	-100.00

net profit = 106.16 - 100.00 = $6.16

S<E	cash outlay
long C : expires worthless	
short P: exercised against you, buy stock @100	-100.00
short S: deliver stock from P	---

net profit = 106.16 - 100.00 = $6.16

So, the arbitrage results in a riskiess profit of $6.16.

Notice that this profit is equivalent to the future value three years hence of the amount that the call is mispriced:

28-14 - 23.00 = 5.14
5.14*e^(.06)(3) = $6.15 (~$6.16 rounded)

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e) Now, suppose that the VP believes that the fair value of the call is too low and is willing to purchase it from you for $32.00. What would you do? Is there an arbitrage opportunity?

Check put-call parity:

C - P = *S* - PV(K)
32.00 -11.67 = (?) 100 - l00e^ - (.06)(3)
20.33 > 16.47

Strategy: Buy low (S - PV(K) @16.47), Sell high (C - P @20.33)

		cash outlay
Short:	sell call	+32.00
	buy put	- 11.67
Long:	buy stock	-100.00
	borrow difference	-79.67

$79.67 borrowed for three years will grow to: 79.67*e^(.06)(3) = $95.3 8

Examine the three possible outcomes at expiration of the option:

S>E	cash outlay
short C: exercised against you, sell stock @100	+100.00
long P : expires worthless	---
long S : deliver S against short C	---

net profit = 100.00 - 95.38 = $4.62

S=E	cash outlay
short C: expires worthless	---
long P : expires worthless	---
long S : sell S @100	+100.00

net profit = 100.00 - 95.38 = $4.62

S<E	cash outlay
short C: expires worthless	---
long P : exercise, sell stock	+100.00
long S : deliver S for P	---

net profit = 100.00 - 95.38 = $4.62

So, the arbitrage results in a riskless profit of $4.62.

Notice that this profit is equivalent to the future value three years hence of the amount that the call is mispriced:

32.00 - 28.14 = 3.86
3.86*e^(.06)(3) = $4.62