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Foundations of Finance (B01.2311)

Simple Numerical Arbitrage Example
Demonstrating Put Call Parity (Assuming r=0 for simplicity)

1. Put call parity states

$$C = S - Ee^{-rt} + P$$

2. Assume S = $110, E = $100, r = 0, t = anything. Then Item (1) implies: C = $110 - $100 + P

Therefore,

If P = $2, then C = $12
If P = $5, then C = $15

3. Assume S = $110, E = $100, r = 0 and t = 1 period. Also assume C = $12 and P = $5

Thus the call is undervalued and/or the put is overvalued, given S = $110 and E = $100

4. Construct an arbitrage today by buying the "cheap" call at $12 and selling the "expensive" put at $5. Recall that buying C and selling P guarantees that you must buy S and pay out E. Therefore, we complete the arbitrage by selling S short at $110 and investing the cash proceeds of the short sale, call purchase, and put sale at r (= 0 here but not in general).

Our position today is

	Cash Flow
Long 1 call	- $ 12
Short 1 put	+ $ 5
Short 1 share	+ $110
Invest Proceeds in Treasuries	- $103
Net	0

5. Examine what happens on expiration if S > E and if S < E

	Cash Flow
S > E	
Receive Proceeds of Treasuries	+$103
Exercise long C (get stock)	-$100
Put expires worthless	0
Return short stock	0
Net	+$3

	Cash Flow
S < E	
Receive Proceeds of Treasuries	+$103
Call expires worthless	0
Put is exercised against you	-$100
Take stock just received from put and return against short	0
	+$3

6. Since the position shown in 4 is profitable and riskless, we do it as often as we can, driving up the value of C and driving down the value of P until they differ by exactly $10. Thus C is driven up from $12 and P is driven down from $5. Whether C is $14 and P is $4 or C is $15 and P is $5 or whether C is $12 and P is $2 depends on the valuation of the call (or the put) from a model such as Black-Scholes. Put call parity just gives the relative value of C versus P given S and E.

7. Two Other Perspectives:

a) The profit of $3 is guaranteed no matter what happens because it equals the "mispricing" of C - P versus S - E

C - P = $12 - $5 = $7
S - E = $110 - $100 = $10

b) Another way of thinking about this is: Buying the call for $12 and selling the put for $5 allows you to be long the underlying stock (including exercise cost of $100) for $107. Since you sold the stock for $110, you are entitled to $3.