American Economic Review: Papers & Proceedings 2008, 98:2, 64–67
http://www.aeaweb.org/articles.php?doi=10.1257/aer.98.2.64

Variable Rare Disasters: A Tractable Theory of Ten Puzzles in Macro-Finance

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Thomas A. Rietz (1988) proposes that the possibility of rare disasters (such as economic depressions or wars) is a major determinant of asset risk premia. Robert J. Barro (2006) shows that, internationally, disasters have been sufficiently frequent and large enough to make the Rietz proposal viable, and they account for a high equity premium. The Rietz-Barro hypothesis is almost always formulated with a constant intensity of disasters, which is fine to analyze the mean equity premium and risk-free rate, but then cannot account for some key features of asset markets, such as volatile price-dividend ratios for stocks, volatile bond risk premia, and return predictability.

In Gabaix (2008), some results of which I report here, I formulate a variable-intensity version of the rare disasters hypothesis, and investigate the impact of time-varying disaster intensity on the prices of stocks, bonds, options, and the predictability of their returns. A later companion paper (Emmanuel Farhi and Gabaix 2008) studies exchange rates, and proposes a theory of the forward premium puzzle.

In the model, the value loss suffered by assets in a disaster varies both in the cross-section and over time. Hence, assets have time-varying risk premia, which generates volatile prices. When agents are optimistic about stocks (and think they will do reasonably well during disasters), stock premia are low, stock valuations are high, and future returns are low. But this optimism changes over time. This yields a time-varying risk premium which generates a time-varying price-dividend ratio, and "excess volatility" of stock prices. It also makes stock returns predictable via measures such as the dividend-price ratio.

This way, the following patterns are not puzzles, but they emerge naturally when the model has two shocks: one real, for stocks, and one nominal, for bonds: (a) equity premium puzzle; (b) risk-free rate-puzzle; (c) excess volatility puzzle; (d) predictability of aggregate stock market returns with price-dividend ratios; (e) value premium; (f) often greater explanatory power of characteristics than covariances for asset returns; (g) upward-sloping nominal yield curve; (h) a steep yield curve predicts high bond excess returns and a fall in long term rates; (i) corporate bond spread puzzle; (j) high price of deep out-of-the-money puts.

The model is presented as rational, but it can be also viewed as a tractable way to model less orthodox things, such as "time-varying perception of risk," or "investor sentiment."[1]

I conclude that the rare disaster hypothesis, augmented by a time-varying intensity of disaster, is a workable additional paradigm for macro-finance. Indeed, within the class of rational, representative-agents frameworks, it may be viewed as a third workable paradigm, along the external habit model of John Campbell and John Cochrane (1999), and the long-run risk model of Ravi Bansal and Amir Yaron (2004). Using the linearity-generating processes of Gabaix (2007), the model is very tractable, and all prices are in closed form.

I. Model Setup

A. *Macroeconomic Environment*

The environment follows Rietz (1988), Francis Longstaff and Monika Piazzesi (2004), and Barro (2006), and adds a stochastic probability and intensity of disasters. I consider an

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[1] It could also be meshed with models of slow adjustment of consumption, such as Gabaix and David I. Laibson (2002).

endowment economy, with C_t as the consumption endowment, and a representative agent with utility: $E_0[\sum_{t=0}^{\infty} e^{-\delta t}((C_t^{1-\gamma} - 1)/(1 - \gamma))]$. Hence, the pricing kernel is $M_t = e^{-\delta t} C_t^{-\gamma}$. The price at t of an asset yielding a stream of dividend of $(D_s)_{s \geq t}$ is: $P_t = E_t[\sum_{s \geq t} M_s D_s]/M_t$.

At each period $t + 1$, a disaster may happen, with a probability p_t. If a disaster does not happen, $C_{t+1}/C_t = e^g$, where g is the normal-time growth rate of the economy. If a disaster happens, $C_{t+1}/C_t = e^g B_{t+1}$, with $B_{t+1} > 0$. For instance, if $B_{t+1} = 0.7$, consumption falls by 30 percent. As the pricing kernel is $M_t = e^{-\delta t} C_t^{-\gamma}$, call $R = \delta + \gamma g$ the associated discount rate; $M_{t+1}/M_t = e^{-R}$ if there is no disaster, and $e^{-R} B_{t+1}^{-\gamma}$ otherwise.[2]

I consider a typical stock i, which is a claim on a stream of dividends $(D_{it})_{t \geq 0}$, which follows:

$$\text{(1)} \quad \frac{D_{i,t+1}}{D_{it}} = \begin{cases} e^{g_{iD}}(1 + \varepsilon_{i,t+1}^D) \\ \quad \text{if there is no disaster at } t+1 \\ e^{g_{iD}}(1 + \varepsilon_{i,t+1}^D) F_{i,t+1} \\ \quad \text{if there is a disaster at } t+1 \end{cases}$$

where $\varepsilon_{i,t+1}^D > -1$ is a mean zero shock that is independent of whether there is a disaster. This shock matters only for the calibration of dividend volatility. In normal times, D_{it} grows at a rate g_{iD}. But, if there is a disaster, the dividend of the asset is partially wiped out, in a way captured by $F_{i,t+1} \geq 0$. I introduce the notion of "resilience" H_{it} of asset i,

$$\text{(2)} \quad H_{it} = p_t E_t[B_{t+1}^{-\gamma} F_{i,t+1} - 1 \mid \text{There is a disaster at } t+1].$$

When the asset is expected to do well in a disaster (high $F_{i,t+1}$), H_{it} is high—investors are optimistic about the asset. In the cross section, an asset with higher resilience H_{it} is safer.

I specify the dynamics of H_{it} directly, rather than by specifying the individual components, $p_t, B_{t+1}, F_{i,t+1}$. I postulate that H_{it} hovers around a central value H_{i*} according to the linearity-generating process (Gabaix 2007):

(3) *Linearity-generating twist:*

$$H_{i,t+1} = H_{i*} + \frac{1 + H_{i*}}{1 + H_{it}} e^{-\phi_H}(H_{it} - H_{i*}) + \varepsilon_{i,t+1}^H$$

where H_{i*} is the steady-state resilience, $E_t \varepsilon_{i,t+1}^H = 0$, and $\varepsilon_{i,t+1}^H, \varepsilon_{i,t+1}^D$, and whether there is a disaster, are uncorrelated variables. Equation (3) means that H_{it} mean-reverts to H_{i*} at speed ϕ_H, but as a "twisted" autoregressive process. The "twist" term $(1 + H_{i*})/(1 + H_{it})$ is close to one, and the process is an AR(1) up to second-order terms. It makes the process very tractable. It is best thought as economically innocuous, and simply an analytical convenience, which will make prices linear in the factors, and independent of the functional form of the noise.

II. Equilibrium Asset Prices and Returns

The next proposition calculates stock prices and expected returns.

PROPOSITION 1: *Defining* $r_i = R - g_D - \ln(1 + H_{i*})$, *the price of a stock i is, in the limit of short time periods,*

$$\text{(4)} \qquad P_{it} = \frac{D_{it}}{r_i}\left(1 + \frac{H_{it} - H_{i*}}{r_i + \phi_H}\right).$$

The expected return on stock i, conditional on no disasters, is

$$\text{(5)} \qquad r_{it}^e = R - H_{it}.$$

The risk-free rate is $r_{f,t} = R - p_t E_t[B_{t+1}^{-\gamma} - 1]$.

The key innovation in Proposition 1 is that it derives the stock price with a *stochastic* resilience H_{it}. As expected, more resilient stocks (higher H_{it}) have a higher price, and a lower expected return. As resilience H_{it} is volatile, price-dividend ratios are volatile, in a way that is potentially independent of innovations to dividends. Hence, the model generates a time-varying equity premium, hence "excess volatility," i.e., volatility of the stocks unrelated to

[2] Gabaix et al. (2003, 2006) offer a theory of the origins of high-frequency fat-tail risk in finance.

Table 1—Predicting Returns with Price-Dividend Ratios

	Data			Model	
Horizon	Slope	S.E.	R^2	Slope	R^2
1	−0.11	(0.053)	0.04	−0.18	0.05
4	−0.42	(0.18)	0.12	−0.65	0.15
8	−0.85	(0.20)	0.29	−1.07	0.22

Notes. Predictive regression $E_t[r_{t \to t+T}] = \alpha_T + \beta_T \ln(P_t/D_t)$, at horizon T (annual frequency). The data are Campbell (2003, Table 10 and 11B)'s calculation for the USA 1891–1997.

cash-flow news. Hence, the model generates predictability in stock returns, given by (5).

Derivation of (4).—I drop the i, call $\hat{H}_t = H_t - H_*$, and observe

$$E_t\left[\frac{M_{t+1}D_{t+1}}{M_t D_t}\right] = e^{-R+g_D}\{\underbrace{(1-p_t)\cdot 1}_{\text{No disaster term}} + \underbrace{p_t \cdot E_t[B_{t+1}^{-\gamma} F_{t+1}]}_{\text{Disaster term}}\}$$

$$= e^{-R+g_D}(1 + H_t).$$

For the rest, the rigorous proof is in Gabaix (2008). Here I just plug and verify the functional form $P_t = D_t(a + b\hat{H}_t)$. The price must satisfy $P_t = D_t + E[M_{t+1}P_{t+1}/M_t]$, i.e., for all $\hat{H}_t$,

$$a + b\hat{H}_t = 1 + E_t\left[\frac{M_{t+1}D_{t+1}}{M_t D_t}(a + b\hat{H}_{t+1})\right]$$

$$= 1 + E_t\left[\frac{M_{t+1}D_{t+1}}{M_t D_t}\right](a + bE_t[\hat{H}_{t+1}])$$

$$= 1 + e^{-R+g_D}(1 + H_t) \times \left(a + be^{-\phi_H}\frac{1 + H_*}{1 + H_t}\hat{H}_t\right)$$

$$= 1 + e^{-R+g_D}\left(a(1 + H_* + \hat{H}_t) + be^{-\phi_H}(1 + H_*)\hat{H}_t\right)$$

where the "twist" term in (3) was crucial to have a right-hand side that is affine in $\hat{H}_t$. Solving for a and b, we get $a = 1 + e^{-r_i}a$, $b = e^{-R+g_D}a + be^{-r_i-\phi_H}$, and:

$$P_t = \frac{D_t}{1 - e^{-r_i}}\left(1 + \frac{e^{-R+g_D}\hat{H}_t}{1 - e^{-r_i-\phi_H}}\right)$$

whose continuous time limit is (4).

III. Calibration and Return Predictability

A. *Postulates*

I use yearly units, and take $\gamma = 4$, $\delta = 0.045$, $g_c = 0.025$. The probability and conditional intensity of macro disasters are constant, and are taken from Barro (2006). The disaster probability $p = 0.017$. I take $E[B^{-\gamma}] = 10$, for a certain-equivalent recovery rate of consumption is $E[B^{-\gamma}]^{-1/\gamma} = 0.56$. Disaster events get a weight that is ten times their risk-neutral weight (see Martin Weitzman (2007) for an even higher weight).

I set $\sigma_D = 0.11$, and $\phi_H = 0.15$. The variability of the recovery rate F_t is $\sigma_F = 0.11$.

B. *Implications*

Average Levels.—The normal-times equity premium is $R^e - r_f = p(E[B^{-\gamma}](1 - F_*)) = 5.3$ percent. The unconditional equity premium (i.e., in long samples that include disasters) is 4.5 percent. The central price/dividend ratio is $P/D = 18$ (equation (4) when $\hat{H}_t = 0$).

"Excess" Volatility.—As stock market resilience H_{it} is volatile, stock market prices, and P/D ratios, are volatile. Volatile resilience yields a volatility of the log of the price/dividend ratio equal to 9.2 percent. The volatility of equity returns is 14.3 percent. I conclude that the model can quantitatively account for an "excess" volatility of stocks.

Predictability.—When $\hat{H}_t$ is high, (5) implies that the risk premium is low, and P/D ratios (4) are high. Hence, when the market-wide P/D ratio is low, stock market returns will be higher than usual. Consider the predictive regression of the return from holding the stock from t to $t + T$, $r_{t \to t+T}$, on the initial P/D ratio: $r_{t \to t+T} = \alpha_T + \beta_T \ln(P_t/D_t) + \text{noise}$.

In the model, for small holding horizon T's, the slope is, to the leading order: $\beta_T = (r_i + \phi_H)T$.

Using the paper's calibration of $r_i = 5$ percent and $\phi_H = 15$ percent, this predicts a slope coefficient $\beta_1 = 0.20$ at a one-year horizon, in line with the return predictability literature. Gabaix (2008) derives similar expressions, and analytics of predictability regressions, for bonds and options.

IV. Conclusion

This paper sketches some results from Gabaix (2008), which yields a tractable way to handle a time-varying intensity of rare disasters, and derives its impact on stock and bond prices, and its implication for time-varying risk premia and asset predictability. I was surprised by how many finance puzzles could be understood with the lens of such a simple model. Given that the model is quite simple to state and to solve in closed form, it can serve as a simple benchmark for various questions in macroeconomics and finance. On the other hand, the model does suffer from several limitations, and suggests several questions for future research.

First, it would be interesting to examine empirically the predictions of the model on the joint behavior of stocks, bond, and options.

Second, linking updates of resiliences to development in the real economy (e.g., political upheaval, state of the business cycle) is important.

Third, a companion paper (Farhi and Gabaix 2008) suggests that various puzzles in international macroeconomics (including the forward premium puzzle) can be accounted for in an international version of the present framework.

Fourth, I used an endowment economy. In ongoing work, I show how to embed the rare disasters idea in a production economy, in a way that does not at all change its business cycle properties, but changes its asset pricing properties. It is possible, because the disaster framework uses the same isoelastic utility as the rest of macroeconomics. Hence, the rare disasters idea may bring us closer to the long-sought goal of a joint, tractable framework for macroeconomics and finance.

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