Inventory Decisions and Signals of Demand Uncertainty to Investors

This paper examines how managerial short-termism can affect a firm’s inventory decision when external investors only have partial information about the firm’s demand uncertainty. We first study the scenario where the manager’s short-termism is exogenously given. We derive the full equilibrium spectrum with both stable separating and pooling equilibria, which yield insights for learning firms’ demand uncertainty from their inventory and sales information and for understanding the impact of managerial short-termism on firm performance. We then analyze the scenario where the manager’s short-termism is endogenous. We find that unlike the scenario with exogenous short-termism, the first-best inventory decisions might be achieved in equilibrium by an alternative signal.

Key words: Short-termism, Market Valuation, Newsvendor, Signaling

1. Introduction

The financial market not only serves as a source of capital but can also influence firms’ strategies. Specifically, because firms’ decisions reflect their characteristics, the financial market may try to infer information from their decisions to predict their future performance. On the other hand, aiming of a better valuation, firms may purposely alter their decisions. For instance, it has been investigated in prior literature that firms’ capital structure and dividend decisions can be signals of their inside information for the financial market and this in turn may influence their strategies (e.g., Ross 1977, Myers and Majluf 1985, Miller and Rock 1985). Beyond these financial decisions, firms’ operations can also be affected. One example is about firms’ procurement orders. In certain circumstances (e.g., new product introduction, market expansion and firm reorganization), the financial market may respond positively to announcements of large procurement orders that firms place at their suppliers and negatively to any reduction or cancelations of the orders. Because of such market responses, recent studies show that firms with truly high demand expectation may over-order to signal their information (see, e.g., Lai et al. 2012, Schmidt et al. 2014).

However, firms’ inventory operations are complex. More generally, a higher inventory level does not necessarily indicate a better performance. In fact, the financial market pays close attention to the operational information from firms’ financial statements such as the sales, inventory markdowns and shortages (Raman et al. 2005). Empirical studies find that writeoffs resulted in from excess inventory can lead to significant drops of firms’ market value in the short run, which can often exceed the actual inventory costs amid concerns of their future prospects (Hendricks and Singhal 2009). Excess inventory is also associated with negative long-term stock returns (Chen et al. 2005, 2007). As such, firms sometimes may artificially reduce inventory to release a more
balanced financial statement (Lai 2006, Monga 2012). However, insufficient inventory can be costly too. Abercrombie & Fitch’s stock price suffered an almost 10% dive when its financial statement revealed that its operations “went the other way” to the lack of inventory (from excess inventory in the earlier years) which led to significant sales reduction (Trefis Team 2013). Similar scenarios have occurred at other firms (The Associated Press 2006, Fisher and Raman 2010). Therefore, firms’ market value can be penalized with either more or less inventory, which reflects their capability of matching supply with demand. While the investors have been taking into account such operational information for their valuation, it has not yet been fully understood how the reported information relates to the firms’ true performance (Raman et al. 2005). On the other hand, the classic inventory literature has mainly focused on the operational tradeoffs (e.g., overage and underage costs), whereas the impacts of the financial market have been little investigated.

To enrich the understanding of the above problem, in this study, we develop a stylized newsvendor model that incorporates both the interaction with the financial market and asymmetric information of the firm’s capability of matching supply with demand. We use the firm’s demand uncertainty as a surrogate of its capability to match supply with demand while setting the expected demand at a fixed level. Hence, all else equal, a more uncertain demand is associated with a higher chance to have either leftovers or shortages, and we assume that this basic characteristic of the firm will affect its future performance which we model in a reduced form by an expected profit depending on the current demand uncertainty level. The firm is run by a manager who has better knowledge of the demand uncertainty than the external investors in the sense that he observes whether the true uncertainty is high or low while the investors only know the prior distribution. The manager is interested in not only the firm’s actual value but also its market value. As such, the model allows us to investigate the impact of the financial market and asymmetric information on the firm’s inventory decision as well as the relationship between the reported operational information and the firm’s actual value.

We first study a commonly adopted setting in the literature, in which the magnitude of the manager’s interest in the firm’s market value is exogenous. We find that a nonstandard signaling game will arise for this problem and we derive the full spectrum of perfect Bayesian equilibria consisting of both stable separating and pooling equilibria. Our analysis yields interesting findings. In particular, the manager may either over- or understock inventory under the same demand uncertainty. For instance, when the firm has a large newsvendor critical ratio (i.e., a high intrinsic service level), observing a low demand uncertainty, the manager may purposely reduce the firm’s safety stock to signal his information. However, we also find that if the cost tied to such signaling is substantial in particular when the interest in the firm’s market value is very strong, he may forego lowering the inventory to signal and sometimes may even be forced to stock more inventory in the
pooling equilibrium. On the contrary, if the firm has a high demand uncertainty, the manager will not distort the inventory level when the interest in the market value is weak, but when the interest is strong he may purposely reduce the safety stock to imitate the low uncertainty counterpart. We derive symmetric results for small newsvendor critical ratios. Hence, to interpret the inventory level, it is critical to segregate the firms by their newsvendor critical ratio as well as their market value interest. A higher inventory level does not necessarily indicate that the firm is either more or less efficient. Furthermore, in the separating regime, the inventory level can be a perfect signal of the firm’s type, whereas in the pooling regime the realized sales information including leftovers and shortages will be critical to fairly value the firm. On the other hand, our results show that the impact of the financial market can undermine the operational efficiency for both types of firms with inventory distortions in different regions, unlike the prediction of the existing literature that only the more efficient type gets hurt.

Second, departing from prior literature, we consider a setting where the manager’s interest in the firm’s market value is endogenous. In practice, it is possible that managers may strategically plan the timing and amount of their shares to sell which investors often pay close attention to (Krantz 2013, Cahill 2013). For instance, the stock price of Facebook jumped up 4.8 percent on September 5th, 2012, when Mark Zuckerberg pledged not to sell his stocks at least for a year (Risberg 2012). As such, a multi-dimensional signaling game will arise in our model. Interestingly, we find that there are scenarios where the inventory distortions characterized above might be eliminated by the manager’s decision of share selling. Observing a high demand uncertainty may lead the manager to sell more shares in the short term, whereas he keeps more shares to the long term if the demand is less uncertain. This gives rise to an alternative signal to the investors about the firm’s efficiency that can sometimes avoid the inventory distortions.

The remainder of this paper is organized as follows. Section 2 reviews the relevant literature, and section 3 describe the model. We analyze the case with exogenous short-term objective in section 4 and investigate the endogenous case in section 5. We conclude in section 6.

2. Literature

In this section, we discuss several streams of literature that are closely related to our study. First, the economics and finance literature has investigated the impacts of asymmetric information on firms’ financial decisions. For instance, to explain the rationales behind debt financing, Ross (1977) develops a model with asymmetric information of the firm’s capital return. He shows that the manager can use different levels of debt, which has a potential bankruptcy cost, to signal his private information to the investors if his compensation depends on both of the firm’s market value and its actual value. Myers and Majluf (1985) use an adverse selection model to explain the pecking order
theory. When there is asymmetric information about a project’s return, to issue equity to fund the project might be interpreted by the market as a signal of low returns since in equilibrium firms with lower returns tend to have more incentives to do so. As a result, internal sources of funds or issuing guaranteed debt, which can prevent inefficient market valuation, is generally preferred to equity financing. Differently, Miller and Rock (1985) focus on firms’ dividend decisions. They show that when there is asymmetric information about the returns, higher return firms may distribute more dividends to signal their information to the investors, even by reducing investment. John and Williams (1985) use the signaling rationale to explain why firms may simultaneously issue stock and distribute dividends even if there are dissipative dividend costs such as tax. Several follow-up studies (e.g., Ambarish et al. 1987, Ofer and Thakor 1987, Williams 1988, Bernheim 1991) extend this signaling theory of dividend distribution to settings with endogenous liquidity need, real asset investment, stock repurchase and bankruptcy cost.

Second, information asymmetry can also affect firms’ asset liquidation and project investment decisions. Stein (1988) shows that more profitable firms may prematurely sell their long-term assets to signal their type and separate from less profitable firms. In contrast, Stein (1989) portrays a signal jamming scenario where the firm’s actual decision is unobservable while the reported information is noisy. He shows that firms may simply follow investors’ anticipation to “borrow” long-term profits by liquidating immature investments to inflate short-term earnings. Bebchuk and Stole (1993) assert that the type of information asymmetry can play an important role in firms’ investment decisions. They demonstrate with a unified framework that: when the firm’s action is observable while its long-term productivity is private information, a typical signaling game arises in which a more efficient firm may overinvest in the long-term project to signal its type; in contrast, when the firm’s long-term productivity is known but its action is unobservable, under-investment may occur. Clearly, either signaling or signal jamming caused by information asymmetry is costly, which motivates the accounting studies to investigate accounting policies that can mitigate the frictions (e.g., Verrecchia 2001, Dye and Sridhar 2004, Liang and Wen 2007).

Third, similar to ours, several recent studies have also investigated the impacts of information asymmetry and market incentives on firms’ operations. For instance, Lai et al. (2011) study the channel-stuffing phenomenon. They show that in the presence of asymmetric information and short-term interest in the stock price, firms may intentionally inflate their reported sales by stuffing the downstream channels with unauthentic orders. They reveal the roles of the operations metrics in such behaviors. Lai et al. (2012) and Schmidt et al. (2014) focus on the phenomenon that firms may announce large procurement orders they place at their suppliers to the market amid asymmetric information about their demand prospect (such as in the events of new product introduction, market expansion and company reorganization). Based on a standard signaling game, Lai et al.
(2012) show that if a regular supply contract is used, an over-ordering equilibrium may arise in which the firm with a large expected demand will order more than the efficient quantity to signal its information to the investors. With the aim of improving supply chain efficiency, the authors develop alternative supply contract menus that can facilitate signaling and alleviate operational frictions. Schmidt et al. (2014) develop a similar signaling game as in Lai et al. (2012), but they focus on the impacts of operational constraints on the outcome and the comparative statics of the signaling game. They show that if there are capacity and lot size constraints which limit the firm’s quantity choices, a pooling equilibrium with under-ordering may arise when the firm is unable to signal due to the physical constraints. As discussed in the introduction section, our study has a different motivation. We focus on the phenomenon where firms desire to show a more balanced inventory and sales relationship amid the investors’ concerns about their demand uncertainty. Our study also reveals different implications. We find that both over- and under-stocking equilibria can arise even without any physical constraints and a higher inventory level does not necessarily signal higher efficiency. We show the significant roles that the newsvendor critical ratio and the magnitude of short-termism can play in the determination of the equilibrium structure. Technically, our work can also complement prior literature as the classical single-crossing condition assumed in the earlier studies fails to hold for the phenomenon we investigate.

Lastly, our work is related to the supply chain literature on information dissemination. Li (2002) and Zhang (2002) focus on voluntary information sharing. They show that the downstream sellers might be willing to share their private demand information with their common supplier in a competition setting. Ha and Tong (2008) and Ha et al. (2011) reveal that information sharing can be influenced by the contract choice and the production efficiency. These studies typically assume that the shared information is credible. However, there are also supply chain scenarios where information is unverifiable. As a result, signaling may arise. For instance, Cachon and Lariviere (2001) study a manufacturer and supplier setting where the manufacturer that privately learns the demand forecast may signal the information to the supplier through capacity distortion. Such signaling games have also been studied in Kong et al. (2013) and Li et al. (2014) for the downstream competition settings, and compared to information sharing in Tian et al. (2014). Different from voluntary information sharing or signaling, several recent studies have investigated whether supply chain parties can truthfully share information through cheap talk. Chu et al. (2013) and Shamir and Shin (2013) find that credible information dissemination with cheap talk is possible when there are conflicting tradeoffs, e.g., announcing a large demand forecast may induce large capacity investment but may also trigger price increase or downstream competition. Our study differs from this literature as we focus on the interactions between firms and their investors.
3. Model

We consider a public firm that is run by a manager in two periods. Our focus is on the operations of the first period which represents a short-term time horizon. In this period, the firm sells one representative product, and we use the level of demand uncertainty that the firm still faces when it needs to make the inventory decision as the proxy of its management capability of matching supply with demand. Specifically, we assume that the demand follows: \( D = \mu + \varepsilon/\tau \), where \( \mu \) is the mean, \( \varepsilon \) is a standard normal random variable with density \( \phi(\cdot) \) and distribution function \( \Phi(\cdot) \), and \( 1/\tau \) is the standard deviation. This is one of the classical demand settings for the newsvendor problem used in the operations literature (Porteus 2002, Cachon and Terwiesch 2012). Further, we assume that ex ante, \( \tau \) can be either \( \tau_h \) with probability \( \rho \) or \( \tau_l (< \tau_h) \) with probability \( 1 - \rho \). While this distribution is common knowledge, only the manager learns the true realization of \( \tau \).

Here, the demand uncertainty as controlled by \( \tau \) can reflect the firm’s operating environment and management capability, about which the internal managers can often have better knowledge than the external investors. Clearly, a larger \( \tau \) implies less uncertain demand and thus a better prospect. To isolate the effect from the mean and also facilitate analysis and exposition, we assume \( \mu \) is public information. Given this demand setting, the manager decides the regular stocking level \( q \) in the first period which can be procured at \( c \) per unit. The selling price of the products is \( p \) per unit. If the realized demand is greater than the inventory, the unmet portion is backordered and satisfied by more expensive emergent supply at \( c_e \) per unit, where \( c < c_e < p \). If the demand is less than the inventory, the leftover inventory is salvaged at \( s(< c) \) per unit. The inventory and sales information including either salvages or backorders is reported to the investors at the end of the first period. Notice that here we assume a backorder setting so that the true demand can be always learned.

While we expect our qualitative insights can carry over to a lost-sales setting, the equilibrium analysis will become technically challenging because with censored demand information one would need to use the tail distribution to conduct Bayes updating.

The second period represents a long-term horizon. In practice, short-termist behaviors arise often because managers wish to show a better prospect of their firm’s long-term performance. To embed this element in the model, it is sufficient to have a positive association between the firm’s long-term cash flow and the firm’s intrinsic characteristic in the short term. Hence, to facilitate exposition, we use a function \( v(\tau) \) to represent the expected profit the firm will make in the second period and assume \( v(\tau_h) > v(\tau_l) \). Naturally, a currently more efficient firm is likely to also perform better in the future. Let \( K \equiv v(\tau_h) - v(\tau_l) \) which will be the driver of short-termism in our model.\(^1\) This setting is common knowledge to the investors.

\(^1\) \( v(\tau) \) can be characterized by operations similar as in the first period. For instance, if we use a repeated newsvendor setting, then the firm’s second-period problem follows \( \max Q E[pD - c_e(D - q)^+ + s(q - D)^+] - cq \). Clearly, at optimum, the inventory level and the profit are both functions of \( \tau \), and the difference of the profits between the two types is a
Now, we describe the manager’s objective. The manager is risk neutral, but he is interested in not only the firm’s actual value but also the firm’s short-term market value. As discussed in prior literature, such an incentive structure can arise in many circumstances (see, e.g., Ross 1977, Miller and Rock 1985, Stein 1989, Bernheim 1991, Bebchuk and Stole 1993, Dye and Sridhar 2004, Liang and Wen 2007): for instance, the manager may represent both the short-term and long-term shareholders’ interests (in our model, it can be that the short-term shareholders sell their shares at the end of the first period while the long-term shareholders keep their shares to the end of the horizon); the manager may face career concern or the firm may face takeover risks so that to boost the firm’s short-term market value will be beneficial; the manager can also bear short-term liquidity pressure and need to sell a part of his shares; or his compensation package may be simply constructed in a way that it depends on both the firm’s short-term market value and its actual profits. We follow the literature and adopt the common linear objective function. Specifically, the manager’s objective function places a weight $\beta \in (0, 1)$ on the firm’s market value at the end of the first period and another weight $(1 - \beta)$ on the firm’s true value. In section 4, we assume that $\beta$ is exogenous, whereas in section 5, we study the case with endogenous $\beta$. Similar as in prior literature, these weights (once determined) are known to the investors (an extension with $\beta$ being the manager’s private information is provided in Appendix B). It is useful to note that the firm’s market value at the end of the first period depends on the released financial report, from which the investors can learn the sales information as well as the first-period inventory decision. We assume that the firm does not distribute any dividends within the time horizon until at the end of the second period when the firm is liquidated. As such, the firm’s market value is the perceived total profits of the firm in the two periods that the investors can claim at the end of the horizon. Clearly, this model shares the same stylized spirit as those in prior literature; however, it also incorporates some distinct elements to reveal new insights as discussed in the following sections.

The sequence of events is detailed as follows: First, the manager decides the inventory level in the first period. Then, the demand and the sales are realized and reported. The investors value the firm, which determines the manager short-term payoff. After that, the firm’s second-period profit is obtained, the firm is dissolved at its true value which determines the manager’s long-term payoff. In these events, the investors obtain all the information about the firm except for $\tau$.

The standard deviation in the second period can be different from that of the first period as long as they have a positive correlation. Moreover, we can allow a similar game as in the first period to arise in the second period. Then, $v(\tau)$ will be the equilibrium second-period profit derived from backward induction. Likewise, the second period can also be extended to multiple periods with discounting. Lastly, our analysis remains to hold for inventory carryover if the marginal value of the leftover inventory is independent of the inventory quantity, for instance, when the demand in the second period is sufficiently large so that the firm needs to replenish, or there is a spot market where the firm can sell the excess part of the inventory. If the marginal value of the leftover inventory follows a function of the leftover quantity, then the inventory decision of the first period needs to incorporate this function, which can make the analysis intractable even though we do not expect the driving forces of our qualitative insights to change.
4. Operations Signal

This section analyzes the case where the weights ($\beta$ and $(1-\beta)$) in the manager’s objective function are exogenous. We first formulate the investors’ and the manager’s problems.

4.1. Problem Formulation

We focus on pure-strategy equilibria in this paper. At the end of the first period, the investors learn the realization of the firm’s profit, $\pi(q, D) \equiv pD - cq - c_e(D - q)^+ + s(q - D)^+ + s(q - D)^+$, and infer $\tau$ from the reported inventory and sales information. Let $\eta(q, D)$ denote the investors’ posterior belief of $\tau$. Their inference can be either imperfect or perfect. In the former case, $\eta(q, D)$ is a random variable, while, in the latter case, it degenerates to a number. The investors can obtain the firm’s expected value as:

$$P(q, D) = \underbrace{\pi(q, D)}_{\text{First-period profit}} + \underbrace{E_\eta[v(\eta(q, D))]}_{\text{Second-period expected profit}}.$$ (1)

To make the inventory decision, the manager considers the firm’s market value at the end of the first period as well as the true value at the end of the second period. Suppose the manager knows how the investors value the firm. Then, the manager’s decision can be formulated as:

$$\max_{q \in [0, \infty)} \beta E_\varepsilon[P(q, \mu + \varepsilon/\tau_h)] + (1 - \beta) (E_\varepsilon[\pi(q, \mu + \varepsilon/\tau_l)] + v(\tau_l)), \forall i \in \{h, l\}.$$ (2)

Let $q^*(\tau_i)$ denote the manager’s optimal inventory decision. In equilibrium, the investors’ valuation for the firm needs to be consistent with the optimal decision the manager makes. We thus define the following equilibrium concept.

**Definition 1.** A market equilibrium is reached if the following two conditions hold:

(i) The manager’s inventory decision, $q^*(\tau_i)$, is the maximizer of (2).

(ii) The investors’ inference function $\eta(q, D)$ that determines the market value $P(q, D)$ in (1) satisfies: $\Pr(\eta(q^*(\tau_i), D) = \tau_i) = 1$ when $q^*(\tau_h) \neq q^*(\tau_l)$; $\Pr(\eta(q^*(\tau_i), D) = \tau_h) = 1 - \Pr(\eta(q^*(\tau_i), D) = \tau_l) = \frac{\rho \phi((D - \mu)\tau_h)}{\rho \phi((D - \mu)\tau_h) + (1 - \rho) \phi((D - \mu)\tau_l)}$ when $q^*(\tau_h) = q^*(\tau_l)$; and for any $q \neq q^*(\tau_h)$ or $q^*(\tau_l)$, $\Pr(\eta(q, D) = \tau_h) = 1 - \Pr(\eta(q, D) = \tau_l) = 0$.

Definition 1 follows the perfect Bayesian equilibrium concept. Notice that if the manager’s inventory decision is different for different $\tau$, then a separating equilibrium arises, in which the investors infer $\tau$ perfectly; otherwise, a pooling equilibrium occurs, in which the investors use Bayes rule to update their belief of $\tau$ based on the demand realization. Bayes’ rule, however, does not apply to any off-equilibrium action. Definition 1 specifies that for any $q$ that deviates from the equilibrium levels, the investors believe that the firm they face has large uncertainty. This is intuitive because, in our model, it is the manager who, observing large uncertainty, has an incentive to pretend to have small uncertainty through operations distortion.
4.2. The First-Best Benchmark

If the manager is not interested in the firm’s market value or \( \tau \) is publicly known, then the manager’s problem in the first period becomes a classical newsvendor problem, which we use as our first-best benchmark. The objective is to maximize the firm’s expected profit, \( \Pi_i(q) \equiv \mathbb{E}_\varepsilon [\pi(q, \mu + \varepsilon/\tau)] \). Let \( c_u = c_e - c \) and \( c_o = c - s \) denote the underage and overage costs, and \( CR = \frac{c_u}{c_u + c_o} \) be the newsvendor critical ratio. It is not difficult to derive the optimal inventory decision:

\[
q^*(\tau_i) = \mu + \Phi^{-1}(CR)/\tau_i, \forall i \in \{h, l\},
\]

and the corresponding expected profit:

\[
\Pi_i(q^*(\tau_i)) = (p - c)\mu - (c_u + c_o)\phi(\Phi^{-1}(CR))/\tau_i, \forall i \in \{h, l\}.
\]

**Proposition 1.**

(i) \( \Pi^*_{h}(q) > (<)\Pi^*_{l}(q) \) when \( q < (>)\mu \);

(ii) When \( CR > 0.5 \), \( q^*(\tau_l) > q^*(\tau_h) > \mu \); when \( CR < 0.5 \), \( q^*(\tau_l) < q^*(\tau_h) < \mu \); and when \( CR = 0.5 \), \( q^*(\tau_h) = q^*(\tau_l) = \mu \).

We depict the results of Proposition 1 in Figure 1. Proposition 1 provides two important implications. First, it shows that the profit functions of the two types of firms do not satisfy the first-order dominance property. Specifically, their first derivatives change the order at the mean of the distributions. Signaling games typically have multiple equilibria. The intuitive criterion (Cho and Kreps 1987) has been widely used in prior literature as a refinement to obtain a unique separating equilibrium under the single-crossing condition which requires, for any change of the decision variable, there is a monotone ordering of the profit changes among different types (Sobel 2009). Hence, Proposition 1(i) implies that the conventional first-order approach to solve signaling games cannot apply to our setting.

Second, the ordering of the first-best inventory levels switches when the critical ratio crosses 0.5. When \( CR > 0.5 \), a larger uncertainty calls for a higher inventory level, whereas it is the converse when \( CR < 0.5 \). This contrasts to the settings in prior literature where the first-best decisions always have the same ordering. An immediate technical implication is that if one type of firm wants to mimic the other, it may cause either upward or downward distortion in the inventory decision. It also suggests the importance of taking into account the operations characteristics (overage and underage costs) to understand the impacts of managerial short-termism. These intuitions will be useful to derive the equilibria and understand the insights.
4.3. Market Equilibrium

In this subsection, we analyze our model with $\beta > 0$ and $\tau$ being private information of the manager. Notice that the two cases under $CR > 0.5$ and $CR < 0.5$ are symmetric in our model. Therefore, we only show the analysis for the large critical ratio case.

Given $v(\tau_h) > v(\tau_l)$, the manager will always like the investors to believe that the firm’s demand uncertainty is low (i.e., $\tau = \tau_h$). As a result, imitation or signaling may arise. To analyze the manager’s strategy, it is convenient to define:

$$V_{ij}(q) \equiv \beta (E_\varepsilon [\pi(q, \mu + \varepsilon/\tau_i)] + v(\tau_i)) + (1 - \beta) (E_\varepsilon [\pi(q, \mu + \varepsilon/\tau_i)] + v(\tau_i)), \forall i, j \in \{h, l\}. \quad (5)$$

$V_{ij}(q)$ is the expected payoff of the manager when the firm’s true type is $i$ while the investors believe that the firm’s type is $j$. Hence, $V_{ih}(q) - V_{il}(q)$ represents the misvaluation gain (loss) of the manager if the large (small) demand uncertainty firm is considered as the small (large) demand uncertainty firm.

**Lemma 1.** (i) $V_{ij}(q)$ is concave and maximized at $q^*)(\tau_I)$ for $i, j \in \{h, l\}$;
(ii) $V_{ih}(q) - V_{il}(q) = \beta K$ for $i \in \{h, l\}$;
(iii) $V_{ij}(q) > (\leq) V_{ij}(q)$ when $q < (\geq) \mu$ for $j \in \{h, l\}$.

For Lemma 1(i), $V_{ij}(q)$ follows a newsvendor function. Thus, it is concave and has a unique maximizer. Given the composition of $V_{ij}(q)$, its maximizer in fact coincides with the first-best solution $q^*(\tau_I)$. We illustrate the curves of $V_{ij}(q)$ in Figure 2. Lemma 1(ii) shows that under the same inventory level, the gain for the manager if the firm is misvalued when it has a large demand uncertainty equals the loss if the firm has a small demand uncertainty but misvalued.
Furthermore, these gain and loss linearly increase in $\beta$ (recall $K = v(\tau_h) - v(\tau_l)$ which is a constant). Finally, since $q^\ast(\tau_l) > q^\ast(\tau_h) > \mu$ when $CR > 0.5$, the result of Lemma 1(iii) indicates that $V_{h_j}(q)$ will increase faster than $V_{l_j}(q)$ in $q$ when $q$ is below the mean; however, when $q$ exceeds the mean, $V_{h_j}(q)$ will increase slower and then decrease faster than $V_{l_j}(q)$ in $q$. Therefore, together with the result of Lemma 1(i), we can see that for any amount of inventory distortion upward from their corresponding optimums, the payoff of the manager with $\tau_h$ will always decrease more than that with $\tau_l$. In contrast, for a small amount of inventory distortion downward from their corresponding optimums, the payoff of the manager with $\tau_h$ will decrease less than that with $\tau_l$; but, this comparison can again be reversed when the downward distortion is significantly large. In other words, moderate understocking is less costly for the manager with a small demand uncertainty, but overstocking or substantial understocking will be more costly for him. These findings imply that if it is needed to distort the inventory level to credibly signal his information, the manager must understock; however, there might also be scenarios where credibly signaling is not achievable since it can be too costly for the manager, which then gives rise to pooling outcomes.

**Lemma 2.** There exists a unique $q_h(\beta) < q^\ast(\tau_h)$ such that $V_{hh}(q_h(\beta)) = V_{h_l}(q^\ast(\tau_h))$ and a unique $q_l(\beta) < q^\ast(\tau_l)$ such that $V_{lh}(q_l(\beta)) = V_{l_l}(q^\ast(\tau_l))$. Both $q_h(\beta)$ and $q_l(\beta)$ decrease in $\beta$; moreover, there exists $\beta^H_{se}$ such that $q_h(\beta) \leq q_l(\beta)$ when $\beta \leq \beta^H_{se}$ and $q_h(\beta) > q_l(\beta)$ when $\beta > \beta^H_{se}$.

In the above lemma, $q_h(\beta)$ represents the smallest amount of inventory that the manager with $\tau_l$ is willing to stock if, by doing so, he can successfully mislead the market to believe that the firm has a small demand uncertainty. Similarly, $q_h(\beta)$ represents the smallest amount of inventory that the manager with $\tau_h$ is willing to stock to avoid the firm being misvalued. These two threshold inventory levels depend on $\beta$, and they both decrease as $\beta$ increases. This is intuitive because when the interest in the market value increases, the manager learning $\tau_l$ will have more incentive to imitate; likewise, he will also have more incentive to signal with $\tau_h$. Naturally, if $q_h(\beta) \leq q_l(\beta)$, then inventory levels between $q_h(\beta)$ and $q_l(\beta)$ exist such that the manager with $\tau_h$ is willing to stock to credibly reveal his information, while he has no incentive to imitate when $\tau = \tau_l$. Therefore, when this condition holds, a separating equilibrium can arise. However, this condition is not always warranted in our model. In fact, we can find a threshold $\beta^H_{se}$ such that $q_h(\beta) < (>) q_l(\beta)$ when $\beta < (>) \beta^H_{se}$ (see Figure 2 for an illustration). With these intuitions, we derive the following proposition.

**Proposition 2.** When $\beta \leq \beta^H_{se}$, there exists a unique separating equilibrium that survives the intuitive criterion, in which the manager’s inventory decision follows:

$$q^\ast(\tau_i) = \begin{cases} q^\ast(\tau_l), & \text{if } i = l, \\ \min\{q^\ast(\tau_h), q_l(\beta)\}, & \text{if } i = h, \end{cases}$$

(6)

and $q^\ast(\tau_h) = q^\ast(\tau_l)$ iff $\beta$ is no greater than a threshold $\beta^H \in (0, \beta^H_{se})$. When $\beta > \beta^H_{se}$, there does not exist any separating equilibrium.
Proposition 2 affirms the existence of a stable separating equilibrium when $\beta$ is small, in which the manager observing a small demand uncertainty may understock while he always stocks at the first-best level if the demand uncertainty is high. This is aligned with the outcomes of the conventional signaling games studied in prior literature: namely, a distortion by the more efficient type firm in its decision deters imitation of the less efficient type firm, which thus credibly signals the information. However, in contrast to prior literature, when $\beta$ is large, separating equilibrium no longer exists in our study because the difference in the management capability does not offer a perfect ordering of the profit functions. To show whether or not there exists any stable equilibrium in such a scenario is technically challenging, and thus a problem like ours has not been explored in prior literature.

To derive the full equilibrium spectrum, it is helpful to note that pooling equilibria always exist. However, they may not survive the intuitive criterion. In our analysis, we find that pooling equilibria can survive the intuitive criterion only if $\beta$ exceeds a threshold so that the manager’s mimicking incentive when $\tau = \tau_l$ is sufficiently strong and he will forgo signaling when $\tau = \tau_h$. We further find that there might be a continuum of stable pooling equilibria but some will Pareto dominate the rest from the manager’s perspective under both $\tau$ values. We call these equilibria the stable dominant equilibria. In these equilibria, the manager always understocks when $\tau = \tau_l$ but can either overstock or understock when $\tau = \tau_h$. Proposition 3 formally states the results.

**Proposition 3.** There exist two thresholds $\beta_{pe}^H < \hat{\beta}_{pe}^H$. When $\beta \geq \hat{\beta}_{pe}^H$, there are multiple pooling equilibria that survive the intuitive criterion. Among these equilibria, there is one unique pooling equilibrium that Pareto dominates the others when $\beta_{pe}^H < \beta < \hat{\beta}_{pe}^H$ and the equilibrium inventory level $q^*(\tau_h) = q^*(\tau_l) < q^0(\tau_h)$; when $\beta \geq \hat{\beta}_{pe}^H$, there is a continuum of Pareto-dominant pooling equilibria.
(A) Large Critical Ratio \((CR > 0.5)\)

<table>
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<th>(\beta)</th>
<th>Stable Dominant</th>
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<th>Uniqueness</th>
<th>Small Uncertainty</th>
<th>Large Uncertainty</th>
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<tr>
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<tr>
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<td>(understock)</td>
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<tr>
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</table>

(B) Small Critical Ratio \((CR < 0.5)\)

<table>
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<td>Pooling</td>
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<td>overstock</td>
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</tbody>
</table>

Table 1 Summary of the Equilibrium Structure

with \(q^*(\tau_h) \leq q^*(\tau_l) = q^*(\tau_l) < q^*(\tau_l)\). Furthermore, if \(\beta_H^{pe} < \beta_H^{se}\), then the separating equilibrium in (6) is Pareto dominated by at least one pooling equilibrium when \(\beta_H^{pe} \leq \beta < \beta_H^{se}\).

It is worth pointing out that \(\beta_H^{pe}\) in the above proposition can be either smaller or greater than \(\beta_H^{se}\) stated in Proposition 2. In the former case, stable separating and pooling equilibria will coexist, but there is always a stable pooling equilibrium that Pareto dominates the separating equilibrium; in the latter case, there is no equilibrium that survives the intuitive criterion when \(\beta \in (\beta_H^{se}, \beta_H^{pe})\).

Thus, we define \(\hat{\beta}_H^{se} = \min(\beta_H^{se}, \beta_H^{pe})\). We can obtain a symmetric set of thresholds \((\hat{\beta}_L^{se}, \beta_L^{se}, \beta_L^{pe}, \hat{\beta}_L^{pe})\) and corresponding equilibria for the case with \(CR < 0.5\). We present the complete equilibrium spectrum in Table 1.

The above results provide a stark contrast to those in prior literature where only the more efficient type firm may overstock to signal its information, while the less efficient type firm always stocks at the first-best inventory level (Lai et al. 2012, Schmidt et al. 2014). The intuition of the difference boils down to the source of asymmetric information. If the information asymmetry between the manager and the investors is about the demand expectation of the firm, as studied in Lai et al. (2012) and Schmidt et al. (2014), then to stock more inventory will always be less costly for the firm with a large demand expectation than that with a small demand expectation. Hence, overstocking is a credible way for the manager to signal his information. From the investors’ perspective, they only need the inventory information to infer the firm’s type, irrespective of its sales information and the operations characteristics. This is however not true when asymmetric information arises about the firm’s management capability of matching supply with demand which is another important aspect that has been widely investigated in the empirical literature and monitored by investors (Chen 2005, 2007, Hendricks and Singhal 2009, Fisher and Raman 2010, The Associated Press 2006,
Trefis Team 2013). Our study shows that the scenario is much more sophisticated. The manager’s inventory decision depends on the firm’s management capability, the operations characteristics (the service level reflected by the newsvendor critical ratio), as well as the magnitude of his short-term incentive. Clearly, from the investors’ perspective, they cannot interpret the firm’s inventory data as suggested in prior literature. Instead, they need to interpret the inventory information based on the firm’s service level, the magnitude of the manager’s short-term incentive and also the sales information. Specifically, when the manager’s short-term incentive is lower than a threshold, a small inventory distortion will be sufficient to deter imitation, for which the manager with strong management capability will be willing to signal his information with distortion, if needed. Thus, the firm’s inventory levels will be different under different management capabilities. A lower (higher) inventory level can credibly reveal that the firm has a strong management capability if the firm’s service level is large (small). Such an inventory level is closer to the expected demand, which shows the firm’s confidence in matching supply with demand. Differently, when the manager’s short-term incentive is higher than the threshold, the manager, if he wants to signal his information, would need to distort the inventory decision to a large extent which is however more costly under strong management capability (small demand uncertainty) than under weak management capability (large demand uncertainty). Hence, the manager will forgo signaling and stock at the same inventory level under both management capabilities. It is now important for the investors to use the sales information to update the likelihood of the firm’s type in their belief. These implications are useful for the investors to interpret firms’ inventory data to make more accurate valuation.

4.4. Numerical Analysis
To better understand the equilibrium segments and the impacts of managerial short-termism on the firm’s profitability, we conduct an extensive numerical study. We consider 5,000 instances of the following parameters: $p = 1$, $c = 0.4$, $s = 0.2$, $c_e \in \{0.7, 0.75, 0.8, 0.85, 0.9\}$, $\mu = 10$, $\tau_l = 1/3$, $\tau_h \in \{2/5, 1/2, 2/3, 1\}$, $\rho \in \{0.1, 0.3, 0.5, 0.7, 0.9\}$, and $v(\tau_h) - v(\tau_l) \in \{0.02, 0.04, ..., 1\}$. The critical ratios are controlled to be greater than 0.5 in all these instances, and the firm’s expected first-best profit in the first period ranges from 5.29 to 5.81. The difference of the two second-period profits $(v(\tau_h) - v(\tau_l))$ ranges from 0.34 to 18.9 percent of the firm’s expected first-best profit in the first period. Thus, the force that drives short-termism is relatively moderate in our experiments. Based on such a setting, we obtain the $\beta$ thresholds and the equilibrium inventory levels which are demonstrated in Figures 3 and 4.

We discuss several observations from our experiments. First, both the separating and pooling equilibrium regions are significant. Figure 3 provides three representative instances. In particular, the $25^{th}$ to $90^{th}$ percentiles of $\hat{\beta}_{se}^H$ (which divides the separating and pooling equilibrium regions)
range from 0.055 to 0.641 in these instances, with an average at 0.23. In other words, both separating and pooling equilibria can arise quite frequently under a reasonable setting. These observations give an interesting contrast to prior literature (Lai et al. 2012, Schmidt et al. 2014) which has suggested that only the more efficient type firm may overstock while the less efficient firm will make first-best decisions. This is however not true when there is asymmetric information about the firm’s management capability of matching supply with demand. We find that the pooling equilibrium region is wide where the firm with a weak management capability also has distorted inventory levels as demonstrated in Figure 4. The pooling equilibrium region expands as $\rho$ becomes larger (when the prior probability that the firm has a small demand uncertainty increases, the gain for the manager with a large demand uncertainty can gain more from the pooling equilibrium, which makes it more difficult to separate). These observations indicate that managerial short-termism can be concerning for less efficient type firms too.

Second, we find that in our experiments, the $25^{th}$ to $90^{th}$ percentiles of $\beta^H$ (below which no inventory distortion occurs) range from 0.007 to 0.124, with an average at 0.058. $\beta^H$ decreases as $\tau_h$ becomes smaller or the critical ratio becomes closer to 0.5 (the two first-best inventory positions...
Figure 4  The solid curves demonstrate the equilibrium inventory levels. In the multi-pooling equilibrium region, any inventory between the two solid curves can be an equilibrium. The flat dashed and dotted lines are the first-best inventory levels. The other parameters are: $\beta = 0.2$, $p = 1$, $c = 0.4$, $s = 0.2$, $\mu = 10$, and $\tau_l = 1/3$.

will be less distinct and thus mimicking incentive will be stronger). These observations indicate that inventory distortion can occur even under a relatively small $\beta$, and the chance of distortion is higher when the two firm types are more similar or the overage and underage costs turn closer. We further observe that compared to the first best, the firm of the two types suffers an average 1 percent profit loss in the first period, with the 75th to 90th percentiles ranging from 1.3 to 2.8 percent. The loss increases in $\beta$. Hence, managerial short-termism can cause relatively significant losses to the firm.

Finally, we find that the region where no stable equilibrium exists (i.e., $(\hat{\beta}^H_{se}, \hat{\beta}^H_{pe})$) does not occur in our entire experiments. In fact, for such a region to appear, the prior probability ($\rho$) that the firm is the more efficient type needs to be very small (e.g., 0.01) in our experiments. Hence, the main analytical results hold for quite common settings.

5. **Endogenous Short-termism**

Thus far, we have considered the case where the weights $\beta$ and $(1 - \beta)$ in the manager’s payoff function are exogenous, same as in prior literature. However, in practice, there are also scenarios
where the managers’ short-term incentives are endogenous. For instance, managers may receive share-based compensation and they may have certain flexibility to decide both the timing and the proportion of their shares to sell. In this section, we investigate the impacts of such endogenous short-termism. Specifically, we assume $\beta \in [0, \bar{\beta}]$ as the proportion of shares that the manager decides to sell at the end of the first period and $1 - \beta$ as the proportion that the manager will keep to the second period. The upper limit $\bar{\beta}$ reflects the constraint that might be in place. When $\beta$ becomes a decision variable, it may also provide information about $\tau$. Thus, we use $\eta(\beta, q, D)$ to denote the investors’ inference and rewrite the market value as:

$$P(\beta, q, D) = \pi(q, D) + E_{\eta}[v(\eta(\beta, q, D))].$$

The manager’s decision can be reformulated as:

$$\max_{\beta \in [0, \bar{\beta}], q \in [0, \infty]} \beta E_{\varepsilon}[P(\beta, q, \mu + \varepsilon/\tau_i)] + (1 - \beta) (E_{\varepsilon}[\pi(q, \mu + \varepsilon/\tau_i)] + v(\tau_i)), \forall i \in \{h, l\}. \quad (8)$$

Let $(\beta^*(\tau_i), q^*(\tau_i))$ denote the manager’s optimal decision. In equilibrium, it shall be consistent with the investors’ valuation for the firm. We redefine the equilibrium concept as follows.

**Definition 2.** A market equilibrium is reached if the following two conditions hold:

(i) The manager’s decision, $(\beta^*(\tau_i), q^*(\tau_i))$, is the maximizer of (6).

(ii) The investors’ inference function $\eta(\beta, q, D)$ that determines the market value $P(\beta, q, D)$ in (7) satisfies: $Pr(\eta(\beta^*(\tau_i), q^*(\tau_i), D) = \tau_i) = 1$ when $(\beta^*(\tau_h), q^*(\tau_h)) \neq (\beta^*(\tau_l), q^*(\tau_l))$ for one or both terms; $Pr(\eta(\beta^*(\tau_i), q^*(\tau_i), D) = \tau_h) = 1 - Pr(\eta(\beta^*(\tau_i), q^*(\tau_i), D) = \tau_l) = \frac{\rho_{\tau_h} \phi((D - \mu)\tau_h) + (1 - \rho_{\tau_l}) \phi((D - \mu)\tau_l)}{\rho_{\tau_h} \phi((D - \mu)\tau_h) + (1 - \rho_{\tau_l}) \phi((D - \mu)\tau_l)}$ when $(\beta^*(\tau_h), q^*(\tau_h)) = (\beta^*(\tau_l), q^*(\tau_l))$; and for any $(\beta, q) \neq (\beta^*(\tau_i), q^*(\tau_i))$, $Pr(\eta(\beta, q, D) = \tau_h) = 1 - Pr(\eta(\beta, q, D) = \tau_l) = 0$.

This reformulated game becomes a multi-dimensional signaling game where the manager can use either $\beta$ or $q$ to signal his information to the investors. Interestingly, the following proposition reveals that operational efficiency can be reached.

**Proposition 4.** There exists an efficient separating equilibrium, in which $\beta^*(\tau_l) = \bar{\beta}$, $\beta^*(\tau_h) < \bar{\beta}$, and $q^*(\tau_i) = q^*(\tau_l)$ for $i \in \{h, l\}$.

Proposition 4 derives an efficient separating equilibrium, in which the manager always stocks the inventory at the first-best levels. Instead of distorting the inventory decisions, the manager signals his information to the investors by his choice of share selling. Specifically, if the manager learns a high demand uncertainty, he sells all the permitted amount of shares early; in contrast, if the demand uncertainty is low, he keeps some shares to the future. This finding provides an interesting implication. While exogenous short-termism may arise for a variety of reasons (e.g., need of capital, reputation and career concern, threat of takeovers, shareholder pressure; see, Narayanan 1985,
Stein 1988, Holmstrom 1999), there are also scenarios in practice where managers are rewarded shares and have the flexibility of deciding the amount and timing to sell (Krantz 2013, Cahill 2013). In the latter scenarios, a manager’s share selling decision might be able to act as a signal of the firm’s characteristics and resolve the operations distortions that would otherwise occur. Clearly, to have this effect, the transparency of the information of the manager’s share selling decision is critical. It needs to be conveyed to the external investors accurately and timely. From the investors’ perspective, monitoring and understanding such share selling announcements can be very helpful to infer a firm’s characteristics.

It is worth noting that for ease of notation we have assumed in this paper that there is no time discount. Intuitively, if the value of the shares kept to the second period is discounted, then the manager would have a stronger incentive to sell his shares early. We can show that with a time discount an efficient separating equilibrium will exist only if the time discount is no larger than a threshold so that the manager observing a small demand uncertainty is still willing to keep a sufficient amount of his shares to the long term (the detailed analysis is provided in the proof of Proposition 4). This suggests that reducing the time discount (such as alleviating the manager’s pressure on early share selling) can be helpful to prevent operations distortions.

6. Conclusion

This paper studies the impacts of managerial short-termism on the inventory decisions. We find that the source of information asymmetry is critical to understand managers’ short-termist behaviors. In contrast to prior literature, we find that in a setting where asymmetric information arises about the firm’s capability of matching supply with demand, the manager may either over- or understock inventory depending on the firm’s operational characteristics and the magnitude of his short-term incentive. The possible distortion of the inventory level under a large service level can be completely opposite to that under a small one. Finally, we study the scenario where the manager’s short-termism is endogenous. Interestingly, we find that to make the managers’ share selling decisions transparent and monitor such decisions can be helpful to resolve operational distortions that might otherwise occur.

References


Appendix A: Proofs

**Proof of Proposition 1.** Part (i) and (ii) follow directly from the equations in (3) and (4). ■

**Proof of Lemma 1.** (i) By (5), we can rewrite \( V_{ij}(q) = E_c [\pi(q, \mu + \varepsilon / \tau_j)] + \beta \nu(\tau_j) + (1 - \beta) \nu(\tau_j) \). Note that only the first term on the right hand side depends on \( q \). Therefore, \( V_{ij}(q) \) is concave and maximized at \( q = q^*(\tau_i) \) for \( i, j \in \{h, l\} \). (ii) is straightforward. (iii) It is verifiable from (5) that

\[
V_{ij}'(q) = (c_e - s) \Phi(\tau_i(q - \mu)) - c + s. 
\]

Thus, \( V_{ij}'(q) - V_{ij}''(q) = (c_e - s)[\Phi(\tau_h(q - \mu)) - \Phi(\tau_l(q - \mu))] \), which is strictly positive (negative) when \( q < (>) \mu \). ■

**Proof of Lemma 2.** By (5), \( V_{ih}(q) = V_{il}(q) + \beta K \), where \( K = v(\tau_h) - v(\tau_l) \). As \( q \) increases over \( q \leq q^*(\tau_i) \), \( V_{ih}(q) \) strictly increases from \(-\infty\) to \( V_{il}(q^*(\tau_i)) + \beta K \). Hence, there exists a unique value \( q_i(\beta) < q^*(\tau_i) \) such that \( V_{ih}(q_i(\beta)) = V_{il}(q^*(\tau_i)) \), or equivalently \( V_{li}(q_i(\beta)) = V_{il}(q^*(\tau_i)) - \beta K \). Clearly, as \( \beta \) increases, \( V_{ih}(q_i(\beta)) \) decreases and hence \( q_i(\beta) \) decreases (because \( V_{il}(q) \) is an increasing function over \( q < q^*(\tau_i) \)). Similarly, there exists a unique value \( q_h(\beta) < q^*(\tau_h) \) such that \( V_{hh}(q_h(\beta)) = V_{hl}(q^*(\tau_h)) - \beta K \), and \( q_h(\beta) \) decreases in \( \beta \).

The inverse function of \( q_i(\beta) \), denoted by \( \beta_i(q) \), for \( i = l, h \), is

\[
\beta_i(q) = [V_{ih}(q^*(\tau_i)) - V_{il}(q)] / K,
\]

defined over \( q \leq q^*(\tau_i) \). Because \( q^*(\tau_i) < q^*(\tau_l) \), both \( h \) and \( i \) are well defined over \( q \leq q^*(\tau_h) \). Consequently, for \( q \leq q^*(\tau_h) \),

\[
\beta_h(q) - \beta_l(q) = [-V_{hh}'(q) + V_{il}'(q)] / K
= (c_e - s)[\Phi(\tau_l(q - \mu)) - \Phi(\tau_h(q - \mu))] / K,
\]

implying that \( \beta_h(q) - \beta_l(q) > 0 \) for \( q \in (\mu, q^*(\tau_h)) \) and \( \beta_h(q) - \beta_l(q) < 0 \) for \( q < \mu \). This, together with the fact that \( \beta_h(q^*(\tau_h)) = 0 \) and \( \beta_l(q^*(\tau_h)) > 0 \), implies that there exists a threshold \( \tilde{q} < \mu \) such that \( \beta_h(q) < \beta_l(q) \) for \( q \in (\tilde{q}, q^*(\tau_h)) \) and \( \beta_h(q) > \beta_l(q) \) for \( q < \tilde{q} \). Let \( \beta_{se}^H = \beta_h(\tilde{q}) \) (which is also equal to \( \beta_l(\tilde{q}) \)). Note that \( \beta_{se}^H = 0, 1 \). Because both \( \beta(\cdot) \) and its inverse function \( q(\cdot) \) are monotone, \( q_h(\beta) < q_l(\beta) \) for \( \beta \in [0, \beta_{se}^H] \) and \( q_h(\beta) > q_l(\beta) \) for \( \beta > \beta_{se}^H \). ■

**Proof of Proposition 2.** We first prove by contradiction that when \( \beta > \beta_{se}^H \), there does not exist any separating equilibrium. Suppose \( \{q_l, q_h\} \) is a separating equilibrium. Then \( q_l = q^*(\tau_l) \) because otherwise the manager of the inefficient firm is better off by deviating to \( q^*(\tau_l) \). Further, by definition of \( q_h(\beta) \), \( q_h \geq q_h(\beta) \) because otherwise the manager of the efficient firm is better off under \( q^*(\tau_h) \). It follows from Lemma 2 that when \( \beta > \beta_{se}^H \), \( q_h(\beta) > q_l(\beta) \). Hence, \( q_h > q_l(\beta) \), which, by definition of \( q_l(\beta) \) and the fact that \( q_l = q^*(\tau_l) \), suggests that the manager of the inefficient firm is better off by deviating to \( q_h \). This is in contradiction with the assumption that \( \{q_l, q_h\} \) is a separating equilibrium.
Next we prove that when $\beta \leq \beta_{se}^H$, $\{q^*(\tau_i), q^*(\tau_h)\}$ is the unique stable separating equilibrium. The proof is carried out in two steps. First, we show that $\{q^*(\tau_i), q^*(\tau_h)\}$ is a stable separating equilibrium. Second, we show that there does not exist any other stable separating equilibrium.

First, consider the following market belief: $\Pr(\eta(q, D) = \tau_h) = 1$ if $q = q^*(\tau_h)$, and 0 otherwise, where $q^*(\tau_h) = \min\{q^0(\tau_h), q_l(\beta)\}$. It follows from Lemma 2 that the inefficient manager stocks $q^0(\tau_i)$ and has no incentive to stock $q^*(\tau_h)$ to mimic the efficient type; the efficient type also has no incentive to deviate from $q^*(\tau_h)$. The market belief is consistent with the manager’s strategies. Thus, we have proved that $\{q^*(\tau_i), q^*(\tau_h)\}$ is a separating equilibrium. This equilibrium survives the intuitive criterion because we make the following claim: There does not exist any quantity $q$ such that the market believes the manager ordering $q$ is of the efficient type, and that the efficient manager is strictly better off and the inefficient manager is worse off by choosing $q$ relative to their performance under the equilibrium $\{q^*(\tau_i), q^*(\tau_h)\}$. We prove this claim by contradiction. Suppose such a quantity $q$ exists. Recall that $q^*(\tau_h) = \min\{q^0(\tau_h), q_l(\beta)\}$. In order for the efficient manager to be strictly better off under $q$ relative to $q^*(\tau_h)$, $q^*(\tau_h)$ can not be equal to $q^0(\tau_h)$, and further $q > q^*(\tau_h) = q_l(\beta)$ because of the concavity property of the manager’s objective as a function of the order quantity. Thus, we have that $q > q_l(\beta)$, which, together with the definition of $q_l(\beta)$, suggests that the inefficient manager is strictly better off under $q$ relative to $q^*(\tau_i)$. This is in contradiction with the earlier statement that the inefficient manager is worse off by choosing $q$ relative to her performance under the equilibrium. Thus, we have proved the claim, and therefore the equilibrium $\{q^*(\tau_i), q^*(\tau_h)\}$ is stable.

Second, we show that there does not exist any other stable separating equilibrium. We prove by contradiction. Suppose there exists another stable separating equilibrium $\{q_l, q_h\}$, which is not the same as $\{q^*(\tau_i), q^*(\tau_h)\}$. Then $q_l = q^0(\tau_i)$ because otherwise the inefficient manager is better off by deviating to $q^0(\tau_i)$. Because the inefficient manager has no incentive to deviate to $q_h$ at which the market’s belief is equal to the efficient type, $q_h \leq q_l(\beta)$ (see the definition of $q_l(\beta)$). This suggests that the efficient (inefficient) manager has (no) incentive to deviate to $q^*(\tau_i) = \min\{q^0(\tau_h), q_l(\beta)\}$. Thus this separating equilibrium survives the intuitive criterion if and only if $q_h = q^*(\tau_h)$. This is in contradiction with the assumption that $\{q_l, q_h\}$ is not the same as $\{q^*(\tau_i), q^*(\tau_h)\}$.

Finally, as $\beta$ increases from 0 to $\beta_{se}^H$, $q_l(\beta)$ decreases from $q^0(\tau_i)$ to some value that is lower than $q^0(\tau_h)$. Hence, there exists a threshold $\beta^H \in [0, \beta_{se}^H]$ such that $q_l(\beta^H) = q^0(\tau_h)$. This, together with the definition $q^*(\tau_h) = \min\{q^0(\tau_h), q_l(\beta)\}$, implies that the manager who faces a small demand uncertainty stocks at its first-best inventory level, i.e., $q^*(\tau_h) = q^0(\tau_h)$ if $0 < \beta \leq \beta^H$, and it understocks, i.e., $q^*(\tau_h) = q_l(\beta) < q^0(\tau_h)$, if $\beta^H < \beta \leq \beta_{se}^H$.

Next we present and prove four lemmas that will be useful for the proof of Proposition 3. Let $V_{im}(q)$ be the type $i$ manager’s expected profit given that the two types order the same quantity.
q in the first period. Observing the order quantity q, the market’s belief remains the same as its prior belief. However, the market will update its belief after observing the realized demand D in the first period by using Bayes’ rule. That is, Pr(η(q, D) = τ_i) = \frac{\rho \beta K \phi((D - \mu) / \tau_i)}{\rho \beta K \phi((D - \mu) / \tau_i) + (1 - \rho) \phi((D - \mu) / \tau_i)}. Let \rho_i = E_D Pr(\eta(q, D) = \tau_i), where D = \mu + \epsilon / \tau_i. Note that \rho_i > \rho_i and V_{lm}(q) - V_{li}(q) = \beta K_i where K_i = \rho_i(v(\tau_i) - v(\tau_i)). Intuitively, \beta K_i is the type i manager’s profit gain from the market valuation due to the change of the market belief from the inefficient type to the mixed type.

Define \overline{q}_i(\beta) = \{q < q^*(\tau_i) | V_{li}(q) = V_{li}(q^*(\tau_i)) - \beta K_i\} and \overline{q}_i(\beta) = \{q > q^*(\tau_i) | V_{li}(q) = V_{li}(q^*(\tau_i)) - \beta K_i\}. Intuitively, \overline{q}_i(\beta) (\overline{q}_i(\beta)) represents the smallest (largest) amount of inventory that the manager with \tau_i is willing to stock, if, by doing that, he can successfully lead the market to believe that the manager is of a mixed type. Define q(\beta) = \max\{\overline{q}_i(\beta), \overline{q}_i(\beta)\} and \overline{q}(\beta) = \min\{\overline{q}_i(\beta), \overline{q}_i(\beta)\}.

**Lemma A1.** The complete set of pooling equilibria is given by \{q(\beta), \overline{q}(\beta)\}.

**Proof of Lemma A1.** For any q \in [q(\beta), \overline{q}(\beta)], we can construct the following market belief: if the manager orders q, then the market’s belief remains the same as its prior; otherwise, the market believes the manager is of the inefficient type. Under such a belief, the type i’s best possible deviation is to choose q^*(\tau_i) and its expected profit is V_{li}(q^*(\tau_i)), which is less than V_{li}(q) + \beta K_i because q \in [q(\beta), \overline{q}(\beta)]. This, together with the fact that V_{lm}(q) = V_{li}(q) + \beta K_i, implies the type i manager is worse off by deviating from q. Hence, any q \in [q(\beta), \overline{q}(\beta)] is a pooling equilibrium.

Next we prove by contradiction that if q \notin [q(\beta), \overline{q}(\beta)], then q can not be a pooling equilibrium. If q < q(\beta), then either q < q(\beta) or q < \overline{q}(\beta). In the former case, V_{lm}(q) = V_{li}(q) + \beta K_i < V_{li}(q^*(\tau_i)), implying that the inefficient manager is better off by deviating to q^*(\tau_i), while in the latter case, V_{lm}(q) = V_{li}(q) + \beta K_i < V_{li}(q^*(\tau_i)), implying that the efficient manager is better off by deviating to q^*(\tau_i). Thus, in either case, we find contradiction with the definition of pooling equilibrium. Using the similar arguments, we can show that if q > \overline{q}(\beta), q can not be the pooling equilibrium quantity.

**Lemma A2.** The pooling equilibria exist if and only if \beta \geq \beta_1 where \beta_1 = \{\beta | \overline{q}_i(\beta) = q(\beta)\}.

**Proof of Lemma A2.** Note that the interval [q(\beta), \overline{q}(\beta)] is nonempty if and only if \overline{q}_i(\beta) > q(\beta). Because \overline{q}_i(\beta) increases and q(\beta) decreases in \beta, there exists a threshold \beta_1 = \{\beta | \overline{q}_i(\beta) = q(\beta)\} such that the interval [q(\beta), \overline{q}(\beta)] is nonempty if and only if \beta \geq \beta_1.

**Lemma A3.** For any pooling equilibrium q, there exists a unique \hat{\beta}(q) such that q is a stable pooling equilibrium if \beta \geq \hat{\beta}(q).

**Proof of Lemma A3.** Recall that a pooling equilibrium q does not survive the Intuitive Criterion if and only if there exists q’ such that the efficient (inefficient) type is better (worse) off by deviating from q to q’ assuming that the market belief under q’ is of the efficient type. Therefore, to show
that $q$ is stable, we need to show that such $q'$ does not exist. We first show that such $q'$ does not exist in the range $q' > q$. We then show that such $q'$ does not exist in the range $q' < q$ if $\beta \geq \hat{\beta}(q)$.

Define $q_i^n(\beta) = \overline{\beta} > q | V_i(q) = V_i(q) - \beta \hat{K}_i \} \text{ where } \hat{K}_i = K - K_i$. Intuitively, $\beta \hat{K}_i$ is the type $i$ manager’s profit gain from the market valuation due to the change of the market belief from the mixed type to the efficient type. Thus $q_i^n(\beta)$ is the threshold quantity above which the type $i$ manager can not make a profitable deviation from $q$ even if such a deviation will alter the market belief from being the mixed type to being the efficient type. Next we show that $q_i^n(\beta) > q_i^m(\beta)$ for any $\beta$, which implies that there does not exist any $q' > q$ such that the efficient (inefficient) type is better (worse) off by deviating from $q$ to $q'$ assuming that the market belief under $q'$ is of the efficient type. We consider two cases. Case 1. $q \geq \mu$. It follows from Lemma 1(iii) that $V_i'(q) > V_i'(q)$ for $q > \mu$. This, together with the fact that $\hat{K}_h < \hat{K}_i$, implies that $q_i^n(\beta) > q_i^m(\beta)$ for any $\beta$. Case 2. $q < \mu$. Let $q' = \{ q > \mu | V_i(q) = V_i(q) \}$. This is well defined because $V_i(\cdot)$ is concave and its maximizer is larger than $\mu$. If $q' > q_h$, then by following the proof in Case 1, we have $q_i^n(\beta) > q_i^m(\beta)$ for any $\beta$. Thus, it remains to prove $q' > q_h$. It is verifiable that $V_i'(q + \Delta) < V_i'(q - \Delta)$ for any $\Delta > 0$. This, together with the definition of $q_h$, implies that $q_h - \mu > q - \mu$, i.e., $q_h$ is further away from $\mu$ relative to $q$. Recall that $V_i'(q) = V_i'(q) = (c_e - s) / \overline{\beta}(\sigma(q - \mu)) - \overline{\beta}(\sigma(q - \mu))$, implying that $V_i'(q + \Delta) - V_i'(q - \Delta) = V_i'(q - \Delta) - V_i'(q + \Delta)$ for any $\Delta > 0$ and $V_i'(q + \Delta) - V_i'(q - \Delta)$ decreases as $\Delta$ increases. This, together with the fact that $q_h - \mu > q_h - q_i(q) = V_i'(q) < V_i'(q) - V_i'(q)$. Recall that $V_i'(q) = V_i'(q)$, we have $V_i'(q) > V_i'(q) = V_i'(q)$, implying that $q' > q_h$ by the concavity property of $V_i(\cdot)$.

Next we show that there exists a threshold $\hat{\beta}(q)$ so that such $q'$ does not exist in the range $q' < q$ if $\beta \geq \hat{\beta}(q)$. Define $q_i^n(\beta) = \{ q < q | V_i(q) = V_i(q) - \beta \hat{K}_i \}$. It suffices to show that $q_i^n(\beta) < q_i^m(\beta)$ for $\beta \geq \hat{\beta}(q)$. The inverse function of $q_i^n(\beta)$ is $\beta(q_i^n) = [V_i(q) - V_i(q)] / \hat{K}_i$.

We first show that $\beta_i^n(q_i^n) - \beta_i^m(q_i^n)$ changes sign only once from negative to positive. Note that $\beta_i^n(q_i^n) - \beta_i^m(q_i^n) = [(c_e - s) / \overline{\beta}(\sigma(q_i^n - \mu)) - c + s] / \hat{K}_i - [(c_e - s) / \overline{\beta}(\sigma(q_i^n - \mu)) - c + s] / \hat{K}_i$. Note that $\beta_i^n(q_i^n) - \beta_i^m(q_i^n) = (c_e - s) / \overline{\beta}(\sigma(q_i^n - \mu)) / \hat{K}_i - (c_e - s) / \overline{\beta}(\sigma(q_i^n - \mu)) / \hat{K}_i$. It follows from the property of the standard normal density function $\phi(\cdot)$ that the equation $\beta_i^n(q_i^n) - \beta_i^m(q_i^n) = 0$ has only one solution on each side of $\mu$; say $q_L$ and $q_H$. Hence, when $q_i^n < q_L$, $\beta_i^n(q_i^n) - \beta_i^m(q_i^n) < 0$, when $q_L < q_i^n < q_H$, $\beta_i^n(q_i^n) - \beta_i^m(q_i^n) > 0$, and when $q_i^n > q_H$, $\beta_i^n(q_i^n) - \beta_i^m(q_i^n) < 0$. This implies that $\beta_i^n(q_i^n) - \beta_i^m(q_i^n)$ decreases when $q_i^n < q_L$, increases in between $q_L$ and $q_H$, and then decreases when $q_i^n > q_H$. Note that $\beta_i^n(q_i^n) - \beta_i^m(q_i^n) < 0$ when $q_i^n \rightarrow -\infty$ and $\beta_i^n(q_i^n) - \beta_i^m(q_i^n) > 0$ when $q_i^n \rightarrow +\infty$. Therefore, we have shown that $\beta_i^n(q_i^n) - \beta_i^m(q_i^n)$ changes sign only once, and $\beta_i^n(q_i^n) < \beta_i^m(q_i^n)$ when $q_i^n$ is small and $\beta_i^n(q_i^n) > \beta_i^m(q_i^n)$ when $q_i^n$ is large.

The result that $\beta_i^n(q_i^n) - \beta_i^m(q_i^n)$ changes sign only once from negative to positive implies that there exists a threshold $\hat{q} \leq q$ such that $\beta_i(\hat{q}) - \beta_l(\hat{q}) = 0$. Further when $q_i^n < \hat{q}$, $\beta_i(q_i^n) - \beta_l(q_i^n) > 0$
and when \( q > q^u > q^v \), \( \beta_h(q^u) - \beta_l(q^v) < 0 \). This, together with the fact that \( \beta_i(\cdot) \) is inverse function of \( q^i(\cdot) \) and the fact that \( \beta_i(\cdot) \) is a decreasing function for \( i = h, l \), implies that \( q^i_h(\beta) > q^i_l(\beta) \) for \( \beta > \beta_h(q) \). Define \( \tilde{\beta}(q) = \beta_h(q) \). Therefore, we conclude that if \( \beta > \tilde{\beta}(q) \), then \( q \) is stable.\]

**Lemma A4.** For any \( q^H > q^L \), \( \tilde{\beta}(q^H) > \tilde{\beta}(q^L) \).

**Proof of Lemma A4.** Let 
\[
\beta_i^H(q) = \frac{V_i(q^H) - V_i(q)}{\widehat{K}_i} \\
\beta_i^H(q) = \frac{V_h(q^H) - V_h(q)}{\widehat{K}_h}
\]
for \( q \leq q^H \), and let 
\[
\beta_i^L(q) = \frac{V_i(q^L) - V_i(q)}{\widehat{K}_i} \\
\beta_i^L(q) = \frac{V_h(q^L) - V_h(q)}{\widehat{K}_h}
\]
for \( q \leq q^L \).

It follows from the arguments in the proof Lemma A3 that for \( i = H, L \), there exists a threshold \( \bar{q}(q^i) \leq q^i \) such that \( \beta_h^i(q) - \beta_l^i(q) = 0 \) when \( q = \bar{q}(q^i) \). Note that the two functions \( \beta^H_i(q) - \beta^L_i(q) \) and \( \beta^L_i(q) - \beta^H_i(q) \) differ only by a fixed constant for any \( q \), i.e., \( \beta^H_i(q) - \beta^L_i(q) - [\beta^H_i(q) - \beta^L_i(q)] \) is a negative constant value for any \( q \). Hence, the fact that \( q^H > q^L \) implies that \( \tilde{\beta}(q^H) < \tilde{\beta}(q^L) \). This, together with the definition of \( \tilde{\beta}(\cdot) \) in the proof of Lemma A3, implies that \( \tilde{\beta}(q^H) > \tilde{\beta}(q^L) \).\]

**Proof of Proposition 3.** Because \( \bar{q}(\beta) \) decreases in \( \beta \) and \( \tilde{\beta}(q) \) increases in \( q \), \( \tilde{\beta}(q(\beta)) \) decreases in \( \beta \). Thus, we can define \( \beta_2 = \min\{\beta \in (0, 1]|\tilde{\beta}(q(\beta)) \leq \beta\} \). Further, there exists a threshold \( \bar{q}(\beta) \in [\bar{q}(\beta), \bar{q}(\beta)] \) such that for all \( q \in [\bar{q}(\beta), \tilde{\beta}(\beta)] \), we have \( \tilde{\beta}(q) \leq \beta \). By Lemmas A1 and A3, we have that the complete set of stable pooling equilibria is the interval \( [\bar{q}(\beta), \tilde{\beta}(\beta)] \). Such an interval exists if and only if \( \beta \geq \beta^H_{pe} \), where \( \beta^H_{pe} = \max\{\beta_1, \beta_2\} \).

If \( 0 < \beta \leq \min\{\beta^H_{se}, \beta^H_{pe}\} \), then there does not exist any stable pooling equilibrium. Thus, the stable separating equilibrium characterized in Proposition 2 is the only stable equilibrium of this game.

If \( \min\{\beta^H_{se}, \beta^H_{pe}\} < \beta \leq \beta^H_{pe} \), then there does not exist any stable equilibrium. Recall that \( \beta^H_{pe} = \max\{\beta_1, \beta_2\} \). Note that \( \beta^H_{se} \geq \beta_2 \), which follows because when \( \beta = \beta^H_{pe} \), \( q(\beta) \) is stable. Further, \( \beta^H_{se} \) is independent of \( \rho \) whereas \( \beta_1 \) decreases in \( \rho \), implying that there exists a threshold \( \rho^H \) such that \( \beta^H_{se} < \beta_1 \) if and only if \( 0 < \rho < \rho^H \). Therefore, \( \beta^H_{se} < \beta^H_{pe} \) and thus this case exists if and only if \( 0 < \rho < \rho^H \).

Let \( \beta_3 \) be the threshold such that \( \tilde{q}(\beta_3) = q^v(\tau_h) \). Let \( \beta^H_{pe} = \max\{\beta_1, \beta_3\} \). Because \( \tilde{q}(\cdot) \) is an increasing function, \( \tilde{q}(\beta) < q^v(\tau_h) \) for \( \beta^H_{pe} < \beta < \beta^H_{pe} \). Hence, \( \tilde{q}(\beta) \) Pareto dominates any other pooling equilibrium \( q \in [\bar{q}(\beta), \tilde{q}(\beta)] \). Further, \( \tilde{q}(\beta) \) also Pareto dominates the unique separating equilibrium
(if it exists) for the following reasons: the inefficient type is better off by choosing the pooling equilibrium \( q(\beta) \) than by choosing the first-best quantity \( q^*(\tau_i) \), which is in turn better off than mimicking the efficient type by choosing the efficient type’s quantity at the unique separating equilibrium; the efficient type is also better off at the pooling equilibrium \( q(\beta) \) than choosing the efficient type’s quantity at the unique separating equilibrium because otherwise the pooling equilibrium \( q(\beta) \) cannot be stable. Therefore, there exists a unique Pareto dominant and stable pooling equilibrium, at which \( q^*(\tau_h) = q^*(\tau_l) < q^*(\tau_i) \).

When \( \hat{\beta}_{pe} \leq \beta \), then \( q(\beta) \geq q^*(\tau_i) \) and thus the set of Pareto dominant and stable pooling equilibria is given by \( [q^*(\tau_h), \min(q(\beta), q^*(\tau_i))] \), in which \( q^*(\tau_h) \leq q^*(\tau_h) = q^*(\tau_l) \leq q^*(\tau_i) \).

Proof of Proposition 4. Before proving this proposition, we want to note that when the manager’s short-term incentive is exogenous, whether or not to add a time discount over the second-period payoff does affect any of the revealed results qualitatively. This is because the magnitude of the manager’s short-term incentive can be easily rescaled to incorporate the time discount, given that the manager’s payoff function follows a linear form of the first- and second-period payoffs. However, when the manager’s short-term incentive is endogenous, the time discount can be an important factor. For simplicity and consistency, we do not introduce the time discount in the paper. But, in this proof, we add the time discount and show the result for a more general setting. That is, we assume here the manager’s payoff in the second period is discounted by \( \delta \in [0,1] \) while his payoff in the first period is not discounted. Thus, the manager’s decision can be reformulated as:

\[
\max_{\beta \in [0,\beta], \eta \in (0,\infty)} \beta E_c [P(\beta, q, \mu + \epsilon/\tau_i)] + (1 - \beta) \delta (E_c [\pi(q, \mu + \epsilon/\tau_i)] + v(\tau_i)), \ \forall i \in \{h, l\}.
\]

Now, we present a more general version of Proposition 4:

There is a threshold \( \delta \) such that if and only if \( \delta \geq \delta \), an efficient separating equilibrium exists, in which \( \beta^*(\tau_l) = \overline{\beta}, \beta^*(\tau_h) < \overline{\beta}, \) and \( q^*(\tau_i) = q^*(\tau_i) \) for \( i \in \{h, l\} \).

We start by deriving the necessary condition for the existence of an efficient separating equilibrium, and then prove that the necessary condition is also sufficient.

Suppose \( \{(\beta^*(\tau_l), q^*(\tau_l)), (\beta^*(\tau_h), q^*(\tau_h))\} \) is an efficient separating equilibrium. By its definition, \( q^*(\tau_i) = q^*(\tau_i) \) for \( i \in \{l, h\} \). Further, \( \beta^*(\tau_l) = \overline{\beta} \) because otherwise the manager with \( \tau = \tau_l \) is better off by deviating to \( \overline{\beta} \).

Let \( \pi_{ij} = E_c [\pi(q^*(\tau_j), \mu + \epsilon/\tau_l)] \) which is the firm’s first-period profit if the manager with \( \tau = \tau_l \) chooses the stocking quantity \( q^*(\tau_j) \), for \( i, j \in \{l, h\} \). To ensure that the manager with \( \tau = \tau_l \) has no incentive to deviate to \( (\beta^*(\tau_h), q^*(\tau_h)) \), the following condition must be satisfied:

\[
\overline{\beta}(\pi_{ll} + v(\tau_i)) + (1 - \overline{\beta})\delta(\pi_{lh} + v(\tau_l)) \geq \beta^*(\tau_h)(\pi_{lh} + v(\tau_h)) + (1 - \beta^*(\tau_h))\delta(\pi_{lh} + v(\tau_l)). \quad (IC)\]
To ensure that the manager with \( \tau = \tau_l \) has no incentive to deviate to \((\overline{\beta}, q^*(\tau_l))\) even if doing so will lead the market to believe \( \tau = \tau_l \), the following condition must be satisfied:

\[
\beta^*(\tau_l)(\pi_{hh} + v(\tau_l)) + (1 - \beta^*(\tau_l))\delta(\pi_{hh} + v(\tau_l)) \geq \overline{\beta}(\pi_{hh} + v(\tau_l)) + (1 - \overline{\beta})\delta(\pi_{hh} + v(\tau_l)). \tag{ICh}
\]

Note that the condition (ICl) imposes an upper bound on \( \beta^*(\tau_l) \), which is
\[
\frac{(\overline{\beta} + (1 - \overline{\beta})\delta(\pi_{hh} + v(\tau_l)) - \delta(\pi_{hh} + v(\tau_l)))}{(1 - \delta)(\pi_{hh} + v(\tau_l)) - \delta(\pi_{hh} + v(\tau_l))} \quad \text{and the condition (ICh) imposes a lower bound on } \beta^*(\tau_l), \text{ which is}
\]
\[
\frac{\overline{\beta}(1 - \delta)(\pi_{hh} + v(\tau_l)) - \delta(\pi_{hh} + v(\tau_l))}{(1 - \delta)(\pi_{hh} + v(\tau_l)) - \delta(\pi_{hh} + v(\tau_l))}.
\]

The existence of \( \beta^*(\tau_l) \) implies that the upper bound must be no lower than the lower bound, i.e.,
\[
\frac{\overline{\beta}(1 - \delta)(\pi_{hh} + v(\tau_l)) - \delta(\pi_{hh} + v(\tau_l))}{(1 - \delta)(\pi_{hh} + v(\tau_l)) - \delta(\pi_{hh} + v(\tau_l))} \leq \frac{(\overline{\beta} + (1 - \overline{\beta})\delta(\pi_{hh} + v(\tau_l)))}{(1 - \delta)(\pi_{hh} + v(\tau_l)) - \delta(\pi_{hh} + v(\tau_l))},
\]

which after some algebra manipulations, is equivalent to
\[
(1 - \delta)\left[\delta(1 - \overline{\beta})A + \frac{\delta}{1 - \delta}(v(\tau_l) - v(\tau_l))^2 - \overline{\beta}B\right] \geq 0
\]

where \( A = (\pi_{hl} - \pi_{lh})(\pi_{hh} + v(\tau_l)) \) and \( B = \pi_{hh}\pi_{hl} + \pi_{lh} v(\tau_l) + \pi_{hh} v(\tau_l) - \pi_{hh}\pi_{hl} - \pi_{hh} v(\tau_l) - \pi_{hl} v(\tau_l) \).

Because the terms inside the square brackets increase in \( \delta \), there exists a threshold \( \delta \) such that the above necessary condition is equivalent to \( \delta \geq \overline{\delta} \). The necessary condition \( \delta \geq \overline{\delta} \) implies that if \( \delta < \overline{\delta} \) then there does not exist any efficient separating equilibrium.

Next we prove that the necessary condition \( \delta \geq \overline{\delta} \) is also sufficient for the existence of an efficient separating equilibrium. Suppose \( \delta \geq \overline{\delta} \). It follows from the prior analysis that \( \delta \geq \overline{\delta} \) implies that there exists a continuous range of \( \beta^*(\tau_l) \) that satisfies both (ICl) and (ICh). In particular, we set \( \beta^*(\tau_l) \) to be the upper bound, i.e.,
\[
\beta^*(\tau_l) = \frac{(\overline{\beta} + (1 - \overline{\beta})\delta(\pi_{hh} + v(\tau_l)) - \delta(\pi_{hh} + v(\tau_l)))}{(1 - \delta)(\pi_{hh} + v(\tau_l)) - \delta(\pi_{hh} + v(\tau_l))}.
\]

Consider the following market belief: \( \Pr(q|\beta, q, D) = \tau_l = 1 \) if \( \beta = \beta^*(\tau_l) \) and \( q = q^*(\tau_l) \), and 0 otherwise. Note that the manager with \( \tau = \tau_l \) is always better off under \((\overline{\beta}, q^*(\tau_l))\) than under any other solution that leads the market to believe \( \tau = \tau_l \). Under the constructed market belief, the only solution under which the market belief is not \( \tau_l \) is \((\beta^*(\tau_l), q^*(\tau_l))\). By (ICl), the manager with \( \tau = \tau_l \) prefers \((\overline{\beta}, q^*(\tau_l))\) to \((\beta^*(\tau_l), q^*(\tau_l))\). Hence, \((\overline{\beta}, q^*(\tau_l))\) is the best response of the manager with \( \tau = \tau_l \).

It remains to prove that the best response of the manager with \( \tau = \tau_l \) is \((\beta^*(\tau_l), q^*(\tau_l))\). If he deviates from \((\beta^*(\tau_l), q^*(\tau_l))\), then under the constructed market belief, the market will always believe \( \tau = \tau_l \), implying that the manager’s best deviation is to set the stocking quantity to be \( q^*(\tau_l) \) and to set \( \beta \) to be \( \arg\max_{\beta \in [0,1]} \beta(\pi_{hh} + v(\tau_l)) + (1 - \beta)\delta(\pi_{hh} + v(\tau_l)) \). Because the maximander is linear in \( \beta \), the maximizer must be either 0 or \( \overline{\beta} \). If the maximizer is \( \overline{\beta} \), then (ICh) implies that the manager with \( \tau = \tau_l \) is worse off by deviating from \((\beta^*(\tau_l), q^*(\tau_l))\) to \((\overline{\beta}, q^*(\tau_l))\). If the maximizer is 0, then the expected payoff of the manager with \( \tau = \tau_h \) under \((\beta^*(\tau_h), q^*(\tau_h))\) is \( \beta^*(\tau_l)(\pi_{hh} + v(\tau_l)) + (1 - \beta^*(\tau_l))\delta(\pi_{hh} + v(\tau_l)) \), which is higher than his expected payoff \( \delta(\pi_{hh} + v(\tau_l)) \) under \((0, q^*(\tau_h))\). Hence, \((\beta^*(\tau_l), q^*(\tau_l))\) is the best response of the manager with \( \tau = \tau_l \).
Appendix B: Asymmetric Information about the Manager’s Short-term Interest in Market Value

Here, we relax the assumption that the manager’s short-term interest $\beta$ is common knowledge to the investors. Specifically, consider the signaling game where $\beta$ may take one of the two values $\beta_h$ and $\beta_l$ ($\beta_h \geq \beta_l$), and the investors do not know the exact value of the manager’s short-term interest $\beta$. Depending on the values of the short-term interest $\beta$ and the forecasting accuracy $\tau$, the manager’s type can have the following four possibilities: $(\beta_h, \tau_h)$, $(\beta_l, \tau_l)$, $(\beta_h, \tau_l)$, and $(\beta_l, \tau_h)$, which we call type $hh$, $hl$, $lh$, $ll$ respectively.

We focus on the scenario where the critical ratio is higher than 0.5. Lemmas 1 and 2 continue to hold under this setting. In particular, there exists $\beta_{se}^H \in (0, 1]$ such that $q_h(\beta) \leq q_l(\beta)$ when $\beta \leq \beta_{se}^H$ and $q_l(\beta) > q_l(\beta)$ when $\beta > \beta_{se}^H$. This implies that if $\beta_i \leq \beta_{se}^H$, then the type $ih$ can successfully separate himself from the type $il$ by choosing the quantity $\min\{q^o(\tau_i), q_i(\beta_i)\}$, for $i = h, l$. However, the condition $\beta_i \leq \beta_{se}^H$ alone can not guarantee that the manager can successfully signal his forecast capability to the investors because this condition does not ensure that the type $hl$ has no incentive to mimic the type $lh$.

To this end, we need to impose a new condition: $\beta_h \leq \Gamma(\beta_l)$ where $\Gamma(\beta_l) = \{\beta | q_l(\beta) = q_h(\beta)\}$, which is well defined over $\beta_l \leq \beta_{se}^H$. The condition $\beta_h \leq \Gamma(\beta_l)$ ensures that the type $hl$ is worse off by choosing the quantity $\min\{q^o(\tau_l), q_l(\beta_l)\}$ even if doing so allows him to be perceived as the efficient type than choosing his first-best quantity $q^o(\tau_l)$. Note that $\Gamma(\beta_l) \leq \beta_{se}^H$. Thus, if (i) $\beta_i \leq \beta_{se}^H$ and (ii) $\beta_h \leq \Gamma(\beta_l)$, then the two efficient types $hh$ and $lh$ can successfully signal their more accurate forecast capability by choosing the quantity $\min\{q^o(\tau_h), q_h(\beta_h)\}$, and the other two inefficient types $hl$ and $ll$ are better off by staying at their first-best quantity $q^o(\tau_i)$. Specifically, condition (i) and (ii) ensure that the type $ll$ and the type $hl$ have no incentive to deviate from $q^o(\tau_i)$, respectively.

Following the arguments in the proof of Proposition 3, we can show that under condition (i) and (ii), the game has a unique stable separating equilibrium at which both the two efficient types stock quantity $\min\{q^o(\tau_h), q_h(\beta_h)\}$ and the other two inefficient types stock their first-best quantity $q^o(\tau_i)$. Further, if any one of the two conditions is violated, then the game does not have any stable separating equilibrium. In such a case, stable partial separating or pooling equilibria may arise.

To see why stable pooling equilibria may arise, we consider the scenario where $\beta_l \geq \beta_{pc}^H$. Proposition 4 implies that there exists at least one stable pooling equilibrium, say $q_{pc}$, for the type $lh$ and the type $ll$, i.e., none of them has incentive to deviate to his first-best quantity at which the market belief is of the inefficient type. The fact that $\beta_h \geq \beta_l$ implies that the type $hh$ and $hl$ benefit more than the type $lh$ and $ll$ by altering the market belief of his type from the inefficient type to the mixed type. This further implies that both the type $hh$ and the type $hl$ have no incentive to deviate from the stable pooling equilibrium $q_{pc}$ to the first-best quantities. Further, it follows from
Lemma A3 that if $q_{pe}$ is stable for the type $lh$ and the type $ll$ with short-term interest $\beta_l$, then $q_{pe}$ is also stable for the type $hh$ and the type $hl$ with a higher short-term interest $\beta_h$. Therefore, if $\beta_l \geq \beta_{pe}^H$, the stable pooling equilibria described in Proposition 4 and associated with $\beta_l$ is also the stable pooling equilibrium in this extended model.

To conclude, we have established the same result as in the base model that the stable equilibrium of this setting with asymmetric information about $\beta$ is separating equilibrium if both $\beta_h$ and $\beta_l$ are small (i.e., $\beta_l \leq \beta_{hc}^H$ and $\beta_h \leq \Gamma(\beta_l)$). Otherwise, there is no stable separating equilibrium, and multiple stable pooling (or partial separating) equilibria may arise. In particular, we have shown that if $\beta_l \geq \beta_{pe}^H$, then the stable pooling equilibria characterized in Proposition 4 remain to be the stable pooling equilibria in this extension. Therefore, most of the qualitative insights (e.g., the equilibrium can be either separating or pooling, stocking quantity can be distorted upward or downward for both the efficient and inefficient type, insights on the value of input and output information) obtained under the base model continue to hold when the assumption of $\beta$ being common information is relaxed.