Procurement from Multiple Suppliers using Options

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This paper studies the design of procurement mechanisms when a manufacturer purchases from multiple suppliers that are heterogeneous in their production costs and salvage values. The option to purchase from different suppliers helps the manufacturer hedge against demand risk. We first consider the case when the set of suppliers’ cost parameters is public information. We propose a simple mechanism that achieves efficiency and allows the manufacturer to extract full surplus from the suppliers.

We then study the case when the suppliers’ cost parameters are privately observed. We establish the revenue equivalence between several widely used procurement mechanisms. We also discuss the optimal mechanism and characterize sufficient conditions under which the manufacturer can extract full surplus.

Subject classifications: games/group decisions: bidding/auctions, noncooperative

Area of review: Manufacturing, Service and Supply Chain Operations

1. Introduction

Firms are beginning to critically analyze their procurement decisions and hedge demand risk by sourcing using supply options. For example, Hewlett-Packard (HP) adopted a new portfolio approach for the procurement of electronic components that emphasized diversification of sources for parts (Billington (2002)). Previously, HP had used similar contracts, usually long-term, to procure from a small set of suppliers. In the new portfolio approach, HP uses a combination of long-term contracts, option contracts, and spot market procurement to satisfy its requirements. In this expanded procurement setting, one of the critical decisions for a supply chain manager is to decide not only how much, but also which “portion” of the demand should be procured from different suppliers. The academic literature in Economics and Operations Management addresses some aspects of such procurement decisions. Martinez-de-Albeniz and Simchi-Levi (2005) provided the general framework for constructing a portfolio of option contracts. In a companion paper (Martinez-de-Albeniz and Simchi-Levi (2003)) they developed a multi-attribute competition model where the firm and its supplier negotiate using two attributes of the option contracts, which they term as the reservation price and the execution price. An implicit assumption in their paper is that the suppliers’ cost parameters are common knowledge.
In this paper, we relax this assumption. Similar to Martinez-de-Albeniz and Simchi-Levi (2003), we consider a supply chain in which multiple suppliers bid for a single manufacturer’s stochastic demand. The manufacturer’s price is assumed to be fixed (determined exogenously). Suppliers are heterogeneous with regard to their (linear) production costs and salvage values of leftover inventory. The contracts considered in this paper specify a wholesale price and a buy-back price. That is, suppliers are obliged to produce the required quantity prior to the realization of the actual demand and ship the entire quantity to the manufacturer. After demand materializes, suppliers buy back unsold inventory. In our setting, the wholesale and buy-back prices are used to model the physical transactions of products between the manufacturer and the suppliers. This is appropriate when the supply lead time is long and products have short life cycles. Therefore, the delivery of product must be made before the selling season. In Section 5.1, we show that this contract form is equivalent to the option contracts in Martinez-de-Albeniz and Simchi-Levi (2005).

Initially, we assume that each supplier is of a distinct “type” determined by his production cost and salvage value. When the cost parameters, i.e., types, of the suppliers are common knowledge (public information), we show that it is optimal to partition the support of the demand distribution into mutually exclusive intervals such that each interval is awarded to at most one supplier and each supplier is awarded at most one interval.

A critical challenge for the manufacturer, however, is to design an optimal procurement mechanism when she is unable to identify the suppliers by their types. Due to lack of information, she has to induce the suppliers to truthfully reveal their cost parameters (types). How should the manufacturer design an optimal procurement mechanism? How should she allocate demand to these suppliers? Can standard auction mechanisms achieve efficiency? Does the form of the procurement mechanism affect the demand allocation? Can the manufacturer extract all the surplus from the suppliers? These are the research questions of this paper.

As a first deviation from the full information case, we assume that the set of supplier types is common knowledge, i.e., it is known that there is a supplier associated with each type in the set. However, the manufacturer is unable to match a supplier with his type. We first study a second-price sealed-bid (Vickrey) auction, since it allows efficient allocation and the suppliers are willing to bid truthfully. Under a second-price auction the manufacturer selects the winner based on her expected payoff given the suppliers’ bids, but the contract terms follow the bid submitted by the loser. We find that it is in the manufacturer’s interest to bundle the entire demand into a single auction. However, the single-bundle auction not only minimizes the total supply chain surplus but also yields the minimum expected payoff for both suppliers! Therefore, efficiency is not achieved in
the second-price auction. We also show that the loss is particularly significant when the suppliers have very different cost parameters, since the second-price auction forces the manufacturer to leave most of the profit to the winning supplier.

We then propose a simple mechanism that alleviates this problem. Using this mechanism, the manufacturer is not only able to elicit the information but also extract “full surplus” from the suppliers. We show that truth-telling is the unique dominance solvable equilibrium. (A collection of strategies for suppliers is called a dominance solvable equilibrium if it survives after iterative eliminations of dominated strategies, see Fudenberg and Tirole (1994).)

Finally, we relax the assumption that the supplier types are common knowledge. We first evaluate several widely-used procurement mechanisms, namely menu auction, scoring rule auction, request-for-quote (RFQ) mechanism, and VCG (Vickrey-Clarke-Groves) mechanism (described in Section 6). Under these mechanisms, the manufacturer has to decide whether to auction the demand as one object, or auction different portions of the demand separately. Moreover, since the suppliers’ cost parameters are privately observed, another important decision for the manufacturer is whether this partition of the demand is made prior to the auction or after the suppliers submit the bids/reports.

To answer the first question, we compare these procurement mechanisms under the assumption that the demand partition is pre-determined. We show that all these mechanisms yield the same expected payoff to the manufacturer when the suppliers are risk neutral. This suggests that the partitioning decision is independent of the choice of the procurement mechanism. We then discuss mechanisms that create the partition depending on all suppliers’ reports (endogenous partition). We first show that with two suppliers, the VCG mechanism with endogenous partition outperforms all mechanisms that use pre-determined partition when one supplier dominates the other. Nevertheless, the general performance ranking is ambiguous when each supplier has a cost advantage. We then introduce an optimal mechanism and characterize sufficient conditions under which the manufacturer can extract full surplus. We also discuss how the optimal mechanism can be implemented.

The rest of this paper is organized as follows. In Section 2, we review related literature. Section 3 introduces the model setting, and Section 4 discusses the efficient allocation. In Section 5, we investigate how bundling can be used in the second-price auction to minimize the manufacturer’s cost, and propose a mechanism under which the manufacturer extracts full surplus from the suppliers. In Section 6, we generalize our results to the case with incomplete information. Finally, we conclude the paper in Section 7. All proofs are in the Appendix.
2. Literature Review

The contractual arrangements between a single supplier and a manufacturer have been well studied, see, e.g., Cachon (2003) for an excellent review. Dasgupta and Spulber (1989) and Chen (2004) consider the design of procurement auctions where multiple suppliers compete for the manufacturer’s supply contract. These two papers assume that suppliers’ costs are privately known to the suppliers and are not accessible to the manufacturer. They mainly focus on the case of single-sourcing and propose different auction mechanisms, namely, the price (entry fee) auction in Chen (2004) and the quantity auction in Dasgupta and Spulber (1989). Seshadri and Zemel (2003) generalize many of these assumptions on the cost and information structure, and show that auditing can be used by the manufacturer to capture a significant portion of the expected supply chain profit. There are some papers that elaborate the benefit of sourcing from multiple suppliers (as in this paper). Martinez-de-Albeniz and Simchi-Levi (2005) assume that the cost parameters are known to the public. They propose to use option contracts to reduce inventory risk, where an option contract specifies a reservation price, a pre-committed reserve capacity and an execution price. Option or option-like contracts are also proposed by other researchers in Operations Management such as Anupindi and Bassok (1999), Burnetas and Ritchken (2007), and Wu et al. (2002). Other surveys of related literature can be found in the book by Tayur et al. (1998) and Elmaghraby (2000).

The profitability of bundling multiple objects has been discussed in the Economics literature. For example, Palfrey (1983) considers a multiple-object auction with additive valuations. He establishes that, in either the first-price or second-price auctions, the auctioneer tends to bundle (partition) the objects when the number of bidders is small (large). In the second-price auction with two bidders, he proves that a single bundle outperforms all other auctions with arbitrary partitions with regard to the auctioneer’s expected revenue. This result is extended in (Krishna 2002, Proposition 16.3) to include the case when bidders possess non-additive valuations. Krishna and Tranæs (2002) focus on a multi-unit first-price auction with general valuations in a complete information setting. They prove that bundling continues to be revenue-maximizing as the number of bidders is small. Similar results have been reported in Chakraborty (1999) in a two-object setting. We show that the intuition carries over to the current paper for the multi-sourcing problem. Anton and Yao (1992) study the first-price procurement auction with two competing suppliers where the auctioneer allocates objects after seeing the suppliers’ bids. Each supplier submits a two-component bid: one indicating the payment she requests if she becomes the sole source, and the other indicating the payment when the supply contract is split between these two suppliers (i.e., the dual source). See (Krishna 2002, Chapter 16) for more discussions on bundling issues in auction designs.
The possibility of full surplus extraction was first investigated in Mirelees (1974) and Myerson (1981). They independently illustrate, via specific examples, the idea of utilizing publicly observable signals that are correlated to an agent’s private information. These observable signals can be the ex-post public information or other agents’ reports when their types are correlated. Cremer and McLean (1985, 1988) formally identify conditions for full surplus extraction in an auction setting. McAfee and Reny (1992) extend the discussion to incorporate continuous type space and find a continuous analogue of Cremer and McLean (1988)’s results. Mezzetti (2005) considers an interdependent-value setting where the agents are able to observe the equilibrium payoffs. Obara (2006) allows the agents to exert effort that affects the probability distribution over types. Riordan and Sappington (1988) consider the contract design problem and show that an ex post public signal helps the principal to extract full surplus.

A few researchers investigate whether it is possible to achieve full surplus extraction as a unique equilibrium outcome when the aggregate distribution among agents’ preferences is known. Piketty (1993) considers the problem of optimal taxation design when the government knows the probability distribution of taxpayers’ characteristics. He shows that full surplus extraction can be achieved as a unique, dominance solvable, equilibrium if the government uses a tax schedule that depends on the entire income profile of society. A similar idea is used by Hamilton and Slutsky (2004) in which they study the monopoly pricing problem with a finite set of consumers. They show that the monopolist can extract full surplus even if consumers can opt not to purchase from the monopolist – the individual rationality condition. In this paper, we show that when the set of supplier cost parameters is publicly known, full surplus extraction can be achieved as a unique equilibrium. We also identify new conditions for full surplus extraction when cost parameters are privately observed.

3. Model

Consider a supply chain where there are $n$ suppliers. Suppliers are labeled as $i \in \{1, \ldots, n\} \equiv I$. The suppliers sell perfectly substitutable products to a single manufacturer, $M$. We assume that supplier $i$ has infinite production capacity and is endowed with a constant marginal cost of production $c_i$. Furthermore, we also assume that the supplier can salvage any leftover inventory, at the end of the selling season, at a constant salvage value $s_i$, where $s_i \leq c_i$ so that the supplier’s profit is bounded. The vector of cost and salvage value tuples is defined as $(c, s) \equiv ((c_1, s_1), \ldots, (c_n, s_n))$. The vector also defines the set of supplier “types.” Production is assumed to have a long lead time and hence must be completed before the demand is realized. The manufacturer faces a stochastic demand (with a cumulative distribution function $F$ and associated density $f$ over a support $K \subseteq R^+$). For simplicity, we assume that the common retail price, $r$, is fixed and independent of the demand.
We first restrict ourselves to the case when the set of supplier types is known but the manufacturer is unable to identify individual suppliers by their types (otherwise social optimality can be achieved by assigning the suppliers in the most cost-efficient manner). This informational scenario coincides with Martinez-de-Albeniz and Simchi-Levi (2005). Later, in Section 6, we relax this informational assumption and consider extensions under incomplete information.

Facing stochastic demand, the manufacturer wants to purchase appropriate quantities from these suppliers at the minimum expected total cost. Her goal is to design an appropriate mechanism to achieve this cost minimization. The realization of demand is observable, and verifiable, and hence contractible. Moreover, we use surplus and profit interchangeably.

4. Efficiency

In this section we consider the efficiency issue. We first define the following terms: demand interval, demand partition, and efficient allocation. For notational ease, we assume that all intervals are half-open and include the right boundary point. Only the lowest interval is a closed interval. Since the demand follows a continuous distribution, the choice of inclusion of endpoints does not affect the supply chain payoff.

**Definition 1.** A demand interval, denoted by $(Q_1, Q_2]$, is defined by a contiguous portion of the support $K$ of the demand distribution that lies in the half open set defined by the two end points $Q_1$ and $Q_2$.

**Definition 2.** A partition of the demand distribution is a collection of disjoint (non-intersecting) demand intervals that cover the entire support of the demand distribution.

**Definition 3.** An allocation is said to be efficient if it maximizes the expected supply chain profit over all feasible assignments of the demand intervals to suppliers. An allocation is said to be constrained efficient for a given demand partition if it maximizes the expected supply chain profit in each demand interval of the partition.

We first derive the marginal benefit of production for the supply chain. To this end, it is helpful to consider an infinitesimal portion of the demand, i.e., a demand interval $(Q, Q + \Delta Q]$, assigned to a specific supplier $i$. The marginal expected supply chain profit from this interval, $(Q, Q + \Delta Q]$, if assigned to supplier $i$, is approximately $[(r_i - s_i)[1 - F(Q)] - (c_i - s_i)]\Delta Q$. Removing the common part, $r_i[1 - F(Q)]\Delta Q$, we know that in an efficient allocation supplier $i$ is assigned to interval $(Q, Q + \Delta Q]$ if and only if $i \in \arg\max_{i\in I}[s_iF(Q) - c_i]\Delta Q$. Ties between suppliers are broken arbitrarily.

Observe that the marginal benefit could be negative for a sufficiently large $Q$ (since $c_i \geq s_i$ and $F(Q) \leq 1$). Moreover, notice that the marginal benefit, for a fixed supplier $i$, is strictly decreasing
in $Q$ due to the strict monotonicity of $F(\cdot)$. Thus, for each supplier $i$, there exists a unique $\bar{Q}_i$ beyond which the marginal benefit of assigning supplier $i$ is strictly negative. For each supplier $i$, the quantity $\bar{Q}_i$ can be defined as

$$\bar{Q}_i = \sup\{Q : (r - s_i)(1 - F(Q)) - (c_i - s_i) = 0, \ Q \geq 0\}. \quad (1)$$

Using similar arguments, we observe that there exists an unique threshold quantity $\bar{Q}^* = \max_{i \in I} \bar{Q}_i$ beyond which the marginal benefit, derived from any supplier, is strictly negative.

First, in Theorem 1, we develop an algorithmic procedure that efficiently allocates demand to a set of suppliers.

**Theorem 1.** Supplier $l$ is said to be the most cost efficient at $Q$ if and only if $l \in \arg \max_{i \in I} [s_iF(Q) - c_i]$. Let $J(Q)$ be the set of suppliers who have ever been the most cost efficient at a certain point before the demand exceeds $Q$. The efficient allocation can be obtained via the following algorithm.

Initialization Set $Q_0 = 0$.

Begin loop For all $k = 1, \ldots, n$

1. Compute $Q_{ka} = \inf\{q : \max_{u \in J(Q_{k-1})} [s_u(1 - F(q)) - c_u] = \max_{l \in J(Q_{k-1})} [s_l(1 - F(q)) - c_l]\};$
2. Compute $Q_{kb} = \sup_{j \in J(Q_{k-1})} \bar{Q}_j;$
3. Compute $Q_k = \min(Q_{ka}, Q_{kb});$
4. Assign interval $(Q_{k-1}, Q_k]$ to supplier $i$ if and only if $\{i\} = J(Q_k) \setminus J(Q_{k-1})$.

End loop

In an efficient allocation, it is not necessary that each supplier is awarded a demand interval. Note that $Q_{ka}$ is the threshold value at which the maximum marginal benefit generated from suppliers, that have been selected up to this point, equals the maximum marginal benefit from the assigned supplier. $Q_{kb}$ is the critical value beyond which all suppliers that have been selected so far find it unprofitable to produce. The demand in interval $(Q_{k-1}, Q_k]$ is assigned to the supplier that has the highest marginal benefit in the interior of $(Q_{k-1}, Q_k]$. Such an allocation guarantees that the most efficient supplier is assigned to each demand interval and no inefficient production takes place.

Furthermore, in an efficient allocation, the demand can be divided into mutually exclusive intervals such that exactly one supplier is awarded an interval and each supplier is awarded at most one interval. The proof, included in the proof of Theorem 1 in the Appendix, is by construction. We formally state this result as a corollary of Theorem 1.

**Corollary 1.** In an efficient allocation either a supplier is awarded exactly one contiguous interval of the demand or none at all.\(^1\)
Having characterized the efficient allocation, we can then define “full surplus extraction” under common knowledge about types:

**Definition 4.** A mechanism $\Gamma$ is said to achieve full surplus extraction under common knowledge about types if

1. The allocation is efficient, i.e., if supplier $i$ is awarded $(Q_{k-1}, Q_k]$, then $i = i_k \equiv \arg\max_{i \in I} \{s_i F(q) - c_i, \forall q \in (Q_{k-1}, Q_k]\}$, $\forall i \in I$.
2. Under $\Gamma$, the expected payoff of each supplier is zero.

If full surplus extraction is achieved, the expected supply chain profit is maximized due to the efficient allocation. Moreover, since no supplier receives surplus, the entire profit is captured by the manufacturer.

### 5. Mechanism Design under Common Knowledge about Types

In this section, we first focus our attention on the design of an auction mechanism to elicit truthful revelation of the suppliers’ cost parameters. We also examine whether the manufacturer has an incentive to auction separate demand intervals. We demonstrate that the manufacturer cannot extract full surplus from suppliers using the auction mechanism. Following this, we propose a simple mechanism using which the manufacturer is able to extract full surplus from the suppliers.

#### 5.1. Bundling issues in second-price auctions

**5.1.1. The auction mechanism.** The manufacturer first has to decide whether she will partition the demand distribution into a number of intervals, and if so, how to create the partition. Since the demand distribution includes no point mass, without loss of generality, we assume that the manufacturer partitions the demand into a finite number of subsets, each of which is the union of pairwise disjoint intervals.

Let a partition of the demand distribution be defined as $\mathcal{K} = \bigcup_{k=1}^{K} (a_{k-1}, a_k]$ ($a_0 = 0$ and $a_K = \infty$ if the demand has an infinite support). Before we proceed we define a “demand object” as follows:

**Definition 5.** A demand object is a collection (subset) of disjoint demand intervals.

Now suppose the manufacturer creates $J$ disjoint “demand objects,” $\{A_1, \ldots, A_J\}$, by combining subsets of demand intervals and holding $J$ auctions with these demand objects. Therefore, $A_j \subseteq \bigcup_{k=1}^{K} (a_{k-1}, a_k]$ and $A_i \cap A_j = \emptyset$, $\forall i \neq j$. Denote $k \in A_j$ if the demand interval $(a_{k-1}, a_k]$ is included in auction $j$. With some abuse of notation the auction and its corresponding demand object are both labeled as $A_j$. In auction $A_j$, the suppliers submit bids, $\{(w_{ij}, b_{ij})\}$’s, where $w_{ij}$ and $b_{ij}$ are the wholesale and buy-back prices respectively specified by supplier $i$. 
Once the manufacturer decides the partition and the demand objects, these auctions take place simultaneously. As we will see later, since a supplier’s expected payoffs from these auctions are additively separable, all our results hold even if these auctions are held sequentially. We restrict our attention to the second-price sealed-bid auction that works as follows. For each auction, \( A_j \), each supplier, \( i \), submits his bid that is composed of a wholesale price, \( w_{ij} \), and a buy-back price, \( b_{ij} \). If the contract is signed with the wholesale price \( w_{ij} \) and the buy back price \( b_{ij} \), then the manufacturer’s expected payoff, from object \( A_j \), is

\[
\pi_j^M = rS_j - w_{ij} \sum_{k \in A_j} (a_k - a_{k-1}) + b_{ij} \left[ \sum_{k \in A_j} (a_k - a_{k-1}) - S_j \right],
\]

where \( S_j = \sum_{k \in A_j} \int_{a_{k-1}}^{a_k} (y - a_{k-1}) f(y) dy + (a_k - a_{k-1}) \int_{a_k}^{\infty} f(y) dy \) is the expected sales for demand object \( A_j \). Since the expected sales \( S_j \) is independent of the contract parameters, \( w_{ij} \) and \( b_{ij} \), the first term \( rS_j \) can be ignored while choosing the suppliers. Therefore, the manufacturer, in auction \( A_j \), awards the contract to a supplier that is \( \arg \max_{i \in I} \{-w_{ij} \sum_{k \in A_j} (a_k - a_{k-1}) + b_{ij} \left[ \sum_{k \in A_j} (a_k - a_{k-1}) - S_j \right] \} \). Ties are broken randomly. For each \( A_j \), no transaction takes place if the score of the second highest bid is negative.

Under the second-price auction, the contract parameters follow from the wholesale and buy-back prices submitted by the second highest bidder. This mechanism is labelled as a “beauty contest” in Asker and Cantillon (2007). It corresponds to the case in which each supplier submits one set of contract terms and the price, and the submitted contract terms are used for the actual transaction. Since the demand realization is contractible (by assumption), the contract imposes enforcement compliance on both the manufacturer and the supplier. After the auctions, the manufacturer orders from the supplier and the supplier delivers the agreed quantities. Once demand is realized, all unsold products (inventory), corresponding to the particular auction, are returned to the supplier at the buy-back price specified in the auction.²

We adopt the dominant strategy equilibrium as the solution concept (Fudenberg and Tirole (1994)). In a dominant strategy equilibrium, each supplier possesses an optimal (dominant) strategy that is independent of other suppliers’ bids. Due to the specific design of the second-price auction, bidding their true cost parameters is the dominant strategy for every supplier. This is independent of whether a supplier knows the cost parameters of other bidders or how he bids in other simultaneous auctions. The latter is because a supplier’s aggregate payoff is the sum of his payoffs under all auctions. Therefore, suppliers are indifferent between simultaneous auctions and sequential auctions.
Before we discuss the bundling issue, we show that our wholesale plus buy-back contracts are in fact equivalent to the option contracts in Martinez-de-Albeniz and Simchi-Levi (2005). The latter are defined as follows. Suppose the manufacturer allocates object \( A_j \) to supplier \( i \). In Martinez-de-Albeniz and Simchi-Levi (2005), the manufacturer pays supplier \( i \) the reservation price \( \tau_{ij} \) to reserve capacity \( \sum_{k \in A_j} (a_k - a_{k-1}) \). After the demand materializes, the manufacturer pays an additional amount \( e_{ij} \) for each unit she purchases. The following lemma establishes the equivalence.

**Lemma 1.** For every option contract with a reservation price and an execution price, \((\tau_{ij}, e_{ij})\), there is an equivalent wholesale contract with a buy-back price, \((w_{ij}, b_{ij})\), where the manufacturer and the suppliers make identical expected profits under both these contracts.

Next, we consider the constrained efficiency for the bundling issue.

**Theorem 2.** Suppose that a set of demand objects (or partition) \( \{A_1, \ldots, A_J\} \) is pre-determined and the manufacturer can only decide whether to merge some of these demand objects into a single auction or conduct separate auctions for all these demand objects. The supply chain surplus is higher when the demand objects are auctioned separately. In general, if \( A, B \) are the two auction choices, with these demand objects, such that \( A \) is a refinement of \( B \), then the total surplus under \( A \) is higher than that under \( B \).

Theorem 2 implies that a single bundle minimizes the supply chain surplus whereas partitioning the demand always (weakly) improves the efficiency. The intuition is as follows: when the manufacturer bundles two intervals of the demand distribution, the bundled demand object is awarded to the supplier that generates the highest expected profit for the bundled object, whereas if they are sold separately, the individual demand intervals may be given to two different suppliers. The latter increases the total surplus.

**5.1.2. The auction with two suppliers.** In this section we consider the case where only two suppliers compete for the manufacturer’s supply contract. We find that each supplier unambiguously prefers separate auctions to a bundled auction.

**Proposition 1.** For any pre-determined partition each supplier prefers separate auctions to a single auction.

Proposition 1 shows that it is in each supplier’s interest to hold these auctions separately. Under separate auctions, a supplier wins the auctions for which his values are higher and pays the other supplier’s valuations. In those auctions where he loses, he receives a null payoff. Nevertheless, if some auctions are bundled, then the winner has to pay for the loser’s aggregate valuation. He loses
surplus in those auctions where his valuations are lower and he would not have lost it if these auctions were held separately.

We next show that it is in the manufacturer’s interest to bundle the demand into a single auction.

**Proposition 2.** *Suppose that a set of demand objects have been pre-selected and the manufacturer can decide whether to bundle demand objects arbitrarily. Then the manufacturer minimizes the expected cost paid to the suppliers by selling all intervals in a single auction.*

Proposition 2 implies that the manufacturer, as opposed to the suppliers, benefits from a single auction. Under the single-bundle auction, each supplier expects to receive a lower payment compared to any other bundling. Hence, the manufacturer reduces the cost she pays to these suppliers. To complete the analysis, we also derive the optimal contracted quantity. It provides insights to the inefficiency that arises from the second-price auction. Recollect the definition of $\bar{Q}_i$ (Equation (1)) for any supplier $i$ ($i = 1, 2$).

**Theorem 3.** *Suppose that there are two suppliers such that $\bar{Q}_1$ and $\bar{Q}_2$ are defined by Equation (1). At optimality the manufacturer adopts a single auction wherein the single auctioned demand object includes the demand interval beginning at $Q = 0$ and ending between $\bar{Q}_1, \bar{Q}_2$."

Note that within $[0, \min(\bar{Q}_1, \bar{Q}_2)]$, the marginal benefits of both suppliers are positive. Therefore, the manufacturer would like to auction every piece of demand as long as it is profitable for both suppliers. Moreover, she may include some portion of demand where only one of the two suppliers finds it profitable to produce. By doing so the manufacturer may potentially award the entire supply contract to a supplier different from the one who would have been the winner under the auction $[0, \min(\bar{Q}_1, \bar{Q}_2)]$. Since the manufacturer may intentionally exclude some portion of demand before $\max(\bar{Q}_1, \bar{Q}_2)$, efficient transactions may be discarded in the auction. This inefficiency is demonstrated in the following example.

**Example 1.** *Suppose that there are two suppliers such that $s_1 > s_2$, and $s_1[1 - F(q)] - c_1 \geq s_2[1 - F(q)] - c_2, \forall q \in (0, \bar{Q}_2)$. In this situation, supplier 1 dominates in the entire region that is of interest and will be the winner regardless of the bundling. Because the marginal benefit for supplier 2 turns negative at $\bar{Q}_2$, going beyond $\bar{Q}_2$ only leads to a net loss of the manufacturer. Thus, the optimal contracted quantity is $\bar{Q}_2$, and the manufacturer’s expected payoff is $\int_0^{\bar{Q}_2} \{(r - s_2)[1 - F(q)] - (c_2 - s_2)\}dq$. Note that it is still profitable for supplier 1, the efficient supplier, to produce beyond $\bar{Q}_2$. However, this benefit is completely captured by supplier 1, and therefore the manufacturer is unwilling to induce production beyond $\bar{Q}_2$.**
5.1.3. Auctions with more than two suppliers. We now consider a setting in which there are more than two suppliers. As the number of suppliers approaches infinity, in all generic cases, the manufacturer prefers to conduct separate auctions to minimize the expected cost. The intuition is as follows. As the number of suppliers increases, the second-highest bid is closer to the first-best. Therefore, by conducting separate auctions, the manufacturer contracts with the most efficient supplier in every separate auction and the contract terms are very close to the winner’s true cost parameters. Since partitioning the demand always raises the total surplus, if the loss by paying according to the second-highest bidder shrinks, the manufacturer (in principle) finds it profitable to hold a number of separate auctions. These auctions may be held simultaneously because at most one unique supplier will be assigned to each demand interval.

The above intuition suggests that in general, with a large number of suppliers, the manufacturer may not be willing to bundle the entire demand. However, whether bundling generates a lower expected payment for the suppliers, when there are only a moderate number of suppliers, remains a difficult problem.

5.2. Optimal mechanism design to extract full surplus under common knowledge about types
In this section we show that the manufacturer can extract all the surplus from suppliers, independent of the number and the cost profile of suppliers. Our goal is to find a direct mechanism, $\mathcal{M}$, in which the strategy space for suppliers is to report their types. The restriction to direct mechanisms is without loss of generality according to the revelation principle, see Fudenberg and Tirole (1994).

The sequence of events is as follows. Since the manufacturer knows supplier types, i.e., $(c, s)$, she can partition the demand distribution a priori and create an efficient allocation using the algorithm described in Theorem 1. Recall that an efficient allocation assigns a demand interval to a supplier who provides the highest marginal benefit in the interior of that demand interval. Furthermore, this efficient allocation is common knowledge as well since the set of suppliers’ types is public information. Once the manufacturer receives the reports from the suppliers, she allocates each demand interval to the most cost efficient supplier. We assume that the manufacturer can commit not to renegotiate with the suppliers.

We now describe our proposed mechanism, $\mathcal{M}$, which we label as the “extraction assignment” mechanism, EAM.

Definition 6. In the extraction assignment mechanism $\mathcal{M}$, each supplier, $i$, is requested to report his cost parameters $(\hat{c}_i, \hat{s}_i)$. The vector of reports is denoted as $(\hat{\mathbf{c}}, \hat{\mathbf{s}})$. After receiving the reports from the suppliers, the manufacturer does the following:
Allocation: If there is a unique supplier, \( i_k \), who reports \((c_{ik}, s_{ik})\), then he is assigned the demand interval \((Q_{k-1}, Q_k] \) for which he is the most cost efficient supplier. If two or more suppliers report the same cost parameters \((c_{ik}, s_{ik})\), then the manufacturer assigns the interval \((Q_{k-1}, Q_k] \) randomly to them with equal probability.\(^3\) A supplier that reports a cost pair that is not a component of \((c, s)\) is not assigned any contract.

Contract terms: Suppose supplier \( i \) is awarded demand interval \((Q_{k-1}, Q_k] \). The manufacturer pays the unit wholesale price \( c_{ik} \) to him for producing \(|Q_k - Q_{k-1}| \) units. After demand realization, all leftover inventory that corresponds to this interval \((Q_{k-1}, Q_k] \) is returned to supplier \( i \) at the buy-back price \( s_{ik} \).

Next, in Theorem 4, given the EAM mechanism, we show that truth-telling is a dominant strategy for every supplier and full surplus extraction is achievable for the manufacturer.

**Theorem 4.** Under the EAM mechanism, truth-telling is a dominant strategy for every supplier. Moreover, full surplus extraction is achieved.

Under this mechanism, telling the truth is a dominant strategy. To see this, notice that the contract terms follow the cost parameters of the most efficient suppliers. Thus, when a supplier is assigned to his own interval, he just breaks even. He may incur a loss if he is assigned to a different interval. Therefore, full surplus extraction can be obtained as an equilibrium outcome. One may be concerned regarding the multiplicity of equilibria. However, such a situation can be easily avoided if the manufacturer leaves a small portion of the expected supply chain profit to each supplier when he is selected.

5.3. **Comparison between the second-price auction and the optimal mechanism**

In Section 5.1.3 we showed that, with a large number of suppliers, the manufacturer is willing to hold separate auctions. This not only leads to supply chain efficiency but also allows the manufacturer to extract almost the entire surplus from the suppliers. This is because in each auction the benefit from sourcing from the second best supplier is very close to that obtained from the most efficient supplier. Our goal in this section is to answer the following question: besides this case, is there any performance guarantee for the second-price auction?

To answer this question, we consider the two-supplier case. For ease of presentation, we assume that \( s_1 - c_1 \geq s_2 - c_2 \), i.e., supplier 1 is more cost efficient than supplier 2 when producing an arbitrarily small quantity. Under the EAM mechanism, the manufacturer captures the entire expected supply chain profit. The following theorem provides lower bounds for the manufacturer’s expected payoff under the second-price auction in these two scenarios.
Theorem 5. Suppose there are two retailers with costs $c_1$ and $c_2$ such that $c_1 \leq c_2$. Further suppose that the manufacturer uses a second-price auction mechanism to allocate demand partitions. If $Q_1 \geq Q_2$, then the manufacturer’s expected profit is $\int_{0}^{Q_2} \{(r - s_2)[1 - F(q)] - (c_2 - s_2)\}dq$. Else the expected profit is at least $\min_{i=1,2} \int_{0}^{\tilde{Q}_1} \{(r - s_i)[1 - F(q)] - (c_i - s_i)\}dq$.

The first part of Theorem 5 ($\tilde{Q}_1 \geq \tilde{Q}_2$) follows from Example 1. This corresponds to the case when one supplier dominates the other. The second part of Theorem 5 provides a lower bound for the manufacturer’s expected payoff when she uses the second-price auction. The manufacturer can always auction up to $\tilde{Q}_1$ units of demand. In this case, she receives an expected payoff that weakly dominates the lower bound stated in Theorem 5.

From the bounds given in Theorem 5, we can discuss whether the performance of the second-price auction is close to the EAM mechanism or not. We consider two polar scenarios. In the first case, the suppliers’ cost parameters are very close, whereas in the second case, they have very distinct cost parameters. Let $\pi^M$ and $\pi^{M*}$ denote the manufacturer’s payoffs under the optimal second-price auction and the EAM mechanism, respectively.

Proposition 3. Suppose that there are two suppliers with $c_1 \leq c_2$ and the manufacturer uses the second-price auction. For any given $r, c_1, s_1$, for any given $\varepsilon > 0$, (1) there exists a $\delta_b > 0$ such that if $\sqrt{(c_1 - c_2)^2 + (s_1 - s_2)^2} \leq \delta_b$, then $\frac{\pi^M}{\pi^{M*}} > 1 - \varepsilon$; (2) there exists a $\delta_a > 0$ such that if $s_2 > r - \delta_a$, and $\frac{r - s_2}{r - s_2} > 1 - \delta_a$, $\frac{\pi^M}{\pi^{M*}} < \varepsilon$.

Proposition 3 shows that when the two suppliers have similar cost parameters (case 1), the manufacturer’s expected payoff is close to the maximum expected supply chain profit under the second-price auction. Even if the manufacturer leaves some surplus for the winning supplier, it is limited because his cost advantage is small. This intuition coincides with the case with many suppliers, since the gap between the most efficient and the second best suppliers is negligible. On the other hand, when the suppliers have very different cost parameters (case 2), the single-bundle auction has to leave a huge surplus for the winning supplier. Although the manufacturer is aware of such cost difference, restricting herself to the second-price auction forces her to give most of the profit to the winning supplier.

6. Mechanism Design under Incomplete Information

In this section we consider the case when neither the suppliers nor the manufacturer has full information regarding the set of cost parameters of other suppliers. Every realization of $(c_i, s_i)$ is privately observed by supplier $i$. Therefore, the manufacturer is unable to decide an optimal partition of the demand distribution. (Without such information, the EAM mechanism becomes
infeasible as well.) We first introduce the model characteristics. Following this, we investigate several procurement mechanisms used in practice when the demand partition is pre-determined. Finally, we consider the case when the manufacturer partitions the demand after seeing suppliers’ bids.

6.1. The model

First, we explicitly specify the model characteristics regarding the informational structure. Let \( \theta \equiv (c,s) \) denote the costs of a supplier drawn from a finite set \( \Theta \equiv \{\theta^1, \ldots, \theta^{|\Theta|}\} \). \( \theta \) is the “type” privately observed by the supplier. The type correlation is modeled as follows. Consider any supplier \( i \). Suppose \( \Theta^{n-1} \) is the product space of \( n-1 \) identical sets of \( \Theta \). For simplicity, we assume that suppliers are ex ante symmetric, but our analysis can be easily extended to the asymmetric case.

We assume that the cost parameters among suppliers are correlated. Specifically, given that supplier \( i \)'s cost parameter is \( \theta^m \in \Theta \), the conditional probability that cost profile of other suppliers is \( \eta \in \Theta^{n-1} \) is denoted by \( p_{mn} \). Therefore, a supplier updates his belief of other suppliers’ cost parameters according to the realization of his own cost parameters. The manufacturer does not observe the true cost profile, but she knows the conditional probability matrices. Note that \( \sum_{\eta \in \Theta^{n-1}} p_{mn} = 1, \forall m = 1, \ldots, |\Theta| \). By symmetry this also defines the conditional probabilities from all other supplier perspectives as well. We define this \( |\Theta| \times (n-1)|\Theta| \) dimensional conditional-probability matrix as \( P \equiv (p_{mn}) \). For notational ease, we denote the number of elements that belong to an arbitrary set \( S \) by \( |S| \). \( S_i \) and \( S_{ij} \) denote the \( i \)-th row and \( j \)-th column of an arbitrary matrix \( S \) respectively, and \( S^T \) denotes its transpose.

Given any realization of the supplier cost profiles (types) \( (\theta_1, \ldots, \theta_n) \), an efficient allocation can be obtained by use of the algorithm in Theorem 1. But, since the manufacturer does not observe \( (\theta_1, \ldots, \theta_n) \), she can only induce these suppliers to reveal their private information by designing an appropriate incentive mechanism. We define \( (\theta_1, \ldots, \theta_n) = (\theta_i, \theta_{-i}), \forall i = 1, \ldots, n \), where \( \theta_{-i} = (\theta_1, \ldots, \theta_{i-1}, \theta_{i+1}, \ldots, \theta_n) \) is the type realization vector of suppliers other than \( i \).

6.2. Procurement mechanisms for a given demand partition

We first consider the case when the manufacturer pre-determines the demand partition. After the demand is partitioned, each demand partition can be regarded as a supply contract. Thus, the manufacturer can use any procurement mechanism to “sell” the contract. Following the notation in Section 5.1, we use \( \mathcal{K} = \bigcup_{k=1}^{K} (a_{k-1}, a_k] \) to denote the demand partition, and \( \{A_1, \ldots, A_J\} \) represent the \( J \) disjoint demand objects the manufacturer sells through the procurement.
6.2.1. The procurement mechanisms. In addition to the beauty contest mechanism introduced in Section 5.1, we consider four other alternatives: menu auction, scoring rule auction, request-for-quote (RFQ) mechanism, and the VCG mechanism, see, e.g., Asker and Cantillon (2007) and Krishna (2002). For ease of exposition, we focus on the “second-price” auction type and explain these procurement mechanisms for a demand object $A_j$.

**Definition 7.** In a menu auction, each supplier submits a schedule $(p_{ij}, w_{ij}, b_{ij})$, where $p_{ij}$ indicates the prices for the demand object, $w_{ij}$ represents the corresponding wholesale prices, and $b_{ij}$ refers to the buy-back prices. The manufacturer then selects the supplier that generates the highest expected payoff for her (based on the schedules). After the auction, the manufacturer chooses a pair of $(w_{ij}, b_{ij})$ from the winning supplier’s schedule, but pays him $\hat{p}_{ij}$ such that her expected payoff matches the highest payoff she can get from the schedule submitted by the highest losing supplier.

**Definition 8.** In a scoring rule auction, each supplier submits a wholesale price $w_{ij}$ and a buy-back price $b_{ij}$. The manufacturer calculates the scores of these suppliers based on the scoring rule, and awards the demand object to the supplier with the highest score. The winning supplier then has to choose the contract terms that match the score of the highest losing supplier.

**Definition 9.** If the RFQ mechanism is used, the manufacturer first announces a maximum wholesale price and a minimum buy-back price. Each supplier then submits a price for the demand object, and the supplier with the highest bid wins the auction. The contract terms of the winning supplier must satisfy the manufacturer’s requirement.

**Definition 10.** In the VCG mechanism with a fixed demand partition, each supplier submits a price for the demand object, and the supplier with the highest bid wins the auction. The winning supplier pays the manufacturer the second-highest price, and takes over the auction object.

From the above description, the suppliers’ bids are the most complicated in a menu auction, since suppliers are requested to submit a complete schedule. In the RFQ and VCG mechanisms, a supplier’s bid is simply the payment he is willing to make for the demand object. These are the simplest to administer. The scoring rule auction is the intermediate case. In it, each supplier is asked to submit one set of payment, wholesale price and buy-back price. These mechanisms also differ in the manner in which the contract terms are determined. In a menu auction, the manufacturer selects the wholesale and buy-back prices, whereas in the scoring rule auction and the RFQ mechanism, it is the winning supplier who chooses these contract terms after the auction. No contract terms are required for a VCG mechanism (except the payment for the supply contract).
These mechanisms are widely used for procurement. For example, the menu auction is very common in e-commerce, since the online bidding environment allows the firms to submit a list of offers (also known as the request-for-proposal), e.g., Compaq, Dell, and IBM. The menu auction is also used in transportation, insurance, and real estate. The scoring rule auction is used for highway construction (e.g., the “A+B bidding” in the United States), and public procurement of the Europe Union. It is also incorporated by a number of procurement software developers such as Digital Union, eBreviate, Oracle Sourcing, see, e.g., Asker and Cantillon (2007). Many electronic marketplaces such as FreeMarkets and Manugistics Group, Inc. provide the option of the request-for-quote mechanism for their customers (Beil and Wein (2003)). The RFQ mechanism is used by the US government for the General Service’s Administrations’ “e-Buy” electronic service. The VCG mechanism is in fact the standard second-price auction when the supply contract of a demand object is treated as a single object. This is also labeled as the “entry-fee” auction in Chen (2004). Since our sourcing problem and these examples share similar contracting and informational issues, these mechanisms are natural candidates for investigation.

6.2.2. Revenue equivalence. Having discussed the differences among these procurement mechanisms, we now show that all these mechanisms, including the beauty contest mechanism, yield the same expected payoff for the manufacturer. This result holds regardless of the demand partition or the sequence of auctions for demand objects.

**Theorem 6.** For any given demand partition, the beauty contest, the menu auction, the scoring rule auction, the RFQ mechanism, and the VCG mechanism yield the same expected payoff to the manufacturer, regardless of whether the auctions are held simultaneously or sequentially.

The above theorem establishes the revenue equivalence between all these mechanisms. The intuition is as follows. First, we observe that the expected supply chain profit is the same across these mechanisms when the same supplier is selected. Moreover, since a supplier’s own bid does not affect his payoff after winning the auction in all these mechanisms, bidding his true valuation of the demand object is a dominant strategy. Therefore, when a supplier wins the auction, his net expected payoff is exactly the difference of the valuations between his and the highest losing supplier. Given that the manufacturer and suppliers are all risk-neutral, the choices of wholesale and buy-back prices only lead to a net transfer of expected payoffs, which can be adjusted by the payment for the supply contract. Combining these observations, the choice of the procurement mechanism does not affect the expected payoff of either the manufacturer or any supplier.
Theorem 6 allows us to focus on the beauty contest mechanism, as long as the manufacturer does not use the suppliers’ bids to partition the demand distribution. Because of the second-price auction, this equivalence is valid even if the suppliers’ cost parameters are ex ante asymmetric, correlated, or the set of cost parameters is publicly known (Section 5). Moreover, this suggests that the bundling decision is independent of the choice of the procurement mechanism due to revenue equivalence. The manufacturer can concentrate on the search for the optimal partition, and then choose any aforementioned mechanism.

According to Krishna and Perry (1998), when the suppliers’ costs are privately known and i.i.d. draws from a common distribution, the VCG mechanism yields the highest expected payoff to the manufacturer among all mechanisms that are constrained efficient (see Definition 3), incentive compatible, and individually rational. Thus, Theorem 6 implies that all above mechanisms provide the best performance guarantee of the manufacturer’s expected payoff for a given demand partition. However, the inefficiency that arises from the pre-determined partition can be arbitrarily high, as demonstrated in Section 5.1. We next study the issue of creating efficient endogenous partition of demand.

6.3. Efficient mechanisms with endogenous demand partition

In this section, we first introduce a mechanism that allows the manufacturer to allocate demand efficiently. Then we derive the necessary and sufficient conditions for full surplus extraction. Finally, we discuss the implementation issues.

6.3.1. The efficient partition. We now describe a mechanism, \( F \), which we label as the “efficient partition” mechanism (EPM). As suggested by the name, this mechanism efficiently partitions the demand.

**Definition 11.** In the efficient partition mechanism \( F \), each supplier is asked to report his cost parameter \( \hat{\theta}_i \). After receiving the reports \( \hat{\theta} = (\hat{\theta}_1, \ldots, \hat{\theta}_n) \) from the suppliers, the manufacturer does the following:

- **Franchise fee:** The manufacturer asks supplier \( i \) to pay a franchise fee \( t_i(\hat{\theta}) \) that depends on all reports by the suppliers.

- **Demand allocation:** The manufacturer uses the algorithm described in Theorem 1 to allocate the demand efficiently according to \( \hat{\theta} \). Let \( \{ (a_k(\hat{\theta}), a_{k+1}(\hat{\theta})], k = 1, \ldots, n \} \) denote the efficient partition of the demand given cost parameters \( \hat{\theta} \).

- **Contract execution:** Suppose under \( \hat{\theta} \), supplier \( i \) is awarded \( (a_i(\hat{\theta}), a_{i+1}(\hat{\theta})] \) of the demand. Then supplier \( i \) is asked to produce \( |a_{i+1}(\hat{\theta}) - a_i(\hat{\theta})| \) units. After the demand is realized, the
revenue generated from this interval is given to the supplier, and the supplier owns all the leftover inventory from the corresponding interval.

From a supplier’s viewpoint, the franchise fee, \( t_i(\hat{\theta}) \), is random since it is jointly determined by other suppliers’ reports to which he has no access. As a supplier, he reports his production cost and salvage value, pays a franchise fee \( t_i(\hat{\theta}) \), and then takes full responsibility for the demand interval. He produces with his true production cost \( c_i \), receives the revenue for every unit sold within this demand interval, and obtains the salvage value from all leftover inventory (with the true value \( s_i \)). This mechanism differs from the auctions studied in Section 5.1 in that no wholesale and buy-back prices are specified in the mechanism.

This mechanism is admittedly more complicated than the aforementioned practical procurement mechanisms. The complexity arises due to two reasons. First, the manufacturer would like to incorporate the suppliers’ bids (reports) to decide the efficient demand allocation, which inevitably requires an algorithm. Second, the manufacturer wants to extract maximum possible surplus from the suppliers. The contingency upon all suppliers’ reports, as we show later, provides the manufacturer the desired flexibility for extracting greater surplus.

Given the report \( \hat{\theta} \), we have an efficient partition of demand \( \{(a_k(\hat{\theta}), a_{k+1}(\hat{\theta}))\}, k = 1, \ldots, n \}. \) Let supplier \( i_k(\hat{\theta}) \) be the most cost efficient one for the interval \( (a_k(\hat{\theta}), a_{k+1}(\hat{\theta})) \) given the suppliers’ reports \( \hat{\theta} \), i.e., \( i_k(\hat{\theta}) = \arg \max_{j \in I} \{ \hat{s}_j F(q) - \hat{c}_j, \forall q \in (a_k(\hat{\theta}), a_{k+1}(\hat{\theta})) \} \}. \) Furthermore, let

\[
v(\hat{\theta}, \theta_i) = \int_{q \in (a_k(\hat{\theta}), a_{k+1}(\hat{\theta})) : i = i_k(\theta)} [(r - s_i)(1 - F(q)) - (c_i - s_i)] dq
\]

denote the expected payoff a type-\( \theta_i \) supplier receives when the reports are \( \hat{\theta} \). Note that the allocation is based on \( \hat{\theta} \), but the supplier produces and salvages the product at his real costs.

Observe that the demand allocation mechanism, described earlier, does not guarantee that each supplier has an incentive to truthfully reveal his true type. Hence, the manufacturer must provide the right incentive such that truthful revelation occurs naturally. Furthermore, the incentive structure must also guarantee that the manufacturer can extract full surplus. Recall that our goal is to develop the necessary and sufficient conditions for both these conditions to apply simultaneously. Let us define the expected payoff \( V_{l\eta m} = v(\theta^l, \eta, \theta^m) \) when the supplier’s true type is \( \theta^m \) but he reports as type-\( \theta^l \), given that other suppliers report \( \eta \in \Theta^{n-1} \).

Assuming that all other suppliers report truthfully, we must ensure that the supplier’s best response is to report his type truthfully as well. Furthermore, a supplier will be willing to participate
only if his expected payoff is non-negative (individual rationality). These conditions can be enforced by constraining the franchise fees, \( \{t_m\eta, \forall \eta \in \Theta^{n-1}, \forall m = 1, \ldots, |\Theta|\} \), as follows:

\[
\sum_{\eta \in \Theta^{n-1}} p_m(V_m\eta - t_m\eta) \geq \sum_{\eta \in \Theta^{n-1}} p_m(V_l\eta - t_l\eta), \ \forall m, l = 1, \ldots, |\Theta| \tag{3}
\]

\[
\sum_{\eta \in \Theta^{n-1}} p_m(V_m\eta - t_m\eta) \geq 0, \forall m = 1, \ldots, |\Theta| \tag{4}
\]

where \( t_u\eta \equiv t_i(\hat{\theta}) \) if \( \hat{\theta}_i = \theta^u \) and \( \hat{\theta}_i = \eta \). The \( \{p_m\} \)'s are the true conditional probabilities. The first condition, Equation (3), guarantees that a supplier’s expected payoff, on truthful revelation of his type, exceeds the expected payoff by not doing so. The second condition, Equation (4), induces the suppliers to participate by ensuring them a positive expected payoff on truthful revelation. Since the second inequality holds for each type, the above constraint set also requires interim individual rationality. These conditions are standard in the screening literature.

We first consider the VCG mechanism with endogenous partition. Let \( \theta^0 \) be the null report such that if a supplier announces \( \theta^0 \), he will never be awarded any demand object.

**Definition 12.** In a VCG mechanism with endogenous partition, the manufacturer uses the EPM mechanism to allocate the demand among suppliers. The franchise fees \( \{t_i(\hat{\theta})\} \)'s satisfy \( t_i(\hat{\theta}) = \sum_{j \neq i} v((\theta^0, \hat{\theta}_{-i}), \hat{\theta}_j) - \sum_{j \neq i} v(\hat{\theta}, \hat{\theta}_j), \forall i = 1, \ldots, n. \)

According to this definition, the franchise fee \( t_i(\hat{\theta}) \) in the VCG mechanism equals the marginal externality a supplier brings to the supply chain. Suppose all suppliers report the cost parameters truthfully. Then, \( t_i(\hat{\theta}) \) is exactly the aggregate expected profit of other suppliers and the manufacturer net the aggregate expected profit when supplier \( i \) is taken away from the system. The VCG mechanism is a special case of the Groves mechanism, and is well known to achieve both incentive compatibility (Equation (3)) and individual rationality (Equation (4)).

Following the argument in Krishna and Perry (1998), the VCG mechanism with endogenous partition gives the manufacturer the highest expected payoff among all efficient, incentive compatible, and individually rational mechanisms when suppliers are ex ante symmetric.\(^4\) Therefore, if the manufacturer aims at finding an efficient mechanism with a performance guarantee with regard to expected payoff, she can simply use the VCG mechanism. Note that the manufacturer may obtain a higher expected payoff by using an inefficient mechanism, but the profitability comes at the expense of forgoing efficient transactions. Also, as demonstrated in Proposition 3, if she is concerned about her payoff in every instance of cost realization, she has to look beyond the VCG mechanism. This motivates us to consider a mechanism that maximizes the manufacturer’s profit.
6.3.2. Extracting full surplus. We now define a mechanism that achieves full surplus extraction using the efficient partition mechanism, described earlier, and Equation (4).

**Definition 13.** An EPM mechanism is said to achieve full surplus extraction under incomplete information if

1. The allocation is efficient, i.e., if supplier $i$ is awarded $(Q_{k-1}, Q_k]$, then $i = i_k \equiv \arg \max_{j \in I} \{ s_j F(q) - c_j, \forall q \in (Q_{k-1}, Q_k] \}, \forall i \in I$.
2. $\sum_{\eta \in \Theta^n-1} p_{m\eta} (V_{m\eta} - t_{m\eta}) = 0, \forall m = 1, \ldots, |\Theta|.

Note that when full surplus extraction is achieved, the efficient allocation maximizes the expected supply chain profit and no surplus is left to the suppliers regardless of their types. Therefore, the manufacturer’s expected profit is maximized.

Next, we characterize the necessary and sufficient conditions for such full surplus extraction under efficient allocation in Proposition 4. We fix the allocation as an efficient one, and verify whether franchise fees can be set such that the mechanism is incentive compatible and individually rational, and at the same time it achieves full surplus extraction. Specifically, we are interested in whether the set of feasible solutions to the following linear system is non-empty:

$$
\sum_{\eta \in \Theta^n-1} p_{m\eta} (V_{m\eta} - t_{m\eta}) \geq \sum_{\eta \in \Theta^n-1} p_{m\eta} (V_{l\eta} - t_{l\eta}), \forall m, l = 1, \ldots, |\Theta|,
$$

(5)

$$
\sum_{\eta \in \Theta^n-1} p_{m\eta} (V_{m\eta} - t_{m\eta}) = 0, \forall m = 1, \ldots, |\Theta|.
$$

(6)

To simplify the notation, we define the weighted average of the payoffs, when a supplier’s true type is $\theta^m$ but he reports type $\theta^l$; as $v_{lm} \equiv \sum_{\eta \in \Theta^n-1} p_{l\eta} V_{m\eta}, \forall m, l = 1, \ldots, |\Theta|$. The corresponding matrix of payoffs, of size $|\Theta| \times |\Theta|$, is defined as $V \equiv (v_{lm})$. Observe that in this definition we replace the conditional probabilities by those under the false type $\theta^l$.

We next characterize necessary and sufficient conditions for full surplus extraction. Recall that the conditional probability matrix is denoted by $P \equiv (p_{mn})$.

**Proposition 4.** Full surplus extraction is feasible if and only if for each $m = 1, \ldots, |\Theta|$, the following linear system is infeasible:

$$
y_m^T \left( P - P \right) = 0, y_m^T \left( \begin{array}{c} V_m \\ v_{mm} \end{array} \right) < 0, y_m \geq 0,
$$

(7)

where $y_m$ is an $(|\Theta| + 1)$-element vector.

A natural question, that follows, is what conditions must be imposed on the conditional probabilities and payoff structures such that the conditions in Proposition 4 can be satisfied? In Theorem 7 we characterize three sets of sufficient conditions that enable full surplus extraction.
Theorem 7. Full surplus extraction is achievable if any of the following three conditions hold:

1. The probability matrix $P$ is such that
   
   (1a) $P$ has full rank, or
   
   (1b) For each $i = 1, \ldots, n$, there does not exist a non-negative solution $\{y_{ml}\}$ to $P_m = \sum_{l \neq m} y_{ml} P_l$, $\forall \ j = 1, \ldots, m$,
   
   or
   
   2. $v_{mm} - v_{ml} \geq 0$, $\forall \ m, l = 1, \ldots, |\Theta|$, or
   
   3. $V_m \in \text{span}(P)$, $\forall \ m = 1, \ldots, |\Theta|$.

The first set of sufficient conditions, i.e., Condition (1), correspond to Riordan and Sappington (1988) and the two conditions proposed by Cremer and McLean (1988) (where they discussed full surplus extraction in the context of auctions). These conditions are imposed only on the conditional probabilities and they formalize the idea of other suppliers’ reports being sufficiently informative. Condition (1a), i.e., the full rank condition, is likely to hold when many suppliers compete for the supply contracts. Condition (1b) implies that no vector $P_m$ can be written as a convex combination of other vectors, which is a relaxation of Condition (1a).

The second condition, Condition (2), states that under the first-best level for a particular type, say type-$n$, all other types are worse off by pretending to be this type. Therefore, the manufacturer can optimally allocate the demand interval to supplier of type-$n$ and make him just break even when she receives such a report. Given this scheme, all other types are unwilling to pretend to be type-$n$ due to the negative expected payoff. Under Condition (2) the manufacturer is able to extract full surplus even when there is a single supplier, hence it is not very interesting. However, the efficient allocation still requires all suppliers report their costs so that the manufacturer can award demand intervals accordingly. Condition (3) implies that the payoff vector falls in the span of the column vectors of $P$, in which case all incentive compatibility and individual rationality constraints are satisfied as equalities under the optimal mechanism.

Finally, we briefly discuss how the manufacturer implements the mechanism if any of the conditions in Theorem 7 is satisfied. According to the EPM mechanism, the implementation requires three steps. The manufacturer first requests the suppliers to report their cost parameters. Next, according to their reports, the manufacturer applies the algorithm in Theorem 1 to partition the demand, and then awards each demand interval to the corresponding cost efficient supplier.

The franchise fees can be obtained by solving a linear system. When either Condition (1a) or (1b) holds, the optimal mechanisms can be obtained from solving the linear system (Equations (5) and (6)). When Condition (2) holds, the optimal mechanism need not include all the reports of
suppliers. In other words, it is without loss of generality to make $t_{mq} \equiv t_m$, $\forall m = 1, \ldots, |\Theta|$ in this case. Under Condition (3), the manufacturer can find franchise fees so that each type of supplier is indifferent between reporting truthfully or lying. The optimal mechanism can be obtained by solving the following system of linear equations: $\sum_{\eta \in \Theta} n_{\eta} - 1 p_{m\eta} (V_{l\eta m} - t_{l\eta}) = 0, \forall m, l = 1, \ldots, |\Theta|$. 

6.4. Comparison between procurement mechanisms

In Theorem 6, we showed that some of the popular procurement mechanisms with a pre-determined demand partition are revenue equivalent. On the other hand, when the manufacturer uses the general EPM mechanism and the conditions in Theorem 7 hold, she is able to extract full surplus from the suppliers. When the conditions do not hold, the VCG mechanism appears to be the most appropriate candidate since it yields the highest expected payoff to the manufacturer among efficient mechanisms. Thus, in this section, we compare the VCG mechanisms with endogenous partition and pre-determined partition. As in Section 5.3, we focus on the two-supplier case.

6.4.1. Analytical comparison. Without loss of generality, we assume that $s_1 - c_1 \geq s_2 - c_2$, i.e., supplier 1 is more cost efficient initially. We find that, when one supplier dominates the other, using an endogenous partition is always better.

Proposition 5. Suppose there are two suppliers and the realized cost parameters satisfy $s_1 - c_1 \geq s_2 - c_2$ and $\bar{Q}_2 \leq \bar{Q}_1$. Then the VCG mechanism with endogenous partition weakly dominates the VCG mechanism with any pre-determined demand partition.

Now consider the case when both suppliers have cost advantages in different demand intervals. We show that the VCG mechanism with endogenous partition is equivalent to conducting two separate auctions.

Proposition 6. Suppose there are two suppliers and the realized cost parameters satisfy $s_1 - c_1 \geq s_2 - c_2$ and $\bar{Q}_2 > \bar{Q}_1$. The VCG mechanism with endogenous partition is equivalent to conducting two separate auctions for demand intervals $[0, Q_{1a})$ and $[Q_{1a}, \bar{Q}_1]$.

Proposition 6 demonstrates the intriguing trade-off the manufacturer faces when deciding between the use of endogenous versus pre-determined partition. In Proposition 2, we observed that the manufacturer prefers a single bundle. However, if the manufacturer allocates the demand after seeing suppliers’ bids, she is forced to conduct two separate auctions. The commitment on the efficient allocation via the VCG mechanism prevents her from extracting more surplus from suppliers. On the other hand, if a pre-determined partition is used, the manufacturer suffers from the lack of information while choosing the single bundle $[0, \bar{Q}]$. It is conceivable that the pre-determined
demand interval may be too narrow, e.g., when $Q < Q_{1a}$. In this case, the manufacturer obtains
less than what she would obtain under the endogenous partition. Likewise, if $Q$ is too large, both
suppliers may find it unprofitable to participate in the auction and therefore no transaction occurs.
The general performance ranking is ambiguous when both suppliers have cost advantages. This is
further demonstrated through the following numerical examples.

6.4.2. Numerical comparison. To illustrate how the manufacturer determines the demand
partition under the VCG mechanism, we consider the following numerical example. The demand
follows a uniform distribution over $[0, 1]$, the retail price $r = 2$, and the cost parameters of supplier
2 are $c_2 = 1, s_2 = 0.5$. We assume that $s_1$ is a known constant and $c_1$ is a random variable that
takes value from $[s_1, r]$. Only supplier 1 knows the realization of $c_1$. This is arguably the simplest
situation with incomplete information regarding suppliers’ cost parameters. With two suppliers,
the single-bundle auction is optimal when the manufacturer pre-determines the demand partition
according to Proposition 2.

We assume that $c_1$ follows a truncated normal distribution in $[s_1, r]$, $\mathbb{E}c_1 = s_1 + \rho(r - s_1)$ and
$Var(c_1) = \sigma(r - s_1)^2$. Note that $\rho$ serves as a proxy of supplier 1’s expected production cost (higher $\rho$
means lower profitability if the manufacturer contracts with supplier 1). The parameter $\sigma$ represents
the variability of the production cost. For example, if $\sigma \to 0$, the manufacturer knows that $c_1$
is quite likely to be $\mathbb{E}c_1$. On the other hand, when $\sigma \to \infty$, the truncated normal distribution is
approximately a uniform distribution over $[s_1, r]$. In the following we change the values of $s_1$, $\rho$, and
$\sigma$ to see how the manufacturer determines the optimal contacted quantity under the pre-determined
partition and when she should use the endogenous partition. We use Mathematica to compute
the optimal contracted quantity and the manufacturer’s expected payoff. Codes are available upon
request.

First, in Figures 1 and 2, we illustrate how the optimal pre-determined contracted quantity
changes with the parameters $\rho$ and $\sigma$. In Figure 1, we observe that the optimal quantity is monotonic
in supplier 1’s expected cost. When $s_1 = 0.8$, the optimal quantity decreases as supplier 1’s expected
production cost increases. This might be because the expected $Q_1$ decreases and therefore the
manufacturer chooses a smaller quantity. However, when $s_1 = 0.2$, the increase in the expected
production cost has a completely opposite effect on the optimal quantity. We next fix $\rho = 0.5$ and
change the value of $\sigma$. From Figure 2, we again observe that no unambiguous prediction can be
obtained when the variability of $c_1$ increases. The optimal bundle critically depends on the set of
cost parameters and the manufacturer must analyze the problem case by case.
Finally, we compare the performance of pre-determined versus endogenous partition in Figures 3 and 4. We apply Proposition 6 to obtain the manufacturer’s expected payoff with the endogenous partition. Figure 3 shows that the manufacturer’s expected payoff is decreasing in \( \rho \), i.e., supplier 1’s expected cost. This is true regardless of whether the demand partition is pre-determined or endogenous. Thus, the manufacturer suffers from the cost increase of supplier 1 because she may contract with him. We also observe in Figure 3 that with these parameters, the manufacturer obtains a higher expected payoff by using an endogenous partition. In this case, the benefit of creating efficient partition outweighs the information rent the manufacturer has to leave for the suppliers. Consequently, the manufacturer can reduce the expected payment to the suppliers when she maximizes the supply chain profit.
Extensive numerical study over combinations of parameters shows that this dominance result appears in most of the cases we examined. However, there are situations where the pre-determined partition performs better, as we illustrate in Figure 4. Note that in this case $s_1$ is very close to $r$, and therefore the value of $c_1$ can be predicted accurately.

Based on our numerical studies, it appears that creating the partition endogenously is preferred in general. However, if the cost uncertainty is small, the manufacturer should consider using the pre-determined partition since the benefit from incorporating suppliers’ bids to determine the endogenous partition is relatively small.

7. Conclusions

This paper studies several procurement mechanisms when the manufacturer purchases products from multiple suppliers with heterogeneous production costs and salvage values. We first consider the situations in which the set of suppliers’ cost parameters is common knowledge. We find that under the second-price auction with a few suppliers, the manufacturer tends to bundle the entire demand. This benefits the manufacturer, but minimizes the expected supply chain profit and the supplier’s expected profits as well. The efficiency loss is particularly large when the suppliers have very different cost parameters. Therefore, when suppliers have significant cost advantages in different regions of the demand distribution, the second-price auction should not be used because it forgoes most of the profit. However, with many (and diverse) suppliers, the manufacturer may prefer to hold several second-price auctions. We then propose a simple mechanism that allows the manufacturer to extract full surplus from the suppliers. Full surplus extraction is possible because the optimal partition of the demand before the auction coincides with the optimal allocation under complete information.

We relax the assumption that the supplier types are common knowledge. We show that the incentive for the manufacturer to bundle or partition the demand is not prone to the choice amongst several popular procurement mechanisms. Moreover, when the diversity of the suppliers increases, the bundling issue becomes less important. Thus, we should expect to see more procurement using options as the number of suppliers (diversity) increases.

We also investigate mechanisms with endogenous partition that depends on all suppliers’ reports. We find that with two suppliers, the VCG mechanism with endogenous partition outperforms all mechanisms that use pre-determined partition when one supplier dominates the other. Nevertheless, the general performance ranking is ambiguous when both suppliers have cost advantages. Our numerical results show that the trade-off is between how accurately one can predict the costs and how much information rent to leave for the suppliers in order to estimate the costs from their bids.
Finally, when the suppliers’ cost parameters are not ex ante symmetric and independent, we show how to set up the mechanism design problem as a linear optimization problem. The solution is implemented using an endogenous partition of demand and franchise fees. We also give conditions where full surplus extraction can be achieved, and discuss how it can be implemented.

Several extensions are in order. We have not examined the optimal endogenous partition while abandoning the requirement of efficiency. This may lead to a higher expected payoff for the manufacturer when the suppliers’ cost parameters are correlated with a general structure. Since the cost parameters of a supplier is naturally multi-dimensional, we have to solve the multi-dimensional screening problem. This has been known to be a real challenge in the mechanism design literature, and no systematic approach has been proposed except for some very restrictive settings. In our problem, the design of optimal contract terms and allocation rules must deal with this challenge as well as the correlation of suppliers’ cost parameters.

Another extension would be to consider a more general cost structure. For example, consider the case when besides the production cost and salvage value, the suppliers are heterogeneous with regard to fixed cost as well. The fixed cost creates further complication on the supplier selection. Our approach to determine the efficient allocation is valid only when there is no fixed cost. When the suppliers must incur a fixed cost before building up the capacity, the selection of suppliers impacts the expected supply chain profit significantly, and ultimately the manufacturer’s revenue. When suppliers privately observe their cost parameters, the supplier selection problem is even more sophisticated, since the manufacturer has to determine the selection of suppliers, the allocation of the demand curve, and the contract terms without knowing the actual cost structure.

Our analysis excludes the possibility that suppliers might collude. When the suppliers can collude (among themselves) either through transfers or tacit agreements (without physical transactions or written contracts), full surplus extraction may be hard to achieve. The manufacturer would need to provide sufficient incentives for suppliers to reveal their cost parameters and reduce the excess payments, taking into account the possibility of coalition formations among suppliers.

Finally, in some situations the risk aversion of the suppliers or the manufacturer is significant. When the suppliers are risk averse, the revenue equivalence result breaks down because payoffs are no longer additive. Moreover, restricting to the second-price auction is not without loss of generality, because suppliers may be willing to receive less in, for example, the first-price auction (Krishna (2002)). In addition, when the manufacturer is risk averse, even if she receives the same expected payoff from different mechanisms, she may still give preference to a specific mechanism.
Appendix
Proof of Theorem 1
We first verify that the algorithm works, and then prove that the resulting allocation is indeed optimal.

The algorithm. Since \( F(\cdot) \) is increasing and continuous, the thresholds \( \{Q_{ka}\} \)’s are well-defined. Moreover, the linear form of \( s_lF(q) - c_l \) and the monotonicity of \( F(q) \) imply that when a supplier is outperformed by another supplier in terms of marginal benefit, he remains dominated by this supplier for the rest of the demand curve. Therefore, in the optimal allocation a supplier should not be awarded two or more disjoint intervals. Moreover, as we keep enlarging the assigned quantity along the demand curve, eventually we either hit the upper bound of \( K \) or we have selected all \( n \) suppliers into the set \( J(Q) \). The procedure ends when either of them occurs first.

Verifying the optimality. The proof is by contradiction. Let \( A \) denote the allocation that results from the algorithm in the theorem. Following the algorithm, as long as a supplier is assigned to a demand interval, he has the highest marginal benefit in each point of that interval. Moreover, his marginal benefit remains positive in his demand interval (otherwise we have to stop producing since the expected supply chain profit goes down).

Suppose that there exists another allocation \( B \) that is different from \( A \) and yields a higher expected supply chain profit. Since the demand distribution includes no point mass, without loss of generality, we let \( \{(a_{j-1}, a_j), j \in J\} \) denote the collection of demand intervals where the manufacturer assigns differently in \( A \) and \( B \).

Let us focus on a specific interval \( (a_{j-1}, a_j) \). Suppose in \( A \) supplier \( i \) is assigned to \( (a_{j-1}, a_j) \) but in \( B \) it is not covered. Then if we keep the allocation in \( B \) but assign interval \( (a_{j-1}, a_j) \) to supplier \( i \) additionally, the expected supply chain profit becomes higher. This is because the profits from all other portions of the demand curve remain unchanged but supplier \( i \) obtains a strictly positive expected profit from \( (a_{j-1}, a_j) \) (because of the definition of \( Q_{kb} \)). Therefore, \( B \) cannot be optimal. Similarly, if \( (a_{j-1}, a_j) \) is assigned to multiple suppliers in \( B \), then replacing them by supplier \( i \) yields a higher expected supply chain profit as well because supplier \( i \) is the most cost efficient supplier in that interval. Moreover, this argument continues to be valid even if in \( A \), \( (a_{j-1}, a_j) \) is decomposed into several mutually disjoint intervals that are assigned to different suppliers. As long as we replace the set of suppliers assigned to this interval \( (a_{j-1}, a_j) \), the expected supply chain profit can only be higher.

We can do this pairwise interchange step by step to replace all the assignments for \( \{(a_{j-1}, a_j), j \in J\} \). In each step we increase the expected supply chain profit, and therefore ultimately allocation
A should outperform \( B \). When a tie occurs at a certain point, these suppliers have identical cost parameters and hence arbitrary re-allocation between them will not change the expected supply chain profit. Thus, tie-breaking rule is irrelevant. Q.E.D.

**Proof of Lemma 1**

Under the option contracts, the manufacturer’s expected profit is 
\[
\tau_i j \sum \in A_j (a_k - a_{k-1}) - e_{ij} S_j,
\]
where \( \tau_i j \sum \in A_j (a_k - a_{k-1}) \) is the cost of reserving the capacity \( A_j \), and \( e_{ij} S_j \) is the expected additional amount for purchasing the products after the demand materializes.

If we let \( \tau_i j = w_{ij} - b_{ij} \) and \( e_{ij} = b_{ij} \), then the manufacturer’s expected profit becomes
\[
r S_j - \tau_i j \sum \in A_j (a_k - a_{k-1}) - e_{ij} S_j = r S_j - w_{ij} \sum \in A_j (a_k - a_{k-1}) + b_{ij} \sum \in A_j (a_k - a_{k-1}) - S_j = \pi^M_j,
\]
and therefore these two contracts are equivalent from the manufacturer’s perspective. Moreover, supplier \( i \)'s expected profit in this case is 
\[
\tau_i j \sum \in A_j (a_k - a_{k-1}) + e_{ij} S_j - c_i \sum \in A_j (a_k - a_{k-1}) + s_i [\sum \in A_j (a_k - a_{k-1}) - S_j],
\]
where the first two terms are expected payments made by the manufacturer, the third term is the production cost, and the last term is the expected value from salvaging the unsold products. From \( \tau_i j = w_{ij} - b_{ij} \) and \( e_{ij} = b_{ij} \), the supplier’s expected profit can be represented as
\[
(w_{ij} - b_{ij} - c_i + s_i) \sum \in A_j (a_k - a_{k-1}) + (b_{ij} - s_i) S_j = (w_{ij} - c_i) \sum \in A_j (a_k - a_{k-1}) + (s_i - b_{ij}) [\sum \in A_j (a_k - a_{k-1}) - S_j].
\]

On the right-hand side of the above equation, the first term is the supplier’s net profit from building the capacity, and the second term is the salvage value net the buy-back price for the unsold products. Therefore, it equals the supplier \( i \)'s expected profit when he signs a wholesale plus buy-back contract \( (w_{ij}, b_{ij}) \) with the manufacturer. In other words, these two contract forms are also identical from supplier \( i \)'s viewpoint. Likewise, we can follow the same argument to show the equivalence between our contracts and the option contracts for any combination of demand object \( \{A_j, j \neq j\} \) and supplier \( i' \neq i \). Q.E.D.

**Proof of Theorem 2**

Since the expected revenue of the supply chain is the sum of all separate auctions, it suffices to show that if in an auction design \( A_1 \) all \( J \) auctions are separately held and the other auction design \( A_2 \) is the single bundle auction, the former generates a higher total surplus to the supply chain.

Recall that in the second-price sealed-bid auction, the winner of each auction is the supplier that values the object most. Hence after the auctions, the total surplus in \( A_1 \) is 
\[
\sum_{j=1}^{J'} \max_{i \in I} \int_{q \in A_j} [(r - s_i)(1 - F(q)) + (s_i - c_i)] dq,
\]
whereas in the single bundle auction it becomes 
\[
\max_{i \in I} \sum_{j=1}^{J'} \int_{q \in A_j} [(r -
\( s_i [1 - F(q)] + (s_i - c_i) dq \). Since interchanging the sum and the maximum operator results in an unambiguous decrease if the summation is taken first, the total surplus in \( A_1 \) is higher.

For general comparison between two nested auction designs \( A \) and \( B \), consider any auction bundle \( A_j \) in \( B \). Since \( A \) is the refinement of \( B \), either object \( A_j \) is also held as a separate auction in \( A \) or there exist several separate auctions in \( A \) such that the aggregation of these auctioned objects become \( A_j \). The comparison is then applied in auction \( A_j \). This relation holds as we consider every auction included in \( B \) and the corresponding counterparts in \( A \), the result goes through for the comparison between these two auction designs \( A \) and \( B \). Q.E.D.

**Proof of Proposition 1**

Let us first consider two auction design \( A \) and \( B \) where \( A \) is composed of two separate auctions \( A_1 \) and \( A_2 \) and \( B \) has a single bundled object \( A_1 \cup A_2 \). Suppose that supplier \( i_1 \) is the winner in both auctions \( A_1 \) and \( A_2 \) and let \( i_2 \) be the other supplier. Then supplier \( i_1 \) is paid according to the other supplier’s cost parameters \( c_{i_1}, s_{i_2} \) in both \( A \) and \( B \), and hence he is indifferent. Supplier \( i_2 \) does not get any contract in both \( A \) and \( B \) and thus feels indifferent as well.

Now suppose the winners of two separate auctions are different, say, supplier \( i_1 \) is the winner in auction \( A_1 \) and \( i_2 \) is the other supplier \( A_2 \). Without loss of generality assume that in the combined auction \( B \) \( i_1 \) outbids \( i_2 \). In auction \( A \) the expected payoff of suppliers \( i_1 \) and \( i_2 \) are respectively

\[
\int_{q \in A_1} [(r-s_{i_1})(1-F(q)) - (c_{i_1} - s_{i_1})] dq - \int_{q \in A_2} [(r-s_{i_2})(1-F(q)) - (c_{i_2} - s_{i_2})] dq
\]

and

\[
\int_{q \in A_2} [(r-s_{i_2})(1-F(q)) - (c_{i_2} - s_{i_2})] dq - \int_{q \in A_1} [(r-s_{i_1})(1-F(q)) - (c_{i_1} - s_{i_1})] dq,
\]

which by design of the second-price auction are both positive. In the combined auction \( B \), supplier \( i_2 \) is left empty-handed and therefore is worse off. Supplier \( i_1 \)'s expected payoff is then

\[
\int_{q \in B} [(r-s_{i_1})(1-F(q)) - (c_{i_1} - s_{i_1})] dq - \int_{q \in B} [(r-s_{i_2})(1-F(q)) - (c_{i_2} - s_{i_2})] dq
\]

\[
= \int_{q \in A_1} [(r-s_{i_1})(1-F(q)) - (c_{i_1} - s_{i_1})] dq - \int_{q \in A_2} [(r-s_{i_2})(1-F(q)) - (c_{i_2} - s_{i_2})] dq
\]

\[
+ \int_{q \in A_2} [(r-s_{i_1})(1-F(q)) - (c_{i_1} - s_{i_1})] dq - \int_{q \in A_1} [(r-s_{i_2})(1-F(q)) - (c_{i_2} - s_{i_2})] dq
\]

\[
\leq \int_{q \in A_1} [(r-s_{i_1})(1-F(q)) - (c_{i_1} - s_{i_1})] dq - \int_{q \in A_1} [(r-s_{i_2})(1-F(q)) - (c_{i_2} - s_{i_2})] dq,
\]

where the inequality follows from that supplier \( i_2 \) outbids \( i_1 \) if auction \( A_2 \) is held separately. Therefore, both suppliers are worse off when the auctions are combined.

For general auctions, we can apply the same argument piece by piece. Note that suppliers possess additive values for the auctioned objects, i.e., the aggregate expected payoff of a supplier in an auction design is the sum of his expected payoffs from those separate auctions. Thus, we can always
combine two separate auctions and leave other auctions unchanged. This makes both suppliers better off. Performing this iteratively, we conclude that a single bundled auction yields lower expected payoffs for both suppliers than any other auction design. Q.E.D.

**Proof of Proposition 2**

Let us consider two auction designs \( A \) and \( B \) where \( A \) is composed of \( A_1, \ldots, A_J \) and \( B = \cup_{j=1}^{J} A_j \) is a single bundled auction. Suppose that under \( A \) supplier 1 wins the set of auctions \( S_1 \subseteq B \) and supplier 2 wins \( B \setminus S_1 \). This implies

\[
\int_{q \in A_j} [(r - s_1)(1 - F(q)) - (c_1 - s_1)] dq \geq \int_{q \in A_j} [(r - s_2)(1 - F(q)) - (c_2 - s_2)] dq, \forall A_j \in S_1
\]

and

\[
\int_{q \in A_j} [(r - s_1)(1 - F(q)) - (c_1 - s_1)] dq \geq \int_{q \in A_j} [(r - s_2)(1 - F(q)) - (c_2 - s_2)] dq, \forall A_j \in B \setminus S_1.
\]

Now if the auctions are bundled into a single auction \( B \), then the manufacturer pays the winning supplier based on the loser’s cost parameters. Therefore, the manufacturer’s expected payoff by offering the single-bundled auction \( B \) will be

\[
\min \left( \sum_{j=1}^{J} \int_{q \in A_j} [(r - s_1)(1 - F(q)) - (c_1 - s_1)] dq, \sum_{j=1}^{J} \int_{q \in A_j} [(r - s_2)(1 - F(q)) - (c_2 - s_2)] dq \right).
\]

Suppose that supplier 1 outbids supplier 2 in the bundled auction \( B \). Then the manufacturer’s expected payoff in auction \( B \) is

\[
\sum_{j=1}^{J} \int_{q \in A_j} [(r - s_2)(1 - F(q)) - (c_2 - s_2)] dq
\]

\[
\geq \sum_{A_j \in S_1} \int_{q \in A_j} [(r - s_2)(1 - F(q)) - (c_2 - s_2)] dq + \sum_{A_j \in B \setminus S_1} \int_{q \in A_j} [(r - s_1)(1 - F(q)) - (c_1 - s_1)] dq,
\]

where the inequality follows from that suppliers 1 and 2 are respectively more cost efficient in auctioned objects inside \( S_1 \) and \( B \setminus S_1 \). Similarly, we can obtain the same inequality when supplier 2 is the winner for the bundled auction \( B \). Since holding separate auctions the manufacturer always pays according to the less cost efficient supplier and hence receives the lower expected payoff in each separate auction, she is better off while bundling them to a single auction. Q.E.D.

**Proof of Theorem 3**

From Proposition 2 we know that it is in the manufacturer’s interest to hold a single auction. We first show that it is never optimal for the manufacturer to include any unprofitable portion of demand in the auctions.

The proof is by contradiction. Suppose there exist an auction \( A_j \) and an interval \((a_{k-1}, a_k] \) that satisfy \((a_{k-1}, a_k] \in A_j \) and \((a_{k-1}, a_k] \in K \setminus [0, Q^*] \). Now consider the alternative auction design where
all separate auctions remain unchanged except that we remove \((a_{k-1}, a_k)\) from auction \(A_j\). Call this auction \(\tilde{A}_j\). It suffices to show that the new auction design with \(\tilde{A}_j\) in place of \(A_j\) raises the manufacturer’s expected revenue.

Since all auctions other than \(A_j\) remain unchanged, we in the following focus on the difference made in auctions \(A_j\) and \(\tilde{A}_j\). Let \(l_1\) denote the winner and \(l_2\) be the highest losing supplier of auction \(\tilde{A}_j\) respectively. Compared to the case where the manufacturer pays supplier \(l_1\) according to his true cost parameters \(c_{l_1}\) and \(s_{l_1}\), the manufacturer now loses \(\int_{q \in \tilde{A}_j} [(r - s_{l_1})(1 - F(q)) - (c_{l_1} - s_{l_1})]dq - \int_{q \in \tilde{A}_j} [(r - s_{l_1})(1 - F(q)) - (c_{l_1} - s_{l_1})]dq\). In auction \(A_j\), the manufacturer’s expected revenue comes from two parts, the one associated with \((a_{k-1}, a_k)\) and the one from \(A_j \setminus (a_{k-1}, a_k) \equiv \tilde{A}_j\). Suppose that in auction \(A_j\) the highest losing supplier \(l\) is not \(l_1\), then compared to the case where the manufacturer pays supplier \(l_1\) according to his true cost parameters, the manufacturer now loses

\[
\int_{q \in \tilde{A}_j} [(r - s_{l_1})(1 - F(q)) - (c_{l_1} - s_{l_1})]dq - \int_{q \in \tilde{A}_j} [(r - s_{l_1})(1 - F(q)) - (c_{l_1} - s_{l_1})]dq \\
+ \int_{q \in (a_{k-1}, a_k)} [(r - s_{l_1})(1 - F(q)) - (c_{l_1} - s_{l_1})]dq \\
\geq \int_{q \in \tilde{A}_j} [(r - s_{l_1})(1 - F(q)) - (c_{l_1} - s_{l_1})]dq - \int_{q \in \tilde{A}_j} [(r - s_{l_2})(1 - F(q)) - (c_{l_2} - s_{l_2})]dq \\
+ \int_{q \in (a_{k-1}, a_k)} [(r - s_{l_1})(1 - F(q)) - (c_{l_1} - s_{l_1})]dq \\
\geq \int_{q \in \tilde{A}_j} [(r - s_{l_1})(1 - F(q)) - (c_{l_1} - s_{l_1})]dq - \int_{q \in \tilde{A}_j} [(r - s_{l_2})(1 - F(q)) - (c_{l_2} - s_{l_2})]dq,
\]

where the first inequality follows from that supplier \(l_2\) is more cost efficient than supplier \(l\) in \(\tilde{A}_j\); the second inequality holds because the last term is negative according to \((a_{k-1}, a_k) \in K \setminus [0, Q^*]\). Therefore, in this case the manufacturer is better off by offering \(\tilde{A}_j\) rather than \(A_j\).

Now suppose that the highest losing supplier is \(l_1\) in auction \(A_j\) and the winner becomes \(i\). Compared to the base case, the manufacturer loses

\[
\int_{q \in (a_{k-1}, a_k)} [(r - s_{l_1})(1 - F(q)) - (c_{l_1} - s_{l_1})]dq \\
\geq \int_{q \in (a_{k-1}, a_k)} [(r - s_{l_1})(1 - F(q)) - (c_{l_1} - s_{l_1})]dq - \int_{q \in (a_{k-1}, a_k)} [(r - s_{l_1})(1 - F(q)) - (c_{l_1} - s_{l_1})]dq \\
\geq \int_{q \in \tilde{A}_j} [(r - s_{l_1})(1 - F(q)) - (c_{l_1} - s_{l_1})]dq - \int_{q \in \tilde{A}_j} [(r - s_{l_1})(1 - F(q)) - (c_{l_1} - s_{l_1})]dq \\
\geq \int_{q \in \tilde{A}_j} [(r - s_{l_1})(1 - F(q)) - (c_{l_1} - s_{l_1})]dq - \int_{q \in \tilde{A}_j} [(r - s_{l_2})(1 - F(q)) - (c_{l_2} - s_{l_2})]dq,
\]

where the first inequality is because \((a_{k-1}, a_k)\) is beyond \(\tilde{Q}^*\). The second inequality follows from that supplier \(i\) outbids supplier \(l_1\), and hence the gain from including \((a_{k-1}, a_k)\) for supplier \(i\) should
outweigh the loss during $\tilde{A}_j$. The last inequality is by the definition of $l_2$: since $i \neq l_1$, in auction $\tilde{A}_j$ supplier $i$ should be less cost efficient than supplier $l_2$. Therefore, the manufacturer gains more while holding auction $\tilde{A}_j$. Therefore, no portion of demand beyond $\bar{Q}_1$ and $\bar{Q}_2$ shall be included in optimality.

We now show that at optimality the manufacturer must auction an interval of demand that covers $[0, \min(\bar{Q}_1, \bar{Q}_2)]$. Let us assume that an optimal auction $\mathcal{A}$ leaves uncovered some intervals between $[0, \min(\bar{Q}_1, \bar{Q}_2)]$. In this case we shall be able to find an interval $(a_{k-1}, a_k) \subseteq [0, \min(\bar{Q}_1, \bar{Q}_2)]$ that is not covered. Consider another auction design $\mathcal{B}$ which is composed of two separate auctions: $\mathcal{A}$ and $(a_{k-1}, a_k)$. Note that the winner and the expected payoffs of the suppliers and the manufacturer remain the same for the separate auction $\mathcal{A}$, and thus the only difference comes from the new piece of demand. Since $(a_{k-1}, a_k) \subseteq [0, \min(\bar{Q}_1, \bar{Q}_2)]$, even if the manufacturer has to pay according to the cost parameters of the less cost efficient supplier for the auction $(a_{k-1}, a_k)$, his expected payoff by auctioning this piece is still positive. Since the expected payoff is additive, the manufacturer is strictly better off by opening the new auction $(a_{k-1}, a_k)$. This implies that auction design $\mathcal{A}$ cannot be optimal, which leads to a contradiction. Q.E.D.

**Proof of Theorem 4**

We first verify that truth-telling is a dominant strategy. We observe that the contract terms are chosen such that only the most cost efficient supplier will break even if assigned an interval of the demand; all other suppliers will incur a loss.

Suppose that supplier $i$ is the most efficient for interval $(Q_{k-1}, Q_k)$. Due to Theorem 1, if he is assigned to an interval other than $(Q_{k-1}, Q_k)$, he will incur a loss. Suppose no other supplier reports supplier $i$’s true cost $(c_i, s_i)$, then by telling the truth supplier $i$ can ensure himself to obtain the contract for $(Q_{k-1}, Q_k)$. If there are other suppliers who also report the cost parameters $(c_i, s_i)$, the supplier $i$ can still win the contract for the interval $(Q_{k-1}, Q_k)$ with positive probability. Thus, in both cases he will get a null profit, whereas by deviating he would either get no contract or incur a loss. This implies that each supplier’s optimal strategy is to tell the truth, regardless of other suppliers’ behavior. Moreover, since each supplier obtains the null profit, his individual rationality is satisfied.

Given the mechanism, while all suppliers tell the truth, the demand distribution is allocated efficiently, and each supplier receives no surplus. In equilibrium, the allocation is also incentive compatible, and the manufacturer keeps all the surplus. Q.E.D.
Proof of Theorem 5
In the first part ($Q_1 \geq Q_2$), supplier 1 dominates supplier 2 in the entire region. Therefore, the manufacturer’s expected payoff follows from Corollary 1. Now we consider the case $Q_1 > Q_2$. Since the manufacturer can always choose $Q_1$ as the end point, her expected payoff is at least the benefit the losing supplier brings to the supply chain under $Q_1$. In other words, $\min_{i=1,2} \int_0^{Q_1} \{(r-s_i)[1-F(q)]-(c_i-s_i)\} dq$ is a lower bound. Q.E.D.

Proof of Proposition 3

Part (1). Our strategy is to show that the manufacturer’s expected payoff (under the second-price auction) and the expected supply chain profit are both close to the maximum expected profit generated by supplier 1, i.e., $\Pi \equiv \int_0^{Q_1} \{(r-s_1)[1-F(q)]-(c_1-s_1)\} dq$. When this is the case, the ratio should be close to 1. Specifically, if (i) $\pi^M \geq \Pi - \frac{1}{2} \Pi \varepsilon$ and (ii) $\pi^{M*} \leq \Pi + \frac{1}{2} \Pi \varepsilon$, then

$$\frac{\pi^M}{\pi^{M*}} \geq \frac{\Pi - \frac{1}{2} \Pi \varepsilon}{\Pi + \frac{1}{2} \Pi \varepsilon} = 1 - \frac{\Pi \varepsilon}{\Pi + \frac{1}{2} \Pi \varepsilon} \geq 1 - \frac{\Pi \varepsilon}{\Pi} = 1 - \varepsilon.$$ 

In the following, we find conditions such that both parts (i) and (ii) are true.

Consider the manufacturer’s expected payoff under the second-price auction. From Theorem 5, $\pi^M$ is lower bounded by $\min_{i=1,2} \int_0^{Q_1} \{(r-s_i)[1-F(q)]-(c_i-s_i)\} dq$, i.e., $\min\{\Pi, \int_0^{Q_1} \{(r-s_1)[1-F(q)]-(c_1-s_1)\} dq\}$. Therefore, to show that the manufacturer’s expected payoff is lower bounded by $\Pi - \frac{1}{2} \Pi \varepsilon$, it suffices to consider $\int_0^{Q_1} \{(r-s_2)[1-F(q)]-(c_2-s_2)\} dq$. Assume that $\sqrt{(c_1-c_2)^2+(s_1-s_2)^2} \leq \delta_1$, where $\delta_1 > 0$ is a constant to be determined. Rewriting this term, we obtain that

$$\pi^M \geq \Pi - \int_0^{Q_1} \{(r-s_1)[1-F(q)]-(c_1-s_1)\} dq + \int_0^{Q_1} \{(r-s_2)[1-F(q)]-(c_2-s_2)\} dq$$

$$\geq \Pi - \int_0^{Q_1} \{\delta_1[1-F(q)]+2\delta_1\} dq \geq \Pi - 3Q_1\delta_1,$$

where the second inequality follows from the assumption $\sqrt{(c_1-c_2)^2+(s_1-s_2)^2} \leq \delta_1$. Note that if we take $\delta_1 = \Pi \varepsilon/6Q_1$, $\pi^M$ is lower bounded by $\Pi - \frac{1}{2} \Pi \varepsilon$ as desired.

Now we switch to the expected supply chain profit $\pi^{M*}$. If $Q_1 \geq Q_2$, $\pi^{M*} = \Pi$, which is exactly the benchmark profit. Otherwise, supplier 1 produces for $[0, Q_{1a})$, and supplier 2 produces for $[Q_{1a}, Q_2]$. We assume that $\sqrt{(c_1-c_2)^2+(s_1-s_2)^2} \leq \delta_b$, where $\delta_b > 0$ and its value is to be determined later. In this case, we obtain

$$\pi^{M*} = \int_0^{Q_{1a}} \{(r-s_1)[1-F(q)]-(c_1-s_1)\} dq + \int_{Q_{1a}}^{Q_2} \{(r-s_2)[1-F(q)]-(c_2-s_2)\} dq$$

$$= \Pi + \int_{Q_{1a}}^{Q_2} \{(s_1-s_2)[1-F(q)]+(c_1-c_2)-(s_1-s_2)\} dq + \int_{Q_1}^{Q_2} \{(r-s_2)[1-F(q)]-(c_2-s_2)\} dq$$

$$\leq \Pi + \int_{Q_{1a}}^{Q_{1b}} \{\delta_b[1-F(q)]+2\delta_b\} dq + \int_{Q_1}^{Q_2} \{(r-s_2)-(c_2-s_2)\} dq.$$
\[ \leq \Pi + 3\delta_b \bar{Q}_1 + (r - c_2)(\bar{Q}_2 - \bar{Q}_1), \]

where the first inequality follows from \( \sqrt{(c_1 - c_2)^2 + (s_1 - s_2)^2} \leq \delta_b \) and \( r - s_2 > 0 \), and the second inequality is because \( \delta_b[1 - F(q)] + 2\delta_b \geq 0. \)

Our goal is to find the condition such that the extra term \( 3\delta_b \bar{Q}_1 + (r - c_2)(\bar{Q}_2 - \bar{Q}_1) \) is bounded by \( \frac{1}{2}\Pi\varepsilon \). To this end, recall that the density \( f \) exists. Therefore, \( F \) is continuous (and monotonic), and its inverse function \( F^{-1} \) exists and is also continuous. Moreover, the function \( g(c, s) = \frac{c - s}{r - s} \) is also continuous in \((c, s)\) around \((c_1, s_1)\). Since the composition of continuous functions remains continuous, the function \( F^{-1} \circ g(c, s) \) is continuous in \((c, s)\) around \((c_1, s_1)\) as well. The function \( F^{-1} \circ g(c, s) \) gives the threshold beyond which the marginal benefit generated by a supplier with cost parameters \((c, s)\) turns negative. Therefore, given \( \frac{\pi_r}{4(r - c_1)} > 0 \), there exists \( \delta_2 > 0 \) such that if \( \sqrt{(c_1 - c_2)^2 + (s_1 - s_2)^2} \leq \delta_2 \), \( \bar{Q}_2 - \bar{Q}_1 < \frac{\pi_r}{4(r - c_1)} \). Now choose \( \delta_b = \min\{\delta_2, 1, \frac{\pi_r}{4(3Q_1 + \frac{\pi_r}{4(r - c_1)})}\} \). When \( \sqrt{(c_1 - c_2)^2 + (s_1 - s_2)^2} \leq \delta_b \), \( \pi^M = \Pi - \frac{1}{2}\Pi\varepsilon \) since \( \delta_b \leq \frac{\pi_r}{4(3Q_1 + \frac{\pi_r}{4(r - c_1)})} < \frac{\pi_r}{12Q_1} < \frac{\Pi}{6Q_1} \varepsilon = \delta_1 \). Moreover, we have

\[
\pi^{M^*} \leq \Pi + 3\delta_b \bar{Q}_1 + (r - c_2)(\bar{Q}_2 - \bar{Q}_1) \leq \Pi + 3\delta_b \bar{Q}_1 + (r - c_1 + \delta_b)(\bar{Q}_2 - \bar{Q}_1)
\]

\[
\leq \Pi + \delta_b(3\bar{Q}_1 + \frac{\pi_\varepsilon}{4(r - c_1)}) + (r - c_1)\frac{\pi_\varepsilon}{4(r - c_1)} \leq \Pi + \frac{1}{2}\Pi\varepsilon.
\]

Therefore, \( \pi^M \geq \Pi - \frac{1}{2}\Pi\varepsilon \) and \( \pi^{M^*} \leq \Pi + \frac{1}{2}\Pi\varepsilon \), which implies \( \pi^M_{\varepsilon, \Pi\varepsilon} \geq 1 - \varepsilon. \)

**Part (2).** Given \( r, c_1, s_1, \bar{Q}_1 \) is a fixed constant that satisfies \( 1 - F(\bar{Q}_1) = \frac{c_1 - s_1}{r - s_1} \). When \( \frac{c_2 - s_2}{r - s_2} > 1 - F(\bar{Q}_1) \), we know that \( \bar{Q}_2 < \bar{Q}_1 \). In this case, the expected supply chain profit is maximized when supplier 1 produces up to \( \bar{Q}_1 \), and the maximum aggregate expected profit is \( \pi^{M^*} = \int_0^{\bar{Q}_1} \{(r - s_1)[1 - F(q)] - (c_1 - s_1)\} dq \). When the second-price auction is used, the manufacturer should use a single auction to sell the demand interval \([0, \bar{Q}_2]\). Suppose that \( \delta_a < \min(F(\bar{Q}_1), \frac{c_1 - s_1}{2r}) \), where \( E[D] \) is the expected demand. When \( s_2 > r - \delta_a \), the manufacturer’s expected payoff under the second-price auction is:

\[
\pi^M = \int_0^{\bar{Q}_2} \{(r - s_2)[1 - F(q)] - (c_2 - s_2)\} dq = (r - s_2)\{q[1 - F(q)]\}^{\bar{Q}_2}_0 + \int_0^{\bar{Q}_2} q[1 - F(q)] dq - (c_2 - s_2)\bar{Q}_2
\]

\[
= (r - s_2)\bar{Q}_2[1 - F(\bar{Q}_2) - \frac{c_2 - s_2}{r - s_2}] + (r - s_2)\int_0^{\bar{Q}_2} q f(q) dq = (r - s_2)\int_0^{\bar{Q}_2} q f(q) dq,
\]

where in the second equality we have applied the integration by parts, and the last equality follows from the definition of \( \bar{Q}_2 \). Since \( \int_0^{\bar{Q}_2} q f(q) dq \leq \int_0^{\infty} q f(q) dq = E[D] \), \( \pi^M \) can be upper bounded by \( \pi^M \leq (r - s_2)E[D] \leq \frac{\pi^{M^*}}{\varepsilon E[D]} E[D] = \varepsilon \pi^{M^*} \), which implies that part (2) is true. Q.E.D.
Proof of Theorem 6

Since each supplier’s expected payoff is additive across auctions of different demand objects, whether the auctions are held simultaneously or sequentially does not affect a supplier’s bidding strategy for a particular demand object. Moreover, the bids submitted in other auctions, including the supplier’s own bid, do not matter as well. Therefore, in the sequel we focus on the bidding behavior of suppliers for a particular demand object \( A_j \).

We in the following take the beauty contest mechanism as a benchmark, and show that all other three mechanisms are equivalent to it in terms of the manufacturer’s expected payoff. As discussed in Section 5.1, if the beauty contest mechanism is used, the manufacturer chooses the most cost efficient supplier. Moreover, the winning supplier’s surplus is the difference between the valuations of her and the highest losing supplier.

**Menu auction.** Suppose the manufacturer uses the menu auction. Since the auction is the “second-price” type, it is a dominant strategy for each supplier to submit the schedule \( \{p_{ij}, w_{ij}, b_{ij}\} \), where for each pair of \((p_{ij}, w_{ij}, b_{ij})\), we have \( p_{ij} = (w_{ij} - c_i) \sum_{k \in A_j} (a_k - a_{k-1}) + (s_i - b_{ij})[\sum_{k \in A_j} (a_k - a_{k-1}) - S_j] \). Given this schedule, the manufacturer can infer the supplier’s true cost parameters. When the manufacturer awards the demand object \( A_j \) to supplier \( i \) and chooses \((p_{ij}, w_{ij}, b_{ij})\), her net expected payoff is \( rS_j - w_{ij} \sum_{k \in A_j} (a_k - a_{k-1}) + b_{ij} [\sum_{k \in A_j} (a_k - a_{k-1}) - S_j] \). Therefore, she will choose the supplier that is the most cost efficient for \( A_j \). The choice of \((w_{ij}, b_{ij})\) is irrelevant because this only leads to a net transfer between the manufacturer and the winning supplier. Thus, it is without loss of generality to assume that the manufacturer chooses \( w_{ij} = c_i \) and \( b_{ij} = s_i \). Moreover, the price should match the highest losing supplier’s bid, and therefore the winning supplier’s surplus is also the difference between the valuations of her and the highest losing supplier. This establishes the equivalence between the beauty contest mechanism and the menu auction.

**Scoring rule auction.** Suppose the scoring rule auction is used. Following a similar argument, for supplier \( i \), submitting \((0, c_i, s_i)\), i.e., her true cost parameters, is a dominant strategy. The manufacturer again should choose the most cost efficient supplier, which maximizes the expected supply chain profit. After winning the auction, the winning supplier is indifferent among any pair of \((p_{ij}, w_{ij}, b_{ij})\), as long as it matches the highest losing supplier’s score. Therefore, she can choose her true cost parameters \( w_{ij} = c_i \) and \( b_{ij} = s_i \), and her surplus is again the same as that in the beauty contest mechanism.
RFQ mechanism. Suppose an RFQ mechanism is used and the manufacturer specifies the maximum wholesale price (denoted by $w_j$) and the minimum buy-back price (denoted by $b_j$). After winning the auction, the winning supplier can always choose exactly $w_j$ and $b_j$ as the wholesale and buy-back prices, respectively. Given this, each supplier bids truthfully his valuation: $p_{ij} = (w_j - c_i) \sum_{k \in A_j} (a_k - a_{k-1}) + (s_i - b_j) \sum_{k \in A_j} (a_k - a_{k-1}) - S_j$. Thus, the manufacturer will select the most cost efficient supplier. The choices of $w_j$ and $b_j$ are irrelevant for the net transfer of expected payoff between the winning supplier and the manufacturer. Therefore, the expected payoffs of the manufacturer and the supplier are exactly the same as those in the beauty contest mechanism.

VCG mechanism. Finally, the supplier determination problem is the same in the RFQ and VCG mechanism. From the above discussion, it does not matter whether the net transfer is completely determined by the payment or the wholesale and buy-back prices. Therefore, these two mechanisms should be identical. This completes the proof. Q.E.D.

Proof of Proposition 4

Since the seller receives no surplus, we can replace the right-hand side of the (IC) conditions in Equation (5) by 0, and replace the equality of the second equation by inequality. Thus, the linear system becomes

$$0 \geq \sum_{\eta \in \Theta^{n-1}} p_{mn}(V_{nm} - t_{mn}), \forall m, l = 1, \ldots, |\Theta|,$$

$$\sum_{\eta \in \Theta^{n-1}} p_{mn}(V_{nm} - t_{mn}) = 0, \forall m = 1, \ldots, |\Theta|. \quad (8)$$

Decomposing Equation (8) into $\sum_{\eta \in \Theta^{n-1}} p_{mn}(V_{nm} - t_{mn}) \geq 0, \forall m = 1, \ldots, |\Theta|$, and $\sum_{\eta \in \Theta^{n-1}} p_{mn}(V_{nm} - t_{mn}) \leq 0, \forall m = 1, \ldots, |\Theta|$, and removing the duplicate equations, we can rewrite the linear system as the following: $\forall m, l = 1, \ldots, |\Theta|$,

$$-\sum_{\eta \in \Theta^{n-1}} p_{mn}t_{\eta} \leq -\sum_{\eta \in \Theta^{n-1}} p_{mn}V_{\eta m}, \sum_{\eta \in \Theta^{n-1}} p_{mn}t_{mn} \leq \sum_{\eta \in \Theta^{n-1}} p_{mn}V_{mn}.$$

Now let us rearrange the sequence of these inequalities to combine all those inequalities associated with $t_m = \{t_{mn}, \forall \eta \in \Theta^{n-1}\}$. That is, $\forall m = 1, \ldots, |\Theta|$,

$$-\sum_{\eta \in \Theta^{n-1}} p_{\eta m} t_{mn} \leq -\sum_{\eta \in \Theta^{n-1}} p_{\eta m} V_{\eta m}, \forall l = 1, \ldots, |\Theta|,$$

$$\sum_{\eta \in \Theta^{n-1}} p_{mn} t_{mn} \leq \sum_{\eta \in \Theta^{n-1}} p_{mn} V_{mn}.$$

Representing it in matrix form, we have $\left(\frac{-P}{P_m}\right) t_m^T \leq \left(\frac{-V_m}{v_{mm}}\right), \forall m = 1, \ldots, |\Theta|$.
Recall that Gale’s theorem of the alternative: for any pair of matrices $A$ and $b$ (with appropriate dimensions), a linear system $(Ax \leq b)$ is feasible if and only if the alternative system \(\left( y' A = 0, y' b < 0, y \geq 0 \right)\) has no solution (see, e.g., Mangasarian (1969)). Therefore, the matrix form presented above gives rise to the alternative system as stated in the theorem. Q.E.D.

**Proof of Theorem 7**

For notational convenience, we define $\Lambda_m = \begin{pmatrix} -P_m \\ P_m \end{pmatrix}$, $B_m = \begin{pmatrix} V_m \\ v_{mm} \end{pmatrix}$, $\forall m = 1, \ldots, |\Theta|$. We also denote $y_{lm}$ as the dual variables corresponding to the truth-telling constraints (Constraint 5). Furthermore, the dual variables, corresponding to the individual rationality for type-$\theta^m$ (Constraint 6), are $w_m$, $\forall m = 1, \ldots, |\Theta|$. Given these dual definitions, the vector $y_m$ in Equation (7) can be expressed as $y_m = [y_{m1}, \ldots, y_{m|\Theta|}, w_m]^T$, $\forall m = 1, \ldots, |\Theta|$. We in the following show that the conditions stated in the theorem indeed make the alternative system (i.e., Equation (7)) infeasible.

Suppose that the only solution to $(y_m^T \Lambda_m = 0, y_m \geq 0)$ is $y_m = 0$. Then $y_m^T B_m < 0$ cannot be satisfied for any $B_m$. Both conditions (1a) and (1b) are sufficient for this case. $(y_m^T \Lambda_m = 0)$ can be rewritten as $(w_m - y_{mm})P_m = \sum_{l \neq m} y_{ml} P_l$. Since $y_{ml} \geq 0, \forall l = 1, \ldots, |\Theta|$, $w_m - y_{mm} \geq 0$ as well. Therefore, when either condition (1a) or condition (1b) holds, we have $w_m - y_{mm} = 0$ and $y_{ml} = 0, \forall l = 1, \ldots, |\Theta|$. We then obtain that $y_m^T B_m = (w_m - y_{mm})v_{mm} = 0$, which implies that the $m$-th alternative system is infeasible, $\forall m = 1, \ldots, |\Theta|$. By Proposition 4, full surplus extraction is achievable in these cases.

Now suppose that for some $m \in \{1, \ldots, |\Theta|\}$, $(y_m^T \Lambda_m = 0, y_m \geq 0)$ has a non-trivial solution. That is, there exist a set of nonnegative parameters $\{y_{ml}\}$’s and $w_m$ such that $-\sum_{l \neq m} p_{l\eta} y_{ml} + p_{m\eta} w_m = 0, \forall \eta \in \Theta^{n-1}$. Rearranging the above equation, we obtain

$$p_{m\eta}(-y_{mm} + w_m) = \sum_{l \neq m} p_{l\eta} y_{ml}, \forall \eta \in \Theta^{n-1} \Rightarrow -y_{mm} + w_m = \sum_{l \neq m} y_{ml}, \quad (9)$$

where we have applied that $\sum_{\eta \in \Theta^{n-1}} p_{\eta} = 1, \forall l \in M$. Now consider $y_m^T B_m < 0$. We can rewrite $y_m^T B_m$ as follows:

$$y_m^T B_m = v_{mm} w_m + \sum_{l = 1, \ldots, |\Theta|} (-v_{ml}) y_{ml} = (-y_{mm} + w_m) v_{mm} + \sum_{l \neq m} (-v_{ml}) y_{ml}$$

$$= \left( \sum_{l \neq m} y_{ml} \right) v_{mm} + \sum_{l \neq m} (-v_{ml}) y_{ml} = \sum_{l \neq m} y_{ml} (v_{mm} - v_{ml}).$$

Since $\{y_{ml}, \forall l = 1, \ldots, |\Theta|\}$ are all nonnegative, if $v_{mm} - v_{ml} \geq 0, \forall l = 1, \ldots, |\Theta|$, then $y_m^T B_m < 0$ cannot be possible. This gives rise to the second set of sufficient conditions.
Finally, if there exists a solution to \((\Lambda_m t_m = B_m)\), it is also feasible to \((\Lambda_m t_m \leq B_m)\). Expressing 
\((\Lambda_m t_m = B_m)\) in algebraic form, we have:

\[-\sum_{\eta \in \Theta^{n-1}} p_{\eta} t_{mn} = -v_{ml}, \forall l = 1, \ldots, |\Theta|,
\]

and \(\sum_{\eta \in \Theta^{n-1}} p_{\eta} t_{mn} = -v_{mm}\). The second equality is redundant since it is equivalent to the special case of the first equality when \(l = m\). Thus, we obtain \(P t_m = V_m\). If \(V_m \in \text{span}(P), \forall m = 1, \ldots, n\), then such franchise fees \(\{t_m\}'s\) are always feasible. Q.E.D.

**Proof of Proposition 5**

When \(\bar{Q}_1 \geq \bar{Q}_2\), the VCG mechanism with endogenous partition yields the expected profit \(\int_{0}^{\bar{Q}_2} \{(r - s_2)\{1 - F(q)\} - (c_2 - s_2)\} dq\) for the manufacturer. Suppose under the pre-determined partition (bundling), the manufacturer chooses to auction the demand interval \([0, \bar{Q}]\). When \(\bar{Q} \leq \bar{Q}_2\), supplier 1 wins, and the manufacturer captures the expected profit \(\int_{0}^{\bar{Q}} \{(r - s_2)\{1 - F(q)\} - (c_2 - s_2)\} dq\). This is less than \(\int_{0}^{\bar{Q}_2} \{(r - s_2)\{1 - F(q)\} - (c_2 - s_2)\} dq\) by definition of \(\bar{Q}_2\). If \(\bar{Q} > \bar{Q}_2\), there are two cases. If supplier 1 wins, then the manufacturer gets

\[
\int_{0}^{\bar{Q}_2} \{(r - s_2)\{1 - F(q)\} - (c_2 - s_2)\} dq
\]

\[
= \int_{0}^{\bar{Q}_2} \{(r - s_2)\{1 - F(q)\} - (c_2 - s_2)\} dq + \int_{\bar{Q}_2}^{\bar{Q}} \{(r - s_2)\{1 - F(q)\} - (c_2 - s_2)\} dq
\]

\[
< \int_{0}^{\bar{Q}_2} \{(r - s_2)\{1 - F(q)\} - (c_2 - s_2)\} dq,
\]

since the second term is negative. If supplier 2 wins, then the manufacturer gets an expected profit that is even less than \(\int_{0}^{\bar{Q}} \{(r - s_2)\{1 - F(q)\} - (c_2 - s_2)\} dq\). Therefore, using the endogenous partition is better. Q.E.D.

**Proof of Proposition 6**

When \(\bar{Q}_2 > \bar{Q}_1\), under the VCG mechanism with endogenous partition, the manufacturer can obtain from supplier 1: \(\int_{0}^{\bar{Q}_2} \{(r - s_2)\{1 - F(q)\} - (c_2 - s_2)\} dq - \int_{\bar{Q}_1}^{\bar{Q}_2} \{(r - s_2)\{1 - F(q)\} - (c_2 - s_2)\} dq = \int_{0}^{\bar{Q}_1} \{(r - s_2)\{1 - F(q)\} - (c_2 - s_2)\} dq\). Similarly, from supplier 2, she gets \(\int_{\bar{Q}_1}^{\bar{Q}_2} \{(r - s_1)\{1 - F(q)\} - (c_1 - s_1)\} dq\). Now consider the case when the manufacturer uses two auctions for \([0, \bar{Q}_1]\) and \([\bar{Q}_1, \bar{Q}_2]\). By definition of \(\bar{Q}_1\), supplier 1 wins \([0, \bar{Q}_1]\) but supplier 2 wins \([\bar{Q}_1, \bar{Q}_2]\). According to Theorem 6, any procurement mechanism yields the same expected payoff for the manufacturer. Since the second-price auction is used, the manufacturer obtains profits \(\int_{0}^{\bar{Q}_1} \{(r - s_2)\{1 - F(q)\} - (c_2 - s_2)\} dq\) and \(\int_{\bar{Q}_1}^{\bar{Q}_2} \{(r - s_1)\{1 - F(q)\} - (c_1 - s_1)\} dq\) from these two auctions respectively. Therefore, they are revenue equivalent to the VCG mechanism with endogenous partition. Q.E.D.
Endnotes

1 The results in this section are similar to those stated in Martinez-de-Albeniz and Simchi-Levi (2003). They show that a supplier is active (efficient) if and only if his bid (reservation price and execution price) falls on the lower envelope (piecewise linear convex hull) of the bids of all suppliers.

2 If the demand realization is not contractible, the manufacturer might have incentive to select the suppliers to return the unsold products according to their buy-back prices. This incentive can be eliminated if we further assume that the ordering of salvage values of suppliers is identical to the ordering of suppliers according to the efficient allocation in Theorem 1.

3 The results are not prone to the choice of these probabilities, as long as each tied supplier is assigned the demand interval with a strictly positive probability.

4 Krishna and Perry (1998) only consider the efficient allocation of finitely many objects, but their results can be easily extended to incorporate the case with an infinitely divisible object (e.g., the supply contract in our procurement problem).

5 This is because the conditional probability vectors \( \{P_m\} \) get longer when more suppliers are involved, whereas the number of vectors remains the same. Therefore, these vectors are more likely to be linearly independent when the manufacturer contracts with more suppliers.

6 In the online appendix, we provide examples of situations to illustrate when these conditions are likely to hold, see http://www.stern.nyu.edu/~ychen0/SupplierOption-Appendix.pdf.

References


