CHAPTER 2

INTRINSIC VALUATION

Every asset that generates cash flows has an intrinsic value that reflects both its cash flow potential and its risk. While many analysts claim that when there is significant uncertainty about the future, estimating intrinsic value becomes not just difficult but pointless, we disagree. Notwithstanding this uncertainty, we believe that it is important that we look past market perceptions and gauge, as best as we can, the intrinsic value of a business or asset. In this chapter, we consider how discounted cash flow valuation models attempt to estimate intrinsic value, estimation details and possible limitations.

Discounted Cash flow Valuation

In discounted cashflows valuation, the value of an asset is the present value of the expected cashflows on the asset, discounted back at a rate that reflects the riskiness of these cashflows. In this section, we will look at the foundations of the approach and some of the preliminary details on how we estimate its inputs.

The Essence of DCF Valuation

We buy most assets because we expect them to generate cash flows for us in the future. In discounted cash flow valuation, we begin with a simple proposition. The value of an asset is not what someone perceives it to be worth but it is a function of the expected cash flows on that asset. Put simply, assets with high and predictable cash flows should have higher values than assets with low and volatile cash flows.

The notion that the value of an asset is the present value of the cash flows that you expect to generate by holding it is neither new nor revolutionary. The earliest interest rate tables date back to 1340, and the intellectual basis for discounted cash flow valuation was laid by Alfred Marshall and Bohm-Bawerk in the early part of the twentieth century. The principles of modern valuation were developed by Irving Fisher in two books that he published – The Rate of Interest in 1907 and The Theory of Interest in 1930. In these

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books, he presented the notion of the internal rate of return. In the last 50 years, we have seen discounted cash flow models extend their reach into security and business valuation, and the growth has been aided and abetted by developments in portfolio theory.

Using discounted cash flow models is in some sense an act of faith. We believe that every asset has an intrinsic value and we try to estimate that intrinsic value by looking at an asset’s fundamentals. What is intrinsic value? Consider it the value that would be attached to an asset by an all-knowing analyst with access to all information available right now and a perfect valuation model. No such analyst exists, of course, but we all aspire to be as close as we can to this perfect analyst. The problem lies in the fact that none of us ever gets to see what the true intrinsic value of an asset is and we therefore have no way of knowing whether our discounted cash flow valuations are close to the mark or not.

**Equity versus Firm Valuation**

Of the approaches for adjusting for risk in discounted cash flow valuation, the most common one is the risk adjusted discount rate approach, where we use higher discount rates to discount expected cash flows when valuing riskier assets, and lower discount rates when valuing safer assets. There are two ways in which we can approach discounted cash flow valuation and they can be framed in terms of the financial balance sheet that we introduced in chapter 1. The first is to value the entire business, with both existing assets (assets-in-place) and growth assets; this is often termed firm or enterprise valuation.
**Figure 2.1: Valuing a Firm (Business)**

<table>
<thead>
<tr>
<th>Assets</th>
<th>Liabilities</th>
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</thead>
<tbody>
<tr>
<td>Assets in Place</td>
<td>Debt</td>
</tr>
<tr>
<td>Growth Assets</td>
<td>Equity</td>
</tr>
</tbody>
</table>

Cash flows considered are cashflows from assets, prior to any debt payments but after firm has reinvested to create growth assets. Discount rate reflects the cost of raising both debt and equity financing, in proportion to their use.

Present value is value of the entire firm, and reflects the value of all claims on the firm.

The cash flows before debt payments and after reinvestment needs are termed **free cash flows to the firm**, and the discount rate that reflects the composite cost of financing from all sources of capital is the **cost of capital**.

The second way is to just value the equity stake in the business, and this is termed equity valuation.

**Figure 2.2: Valuing Equity**

<table>
<thead>
<tr>
<th>Assets</th>
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</tr>
<tr>
<td>Growth Assets</td>
<td>Equity</td>
</tr>
</tbody>
</table>

Cash flows considered are cashflows from assets, after debt payments and after making reinvestments needed for future growth. Discount rate reflects only the cost of raising equity financing.

Present value is value of just the equity claims on the firm.

The cash flows after debt payments and reinvestment needs are called **free cash flows to equity**, and the discount rate that reflects just the cost of equity financing is the cost of equity. With publicly traded firms, it can be argued that the only cash flow equity investors get from the firm is dividends and that discounting expected dividends back at the cost of equity should yield the value of equity in the firm. T
Note also that we can always get from the former (firm value) to the latter (equity value) by netting out the value of all non-equity claims from firm value. Done right, the value of equity should be the same whether it is valued directly (by discounting cash flows to equity at the cost of equity) or indirectly (by valuing the firm and subtracting out the value of all non-equity claims).

**Inputs to a DCF Valuation**

While we can choose to value just the equity or the entire business, we have four basic inputs that we need for a value estimate, though how we define the inputs will be different depending upon whether you do firm or equity valuation. Figure 2.3 summarizes the determinants of value.

The first input is the cashflow from existing assets, defined either as pre-debt (and to the firm) or as post-debt (and to equity) earnings, net of reinvestment to generate future growth. With equity cashflows, we can use an even stricter definition of cash flow and consider only dividends paid. The second input is growth, with growth in operating income being the key input when valuing the entire business and growth in equity income (net income or earnings per share) becoming the focus when valuing equity. The third input is the discount rate, defined as the cost of the overall capital of the firm, when valuing the business, and as cost of equity, when valuing equity. The final input, allowing...
for closure, is the terminal value, defined as the estimated value of firm (equity) at the end of the forecast period in firm (equity) valuation.

For the rest of this section, we will focus on estimating the inputs into discounted cash flow models, starting with the cashflows, moving on to risk (and discount rates) and then closing with a discussion of how best to estimate the growth rate for the high growth period and the value at the end of that period.

**Cash Flows**

Leading up to this section, we noted that cash flows can be estimated to either just equity investors (cash flows to equity) or to all suppliers of capital (cash flows to the firm). In this section, we will begin with the strictest measure of cash flow to equity, i.e. the dividends received by investors, and then progressively move to more expansive measures of cash flows, which generally require more information.

**Dividends**

When an investor buys stock, he generally expects to get two types of cash flows - dividends during the holding period and an expected price at the end of the holding period. Since this expected price is itself determined by future dividends, the value of a stock is the present value of just expected dividends. If we accept this premise, the only cash flow to equity that we should be considering in valuation is the dividend paid, and estimating that dividend for the last period should be a simple exercise. Since many firms do not pay dividends, this number can be zero, but it should never be negative.

**Augmented Dividends**

One of the limitations of focusing on dividends is that many companies, especially in the United States but increasingly around the world, have shifted from dividends to stock buybacks as their mechanism for returning cash to stockholders. While only those stockholders who sell their stock back receive cash, it still represents cash returned to equity investors. In 2007, for instance, firms in the United States returned twice as much cash in the form of stock buybacks than they did in dividends, and focusing only on dividends will result in the under valuation of equity. One simple way
of adjusting for this is to augment the dividend with stock buybacks and look at the cumulative cash returned to stockholders.

Augmented Dividends = Dividends + Stock Buybacks

One problem, though, is that unlike dividends that are smoothed out over time, stock buybacks can spike in some years and be followed by years of inaction. We therefore will have to normalize buybacks by using average buybacks over a period of time (say, 5 years) to arrive at more reasonable annualized numbers.

Potential Dividends (Free Cash flow to Equity)

With both dividends and augmented dividends, we are trusting managers at publicly traded firms to return to pay out to stockholders any excess cash left over after meeting operating and reinvestment needs. However, we do know that managers do not always follow this practice, as evidenced by the large cash balances that you see at most publicly traded firms. To estimate what managers could have returned to equity investors, we develop a measure of potential dividends that we term the free cash flow to equity. Intuitively, the free cash flow to equity measures the cash left over after taxes, reinvestment needs and debt cash flows have been met. It is measured as follows:

\[ \text{FCFE} = \text{Net Income} - \text{Reinvestment Needs} - \text{Debt Cash flows} \]

\[ = \text{Net Income} + (\text{Capital Expenditures} - \text{Depreciation} + \text{Change in non-cash working Capital} - \text{Principal}) - (\text{Repayments} + \text{New Debt Issues}) \]

Consider the equation in pieces. We begin with net income, since that is the earnings generated for equity investors; it is after interest expenses and taxes. We compute what the firm has to reinvest in two parts:

a. Reinvestment in long-lived assets is measured as the difference between capital expenditures (the amount invested in long lived assets during the period) and depreciation (the accounting expense generated by capital expenditures in prior periods). We net the latter because it is not a cash expense and hence can be added back to net income.

b. Reinvestment in short-lived assets is measured by the change in non-cash working capital. In effect, increases in inventory and accounts receivable represent cash tied up in assets that do not generate returns – wasting assets. The reason we done
consider cash in the computation is because we assume that companies with large cash balances generally invest them in low-risk, marketable securities like commercial paper and treasury bills; these investments earn a low but a fair rate of return and are therefore not wasting assets. To the extent that they are offset by the use of supplier credit and accounts payable, the effect on cash flows can be muted. The overall change in non-cash working capital therefore is investment in short term assets.

Reinvestment reduces cash flow to equity investors, but it provides a payoff in terms of future growth. We will come back and consider whether the net effect is positive or negative after we consider how best to estimate growth. The final input into the process are the negative cash flows associated with the repayment of old debt and the positive cash flows to equity investors from raising new debt. If old debt is replaced with new debt of exactly the same magnitude, this term will be zero, but it will generate positive (negative) cash flows when debt issues exceed (are less than) debt repayments.

Focusing on just debt cash flows allows us to zero in on a way to simplify this computation. In the special case where the capital expenditures and the working capital are expected to be financed at a fixed debt ratio $\delta$, and principal repayments are made from new debt issues, the FCFE is measured as follows:

$$FCFE = \text{Net Income} + (1-\delta) (\text{Capital Expenditures} - \text{Depreciation}) + (1-\delta) \Delta \text{Working Capital}$$

In effect, we are assuming that a firm with a 30% debt ratio that is growing through reinvestment will choose to fund 30% of its reinvestment needs with new debt and replace old debt that comes due with new debt.

There is one more way in which we can present the free cash flow to equity. If we define the portion of the net income that equity investors reinvest back into the firm as the equity reinvestment rate, we can state the FCFE as a function of this rate.

Equity Reinvestment Rate

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3 Note that we do not make the distinction between operating and non-operating cash that some analysts do (they proceed to include operating cash in working capital). Our distinction is between wasting cash (which would include currency or cash earning below-market rate returns) and non-wasting cash. We are assuming that the former will be a small or negligible number at a publicly traded company.
\[
\text{FCFE} = \text{Net Income} \left(1 - \text{Equity Reinvestment Rate}\right)
\]

A final note on the contrast between the first two measures of cash flows to equity (dividends and augmented dividends) and this measure. Unlike those measures, which can never be less than zero, the free cash flow to equity can be negative for a number of reasons. The first is that the net income could be negative, a not uncommon phenomenon even for mature firms. The second is that reinvestment needs can overwhelm net income, which is often the case for growth companies, especially early in the life cycle. The third is that large debt repayments coming due that have to funded with equity cash flows can cause negative FCFE; highly levered firms that are trying to bring their debt ratios down can go through years of negative FCFE. The fourth is that the quirks of the reinvestment process, where firms invest large amounts in long-lived and short-lived assets in some years and nothing in others, can cause the FCFE to be negative in the big reinvestment years and positive in others. As with buybacks, we have to consider normalizing reinvestment numbers across time when estimating cash flows to equity. If the FCFE is negative, it is indicative of the firm needing to raise fresh equity.

**Cash Flow to the Firm**

The cash flow to the firm that we would like to estimate should be both after taxes and after all reinvestment needs have been met. Since a firm raises capital from debt and equity investors, the cash flow to the firm should be before interest and principal payments on debt. The cash flow to the firm can be measured in two ways. One is to add up the cash flows to all of the different claim holders in the firm. Thus, the cash flows to equity investors (estimated using one of the three measures described in this section) are added to the cash flows to debt holders (interest and net debt payments) to arrive at the cash flow. The other approach is to start with operating earnings and to estimate the cash flows to the firm prior to debt payments but after reinvestment needs have been met:

\[
\text{Free Cash flow to firm (FCFF)} = \text{After-tax Operating Income} - \text{Reinvestment} = \text{After-tax Operating Income} - (\text{Capital Expenditures} - \text{Depreciation} + \text{Change in non-cash Working Capital})
\]
It is easiest to understand FCFF by contrasting it with FCFE. First, we begin with after-tax operating income instead of net income; the former is before interest expenses whereas the latter is after interest expenses. Second, we adjust the operating income for taxes, computed as if you were taxed on the entire income, whereas net income is already an after-tax number.\(^4\) Third, while we subtract out reinvestment, just as we did to arrive at free cash flows to equity, we do not net out the effect of debt cash flows, since we are now looking at cash flows to all capital and not just to equity.

Another way of presenting the same equation is to cumulate the net capital expenditures and working capital change into one number, and state it as a percentage of the after-tax operating income. This ratio of reinvestment to after-tax operating income is called the reinvestment rate, and the free cash flow to the firm can be written as:

\[
\text{Reinvestment Rate} = \frac{(\text{Capital Expenditures} - \text{Depreciation} + \Delta \text{Working Capital})}{\text{After} - \text{tax Operating Income}}
\]

Free Cash Flow to the Firm = EBIT \((1-t)\) \((1 - \text{Reinvestment Rate})\)

Note that the reinvestment rate can exceed 100%\(^5\), if the firm has substantial reinvestment needs. The reinvestment rate can also be less than zero, for firms that are divesting assets and shrinking capital.

A few final thoughts about free cash flow to the firm are worth noting before we move on to discount rates. First, the free cash flow to the firm can be negative, just as the FCFE can, but debt cash flows can no longer be the culprit; even highly levered firms that are paying down debt will report positive FCFF while also registering negative FCFE. If the FCFF is negative, the firm will be raising fresh capital, with the mix of debt and equity being determined by the mix used to compute the cost of capital. Second, the cash flow to the firm is the basis for all cash distributions made by the firm to its investors; dividends, stock buybacks, interest payments and debt repayments all have to be made out of these cash flows.

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\(^4\) In effect, when computing taxes on operating income, we act like we have no interest expenses or tax benefits from those expenses while computing the cash flow. That is because we will be counting the tax benefits from debt in the cost of capital (through the use of an after-tax cost of debt). If we use actual taxes paid or reflect the tax benefits from interest expenses in the cash flows, we will be double counting its effect.

\(^5\) In practical terms, this firm will have to raise external financing, either from debt or equity or both, to cover the excess reinvestment.
Illustration 2.1: Estimating Cash flows for a firm – 3M in 2007

Minnesota Mining and Manufacturing (3M) is a large market capitalization company, with operations in transportation, health care, office supplies and electronics.

- In 2007, the firm reported operating income, before taxes, of $5,344 million and net income of $4,096 million; interest expenses for the year amounted to $210 million and interest income on cash and marketable securities was $132 million. The firm also paid dividends of $1.380 million during the year and bought back $3,239 million of stock. The effective tax rate during the year was 32.1% but the marginal tax rate is 35%.
- During 2007, 3M reported $1,422 million in capital expenditures and cash acquisitions of $539 million. The depreciation and amortization charges for the year amounted to $1,072 million. The non-cash working capital increased by $243 million during 2007.
- Finally, 3M repaid $2,802 million of debt during the year but raised $4,024 million in new debt.

With this data, we can first estimate the free cashflows to equity:

\[
\text{Net Income} = $4,010 - (\text{Capital expenditures} - \text{Depreciation}) = $4,010 - $889 - \text{Change in non-cash working capital} = $4,010 - $243 + \text{Net Debt Issued (paid)} = $4,010 + $1,222 = $4,100
\]

Note that the net debt issued reflects the new debt issues, netted out against debt repaid.

The free cash flow to the firm for 2007 can also be computed:

\[
\text{EBIT (1-t)} - (\text{Capital expenditures} - \text{Depreciation}) - \text{Change in non-cash working capital} = $3,474 - $889 - $243 = $2,342
\]

Total reinvestment = $889 + $243 = 1132
Equity Reinvestment Rate = 1132/3474= 36.37%
Figure 2.4 summarizes all four estimates of cashflows for 3M for 2007 – dividends, augmented dividends, free cash flows to equity and free cash flows to the firm:

*Figure 2.4: Comparison of Cash Flow Estimates: 3M in 2007*

How can we reconcile these very different numbers? During 2007, 3M increased its borrowing and used the funds from the additional debt and cash accumulated in prior years to buy back stock.

**Risk**

Cash flows that are riskier should be assessed a lower value than more stable cashflows, but how do we measure risk and reflect it in value? In conventional discounted cash flow valuation models, the discount rate becomes the vehicle for conveying our concerns about risk. We use higher discount rates on riskier cash flows and lower discount rates on safer cash flows. In this section, we will begin by contrasting how the risk in equity can vary from the risk in a business, and then consider the mechanics of estimating the cost of equity and capital.
Business Risk versus Equity Risk

Before we delve into the details of risk measurement and discount rates, we should draw a contrast between two different ways of thinking about risk that relate back to the financial balance sheet that we presented in chapter 1. In the first, we think about the risk in a firm’s operations or assets, i.e., the risk in the business. In the second, we look at the risk in the equity investment in this business. Figure 2.5 captures the differences between the two measures:

Figure 2.5: Risk in Business versus Risk in Equity

As with any other aspect of the balance sheet, this one has to balance has well, with the weighted risk in the assets being equal to the weighted risk in the ingredients to capital – debt and equity. Note that the risk in the equity investment in a business is partly determined by the risk of the business the firm is in and partly by its choice on how much debt to use to fund that business. The equity in a safe business can be rendered risky, if the firm uses enough debt to fund that business.

In discount rate terms, the risk in the equity in a business is measured with the cost of equity, whereas the risk in the business is captured in the cost of capital. The latter will be a weighted average of the cost of equity and the cost of debt, with the weights reflecting the proportional use of each source of funding.

Measuring Equity Risk and the Cost of Equity

Measuring the risk in equity investments and converting that risk measure into a cost of equity is rendered difficult by two factors. The first is that equity has an implicit cost, which is unobservable, unlike debt, which comes with an explicit cost in the form of an interest rate. The second is that risk in the eyes of the beholder and different equity
investors in the same business can have very different perceptions of risk in that business and demand different expected returns as a consequence.

The Diversified Marginal Investor

If there were only one equity investor in a company, estimating equity risk and the cost of equity would be a far simpler exercise. We would measure the risk of investing in equity in that company to the investor and assess a reasonable rate of return, given that risk. In a publicly traded company, we run into the practical problem that the equity investors number in the hundreds, if not the thousands, and that they not only vary in size, from small to large investors, but also in risk aversion. So, whose perspective should we take when measuring risk and cost of equity? In corporate finance and valuation, we develop the notion of the marginal investor, i.e., the investor most likely to influence the market price of publicly traded equity. The marginal investor in a publicly traded stock has to own enough stock in the company to make a difference and be willing to trade on that stock. The common theme shared by risk and return models in finance is that the marginal investor is diversified, and we measure the risk in an investment as the risk added to a diversified portfolio. Put another way, it is only that portion of the risk in an investment that is attributable to the broader market or economy, and hence not diversifiable, that should be built into expected returns.

Models for Expected Return (Cost of Equity)

It is on the issue of how best to measure this non-diversifiable risk that the different risk and return models in finance part ways. Let use consider the alternatives:

- In the capital asset pricing model (CAPM), this risk is captured in the beta that we assign an asset/business, with that number carrying the burden of measuring exposure to all of the components of market risk. The expected return on an investment can then be specified as a function of three variables – the risk-free rate, the beta of the investment and the equity risk premium (the premium demanded for investing in the average risk investment):

  Expected Return = Riskfree Rate + Beta_{Investment} (Equity Risk Premium)

  The risk-free rate and equity risk premium are the same for all investments in a market but the beta will capture the market risk exposure of the investment; a beta of one
represents an average risk investment, and betas above (below) one indicate investments that are riskier (safer) than the average risk investment in the market.

• In the arbitrage pricing and multi-factor models, we allow for multiple sources of non-diversifiable (or market) risk and estimate betas against each one. The expected return on an investment can be written as a function of the multiple betas (relative to each market risk factor) and the risk premium for that factor. If there are k factors in the model with $\beta_{ji}$ and Risk Premium$_j$ representing the beta and risk premium of factor j, the expected return on the investment can be written as:

$$\text{Expected Return} = \text{Riskfree Rate} + \sum_{j=1}^{k} \beta_{ji} \times \text{Risk Premium}_j$$

Note that the capital asset pricing model can be written as a special case of these multi-factor models, with a single factor (the market) replacing the multiple factors.

• The final class of models can be categorized as proxy models. In these models, we essentially give up on measuring risk directly and instead look at historical data for clues on what types of investments (stocks) have earned high returns in the past, and then use the common characteristic(s) that they share as a measure of risk. For instance, researchers have found that market capitalization and price to book ratios are correlated with returns; stocks with small market capitalization and low price to book ratios have historically earned higher returns than large market stocks with higher price to book ratios. Using the historical data, we can then estimate the expected return for a company, based on its market capitalization and price to book ratio.

$$\text{Expected Return} = a + b(\text{Market Capitalization}) + c(\text{Price to Book Ratio})$$

Since we are no longer working within the confines of an economic model, it is not surprising that researchers keep finding new variables (trading volume, price momentum) that improve the predictive power of these models. The open question, though, is whether these variables are truly proxies for risk or indicators of market inefficiency. In effect, we may be explaining away the misvaluation of classes of stock by the market by using proxy models for risk.
**Estimation Issues**

With the CAPM and multi-factor models, the inputs that we need for the expected return are straightforward. We need to come up with a risk free rate and an equity risk premium (or premiums in the multi-factor models) to use across all investments. Once we have these market-wide estimates, we then have to measure the risk (beta or betas) in individual investments. In this section, we will lay out the broad principles that will govern these estimates but we will return in future chapters to the details of how best to make these estimates for different types of businesses:

- **The riskfree rate** is the expected return on an investment with guaranteed returns; in effect, you expected return is also your actual return. Since the return is guaranteed, there are two conditions that an investment has to meet to be riskfree. The first is that the entity making the guarantee has to have no default risk; this is why we use government securities to derive riskfree rates, a necessary though not always a sufficient condition. As we will see in chapter 6, there is default risk in many government securities that is priced into the expected return. The second is that the time horizon matters. A six-month treasury bill is not riskfree, if you are looking at a five-year time horizon, since we are exposed to reinvestment risk. In fact, even a 5-year treasury bond may not be riskless, since the coupons received every six months have to be reinvested. Clearly, getting a riskfree rate is not as simple as it looks at the outset.

- **The equity risk premium** is the premium that investors demand for investing in risky assets (or equities) as a class, relative to the riskfree rate. It will be a function not only of how much risk investors perceive in equities, as a class, but the risk aversion that they bring to the market. It also follows that the equity risk premium can change over time, as market risk and risk aversion both change. The conventional practice for estimating equity risk premiums is to use the historical risk premium, i.e., the premium investors have earned over long periods (say 75 years) investing in equities instead of riskfree (or close to riskfree) investments. In chapter 7, we will question the efficacy of this process and offer alternatives.

- To estimate the beta in the CAPM and betas in multi-factor models, we draw on statistical techniques and historical data. The standard approach for estimating the
CAPM beta is to run a regression of returns on a stock against returns on a broad equity market index, with the slope capturing how much the stock moves, for any given market move. To estimate betas in the arbitrage pricing model, we use historical return data on stocks and factor analysis to extract both the number of factors in the models, as well as factor betas for individual companies. As a consequence, the beta estimates that we obtain will always be backward looking (since they are derived from past data) and noisy (they are statistical estimates, with standard errors). In addition, these approaches clearly will not work for investments that do not have a trading history (young companies, divisions of publicly traded companies). One solution is to replace the regression beta with a bottom-up beta, i.e., a beta that is based upon industry averages for the businesses that the firm is in, adjusted for differences in financial leverage. Since industry averages are more precise than individual regression betas, and the weights on the businesses can reflect the current mix of a firm, bottom up betas generally offer better estimates for the future.

*Illustration 2.2: Estimating the cost of equity for a firm – 3M*

Since 3M is a publicly traded stock, with a long history, we can use its price history to run a regression against the market index to derive a regression beta. Figure 2.6 provides a regression beta for 3M against the S&P 500, using 2 years of weekly returns against the S&P 500. The regression (raw) beta is 0.86; the adjusted beta, which is the raw beta moved towards the market average beta of one, is 0.79.

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6 The simplest and most widely used equation relating betas to debt to equity ratios is based on the assumption that debt provides a tax advantage and that the beta of debt is zero.

\[
\text{Beta for equity} = \text{Beta of business} \times (1 + (1 - \text{tax rate}) \times (\text{Debt/Equity}))
\]

The beta for equity is a levered beta, whereas the beta of the business is titled an unlevered beta. Regression betas are equity betas and are thus levered – the debt to equity ratio over the regression period is embedded in the beta.
While we do have a regression beta, all of the normal caveats that we listed in the preceding section apply. It is backward looking (for the last 2 years) and has a standard error (albeit a small one of 0.07). The regression results would have been very different if we had run the regression using a different time period (say 5 years), different return intervals (daily or monthly) and used a different market index.

To yield a contrasting value, we estimated a beta for 3M by breaking it down into individual businesses and taking a weighted average of the business betas:

Table 2.1: Bottom up Beta estimate for 3M

<table>
<thead>
<tr>
<th>Business</th>
<th>Revenues</th>
<th>EV/Sales</th>
<th>Estimated Value</th>
<th>Weight in firm</th>
<th>Unlevered Beta</th>
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</thead>
<tbody>
<tr>
<td>Industrial &amp; Transportation</td>
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<td>$8,265</td>
<td>27.42%</td>
<td>0.82</td>
</tr>
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<td>Health Care</td>
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<td>$7,261</td>
<td>24.09%</td>
<td>1.40</td>
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<td>Display &amp; Graphics</td>
<td>$3892</td>
<td>1.63</td>
<td>$6,344</td>
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<td>1.97</td>
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<td>Consumer &amp; Office</td>
<td>$3403</td>
<td>0.78</td>
<td>$2,654</td>
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<td>0.99</td>
</tr>
<tr>
<td>Safety, Security &amp; Protection</td>
<td>$3070</td>
<td>1.09</td>
<td>$3,346</td>
<td>11.10%</td>
<td>1.16</td>
</tr>
<tr>
<td>Electro &amp;</td>
<td>$2775</td>
<td>0.82</td>
<td>$2,276</td>
<td>7.55%</td>
<td>1.32</td>
</tr>
</tbody>
</table>
Communications 3M as a firm $30,146 100.00% 1.29

The unlevered betas of the businesses are obtained by averaging the regression betas of publicly traded firms in each business, and the EV/Sales ratio measures the typical multiple of revenues that firms in each business trade for. Applying the debt to equity ratio of 8.80% in 2007 (based upon market values for debt and equity) for 3M to the unlevered beta of 1.29 yields an equity beta of 1.36 for 3M:

Levered (Equity) Beta = 1.29 (1 + (1-.35) (8.80%)) = 1.36

Using the ten-year treasury bond rate of 3.72% in September 2007 as the riskfree rate and a 4% equity risk premium yields a cost of equity of 9.16%:

\[
\text{Cost of equity} = \text{Riskfree Rate} + \text{Beta} \times \text{Equity Risk Premium}
\]

\[
= 3.72\% + 1.36 \times 4\% = 9.16\%
\]

Obviously, using a higher equity risk premium would have led to a higher cost of equity.

*The Cost of Debt*

While equity investors receive residual cash flows and bear the bulk of the operating risk in most firms, lenders to the firm also face the risk that they will not receive their promised payments – interest expenses and principal repayments. It is to cover this default risk that lenders add a “default spread” to the riskless rate when they lend money to firms; the greater the perceived risk of default, the greater the default spread and the cost of debt. The other dimension on which debt and equity can vary is in their treatment for tax purposes, with cashflows to equity investors (dividends and stock buybacks) coming from after-tax cash flows, whereas interest payments are tax deductible. In effect, the tax law provides a benefit to debt and lowers the cost of borrowing to businesses.

To estimate the cost of debt for a firm, we need three components. The first is the riskfree rate, an input to the cost of equity as well. As a general rule, the riskfree rate used to estimate the cost of equity should be used to compute the cost of debt as well; if the cost of equity is based upon a long-term riskfree rate, as it often is, the cost of debt should be based upon the same rate. The second is the default spread and there are three approaches that are used, depending upon the firm being analyzed.
• If the firm has traded bonds outstanding, the current market interest rate on the bond (yield to maturity) is used as the cost of debt. This is appropriate only if the bond is liquid and is representative of the overall debt of the firm; even risky firms can issue safe bonds, backed up by the most secure assets of the firms.

• If the firm has a bond rating from an established ratings agency such as S&P or Moody’s, we can estimate a default spread based upon the rating. In September 2008, for instance, the default spread for BBB rated bonds was 2% and would have been used as the spread for any BBB rated company.

• If the firm is unrated and has debt outstanding (bank loans), we can estimate a “synthetic” rating for the firm, based upon its financial ratios. A simple, albeit effective approach for estimating the synthetic ratio is to base it entirely on the interest coverage ratio (EBIT/ Interest expense) of a firm; higher interest coverage ratios will yield higher ratings and lower interest coverage ratios.

The final input needed to estimate the cost of debt is the tax rate. Since interest expenses save you taxes at the margin, the tax rate that is relevant for this calculation is not the effective tax rate but the marginal tax rate. In the United States, where the federal corporate tax rate is 35% and state and local taxes add to this, the marginal tax rate for corporations in 2008 was close to 40%, much higher than the average effective tax rate, across companies, of 28%. The after-tax cost of debt for a firm is therefore:

\[
\text{After-tax cost of debt} = (\text{Riskfree Rate} + \text{Default Spread}) \times (1 - \text{Marginal tax rate})
\]

The after-tax cost of debt for most firms will be significantly lower than the cost of equity for two reasons. First, debt in a firm is generally less risky than its equity, leading to lower expected returns. Second, there is a tax saving associated with debt that does not exist with equity.

**Illustration 2.3: Estimating the Cost of Debt – 3M**

To estimate the synthetic rating for 3M, we begin with an estimate of the interest coverage ratio in 2007:

\[
\text{Interest coverage ratio} = \frac{\text{After - tax Operating Income}}{\text{Interest Expenses}} = \frac{$5,361}{$227} = 23.63
\]

Given its large market capitalization (more than $50 billion), we use table 2.2 to extract the synthetic rating and the default spread on 3M debt:
Table 2.2: Interest coverage ratios, ratings and default spreads

<table>
<thead>
<tr>
<th>Interest Coverage Ratio</th>
<th>Rating</th>
<th>Typical default spread</th>
</tr>
</thead>
<tbody>
<tr>
<td>&gt; 12.5</td>
<td>AAA</td>
<td>0.75%</td>
</tr>
<tr>
<td>9.50 - 12.50</td>
<td>AA</td>
<td>1.25%</td>
</tr>
<tr>
<td>7.50 – 9.50</td>
<td>A+</td>
<td>1.40%</td>
</tr>
<tr>
<td>6.00 – 7.50</td>
<td>A</td>
<td>1.50%</td>
</tr>
<tr>
<td>4.50 – 6.00</td>
<td>A-</td>
<td>1.70%</td>
</tr>
<tr>
<td>4.00 – 4.50</td>
<td>BBB</td>
<td>2.50%</td>
</tr>
<tr>
<td>3.50 - 4.00</td>
<td>BB+</td>
<td>3.20%</td>
</tr>
<tr>
<td>3.00 – 3.50</td>
<td>BB</td>
<td>3.65%</td>
</tr>
<tr>
<td>2.50 – 3.00</td>
<td>B+</td>
<td>4.50%</td>
</tr>
<tr>
<td>2.00 - 2.50</td>
<td>B</td>
<td>5.65%</td>
</tr>
<tr>
<td>1.50 – 2.00</td>
<td>B-</td>
<td>6.50%</td>
</tr>
<tr>
<td>1.25 – 1.50</td>
<td>CCC</td>
<td>7.50%</td>
</tr>
<tr>
<td>0.80 – 1.25</td>
<td>CC</td>
<td>10.00%</td>
</tr>
<tr>
<td>0.50 – 0.80</td>
<td>C</td>
<td>12.00%</td>
</tr>
<tr>
<td>&lt; 0.65</td>
<td>D</td>
<td>20.00%</td>
</tr>
</tbody>
</table>

The rating that we assign to 3M is AAA, with a default spread of 0.75%. Adding this spread to the ten-year treasury bond rate of 3.72% results in a pre-tax cost of debt of 4.49%. Just as a contrast, we computed the book interest rate by dividing the interest expenses in 2007 by the book value of debt:

\[
\text{Book interest rate} = \frac{\text{Interest Expenses}}{\text{Book Value of Debt}} = \frac{\$210}{\$4920} = 4.27\%
\]

Given how sensitive this number is to different definitions of book value of debt, we remain skeptical about its usefulness. Using the marginal tax rate of 35% on the pre-tax cost of debt of 4.49%, we derive an after-tax cost of debt of 2.91% for the company:

\[
\text{After-tax Cost of Debt} = (\text{Riskfree Rate} + \text{Default spread for debt}) (1 - \text{Marginal tax rate})
\]

\[
= (3.72\% + 0.75\%) (1 - 0.35) = 2.91\%
\]

Debt Ratios and the Cost of Capital

Once we have estimated the costs of debt and equity, we still have to assign weights for the two ingredients. To come up with this value, we could start with the mix of debt and equity that the firm uses right now. In making this estimate, the values that we should use are market values, rather than book values. For publicly traded firms, estimating the market value of equity is usually a trivial exercise, where we multiply the
share price by the number of shares outstanding. Estimating the market value of debt is usually a more difficult exercise, since most firms have some debt that is not traded. Though many practitioners fall back on book value of debt as a proxy of market value, estimating the market value of debt is still a better practice.

Once we have the current market value weights for debt and equity for use in the cost of capital, we have a follow up judgment to make in terms of whether these weights will change or remain stable. If we assume that they will change, we have to specify both what the right or target mix for the firm will be and how soon the change will occur. In an acquisition, for instance, we can assume that the acquirer can replace the existing mix with the target mix instantaneously. As passive investors in publicly traded firms, we have to be more cautious, since we do not control how a firm funds its operations. In this case, we may adjust the debt ratio from the current mix to the target over time, with concurrent changes in the costs of debt, equity and capital. In fact, the last point about debt ratios and costs of capital changing over time is worth reemphasizing. As companies change over time, we should expect the cost of capital to change as well.

*Illustration 2.4: Estimating cost of capital – 3M*

In illustration 2.2, we estimated a cost of equity of 9.16% for 3M, based upon a bottom-up beta estimate of 1.36. In illustration 2.3, we concluded that the after-tax cost of debt for 3M is 2.91%, based upon the synthetic AAA rating that we assigned the firm. We estimated the market values of equity and debt for the firm in September 2008 to be (with the resulting weights and the overall costs of capital) and derived the cost of capital for the firm in the table below:

<table>
<thead>
<tr>
<th></th>
<th>Market value</th>
<th>Proportion of capital</th>
<th>Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>Equity</td>
<td>$57,041</td>
<td>91.50%</td>
<td>9.16%</td>
</tr>
<tr>
<td>Debt</td>
<td>$5,297</td>
<td>8.50%</td>
<td>2.91%</td>
</tr>
<tr>
<td>Capital</td>
<td>$62,338</td>
<td>100.00%</td>
<td>8.63%</td>
</tr>
</tbody>
</table>

At its current debt ratio of 8.50%, the cost of capital for 3M is 8.63%.

*Growth Rates*

There is no other ingredient in discounted cash flow valuation that evokes as much angst as estimating future growth. Unlike cash flows and discount rates, where we
often have the security of historical data, growth rates require us to grapple with the future. In this section, we will look first at why growth rates can be different for equity and operating earnings, examine two of the standard approaches for estimating growth (by looking at the past and using analyst estimates) and close with a discussion of the fundamentals that determine growth.

**Equity versus Operating Earnings**

As with cashflows and discount rates, a contrast has to be drawn between growth in equity earnings and growth in operating earnings. To make the distinction, consider the simplified version of an income statement in table 2.3:

**Table 2.3: An Income Statement – Revenues to Earnings per Share**

<table>
<thead>
<tr>
<th>Item</th>
<th>Factors that explain differences in growth</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Revenues</strong></td>
<td></td>
</tr>
<tr>
<td>- Operating Expenses</td>
<td>1. Changes in operating efficiency/ performance</td>
</tr>
<tr>
<td></td>
<td>2. Operating leverage</td>
</tr>
<tr>
<td><strong>EBITDA</strong></td>
<td></td>
</tr>
<tr>
<td>- Depreciation &amp; Amortization</td>
<td>1. Changes in depreciation schedules/ rules</td>
</tr>
<tr>
<td></td>
<td>2. Amortization of intangibles</td>
</tr>
<tr>
<td><strong>EBIT</strong></td>
<td></td>
</tr>
<tr>
<td>- Interest Expenses</td>
<td>1. Changes in financial leverage (debt)</td>
</tr>
<tr>
<td>+ Income from cash holdings</td>
<td>2. Changes in cash holdings/ interest rates</td>
</tr>
<tr>
<td>- Taxes</td>
<td>3. Changes in tax rates/ rules</td>
</tr>
<tr>
<td><strong>Net Income</strong></td>
<td></td>
</tr>
<tr>
<td>/ Number of Shares</td>
<td>1. Stock buybacks and issues</td>
</tr>
<tr>
<td></td>
<td>2. Exercise of past option grants</td>
</tr>
<tr>
<td><strong>Earnings per share</strong></td>
<td></td>
</tr>
</tbody>
</table>

We are assuming that the firm has no minority holdings in other companies, which would result in an additional line item, just above the net income line, for income from these holdings.

The growth rates in different measures of earnings (operating income, net income and earnings per share) will generally be different for most firms, and especially so for growth firms or firms in transition.

- **Share issues and Buybacks:** If the number of shares remains fixed, the growth rate in earnings per share should be the same as the growth rate in net income. Firms that
generate excess cash flows and use these cash flows to buy back stock will register
higher growth rates in earnings per share than in net income. Conversely, firms that
make a practice of raising new equity (issuing new shares) to fund investments or
acquisitions can have higher growth rates in net income than in earnings per share.

- **Financial Leverage**: The growth rates in operating and net income can diverge if the
  net interest expense (interest expense – interest income) grows at a rate different from
  operating income. Firms that use increasing amounts of debt to fund their operations
  will generally report higher growth rates in operating income than net income.
  However, if that debt is used to buy back shares, the earnings per share growth will
  reflect the fewer shares outstanding.

- **Operating Leverage**: The growth in operating income can also be very different from
  the growth in revenues, primarily because some operating expenses are fixed and
  others are variable. The higher the proportion of the costs that are fixed costs (higher
  operating leverage), the greater will be the growth rate in operating income relative to
  the growth in revenues.

In effect, when asked to estimate growth rates, the first question that an analyst has to ask
is “In what item?” If our task is to estimate growth in operating income, we cannot use
growth rates in earnings per share as substitutes.

**Historical and Forecasted Growth Rates**

When confronted with the task of estimating growth, it is not surprising that
analysts turn to the past. In effect, they use growth in revenues or earnings in the recent
past as a predictor of growth in the future. Before we put this practice under the
microscope, we should add that the historical growth rates for the same company can
yield different estimates for the following reasons:

1. **Earnings measure**: As we noted above, the growth rates in earnings per share, net
   income, operating income and revenues can be very different for the same firm over a
   specified time period.

2. **Period of analysis**: For firms that have been in existence for long periods, the growth
   rates can be very different if we look at ten years of history as opposed to five years.
3. **Averaging approach:** Even if we agree on an earnings measure and time period for the analysis, the growth rates we derive can be different, depending upon how we compute the values. We could, for instance, compute the growth rate in each period and average the growth rates over time, yielding an arithmetic average. Alternatively, we could use just the starting and ending values for the measure and compute a geometric average. For firms with volatile earnings, the latter can generate a very different (and lower) value for growth than the former.

A debate how best to estimate historical growth makes sense only if it is a good predictor of future growth. Unfortunately, studies that have looked at the relationship have generally concluded that (a) the relationship between past and future growth is a very weak one, (b) scaling matters, with growth dropping off significantly as companies grow and (c) firms and sectors grow through growth cycles, with high growth in one period followed by low growth in the next.

If historical growth is not a useful predictor of future growth, there is another source that we can use for future growth. We can draw on those who know the firm better than we do – equity research analysts who have tracked the firm for years or the managers in the firm – and use their estimates of growth. On the plus side, these forecasts should be based upon better information than we have available to us. After all, managers should have a clearer sense of how much they will reinvest in their own businesses and what the potential returns on investments are when they do, and equity research analysts have sector experience and informed sources that they can draw on for better information. On the minus side, neither managers nor equity research analysts are objective about the future; managers are likely to over estimate their capacity to generate growth and analysts have their own biases. In addition, both analysts and managers can get caught up in the mood of the moment, over estimating growth in buoyant times and under estimating growth in down times. As with historical growth, studies indicate that neither analyst estimates nor management forecasts are good predictors of future growth.

**Fundamental Growth Rates**

If we cannot draw on history or trust managers and analysts, how then do we estimate growth? The answer lies in the fundamentals within a firm that ultimately
determine its growth rate. In this section, we will consider the two sources for growth – new investments that expand the business and improved efficiency on existing investments.

**Decomposing Growth**

The best way to consider earnings growth is to break it down algebraically into its constituent parts. Define \( E_t \) to be the earnings in period \( t \), \( I_t \) to be the investment at the start of period \( t \) and \( \text{ROI}_t \) as the return on that investment. Thus, we can rewrite \( E_t \) as:

\[
E_t = \text{ROI}_t \times I_t
\]

The change in earnings from period \( t-1 \) to \( t \), \( \Delta E \), can then be written as follows

\[
\Delta E = E_t - E_{t-1} = \text{ROI}_t \times I_t - \text{ROI}_{t-1} \times I_{t-1}
\]

The growth rate is written in terms of \( \Delta E \) and \( E_{t-1} \):

\[
g = \frac{\Delta E}{E_{t-1}} = \frac{\text{ROI}_t \times I_t - \text{ROI}_{t-1} \times I_{t-1}}{E_{t-1}}
\]

Consider the simplest scenario, where the ROI is stable and does not change from period to period (\( \text{ROI} = \text{ROI}_t = \text{ROI}_{t-1} \)). The expected growth rate in earnings for this firm is:

\[
g = \frac{\Delta E}{E_{t-1}} = \frac{\text{ROI} \times (I_t - I_{t-1})}{E_{t-1}}
\]

\[
= \frac{\text{ROI} \times \Delta I_{t}}{E_{t-1}}
\]

In other words, the growth rate for this firm will be a function of only two variables – the return it makes on new investments (\( \text{ROI} \)) and the proportion of its earnings that are put into new investments (\( \Delta I/E_{t-1} \)).

The more general scenario is one where the return on investment does change from period to period. In this case, the expected growth rate can be written as:

\[
g = \frac{\Delta E}{E_{t-1}} = \text{ROI}_t \times \frac{\Delta I}{E_{t-1}} + \frac{(\text{ROI}_t - \text{ROI}_{t-1})}{\text{ROI}_{t-1}}
\]

This equation is based on the assumption that the return on new investments in period \( t \) is identical to the return earned on existing investments in that period. In fact, this can be generalized even further, if we allow the return on new investments, \( \text{ROI}_{\text{New},t} \), to be different from the return on existing assets, \( \text{ROI}_{\text{Existing},t} \), the expected growth rate can be written as:

\[
g = \frac{\Delta E}{E_{t-1}} = \text{ROI}_{\text{New},t} \times \frac{\Delta I}{E_{t-1}} + \frac{\text{ROI}_{\text{Existing},t} - \text{ROI}_{\text{Existing},t-1}}{\text{ROI}_{\text{Existing},t-1}}
\]

The first term in this equation captures the growth from new investments, determined by the marginal return on those investments and the proportion invested in these
investments. The second term captures the effect of changes in the return on investment on existing assets, a component that we will title “efficiency growth”. Increasing the return on investment (improving efficiency) will create additional earnings growth, whereas declining efficiency (with drops in the return on investment) will reduce earnings growth.

_Growth from new investments_

While investment and return on investment are generic terms, the way in which we define them will depend upon whether we are looking at equity earnings or operating income. When looking at equity earnings, our focus is on the investment in equity and the return is the return on equity. When looking at operating earnings, the focus is on the investment in capital and the return is the return on capital. In the cash flow definitions introduced at the start of this chapter, the change in investment is computed as the reinvestment, with the measurement of the reinvestment again varying depending upon the cash flow being discounted. In dividend discount models, reinvestment is defined as retained earnings (i.e., any income not paid out as dividends). In free cash flow to equity (firm) models, reinvestment is defined in terms of the equity reinvestment rate (reinvestment rate).

Central to any estimate of fundamental growth is the estimate of return on capital or equity. Table 2.4 summarizes the inputs for each measure depending on the measure of cash flow that we are focused on:

_Table 2.4: Measuring Investment and Return on Investment_

<table>
<thead>
<tr>
<th>Change in Investment</th>
<th>Return on Investment</th>
</tr>
</thead>
</table>
| **Operating Income** | Reinvestment Rate = \[
\frac{(\text{Cap Ex} - \text{Deprec' n} + \Delta WC)}{\text{EBIT}(1 - t)}\] | Return on Invested Capital (ROC or ROIC) |
| **Net Income (Non-cash)** | Equity Reinvestment Rate = \[
\frac{(\text{Cap Ex} - \text{Deprec' n} + \Delta WC - \Delta Debt)}{\text{Net Income}}\] | Non-cash Return on Equity (NCROE) |
| **Earnings per share** | Retention Ratio = \[
\frac{\text{Dividends}}{\text{Net Income}}\] | Return on Equity (ROE) |
It is conventional practice to use accounting measures of investment and return on investment. Thus, the book values of equity and invested capital and accounting earnings are used to compute returns on equity and capital:

\[
\text{Return on Capital (ROIC)} = \frac{\text{Operating Income}_t (1 - \text{tax rate})}{\text{Book Value of Invested Capital}_{t-1}}
\]

\[
\text{Non-cash Return on Equity (NCROE)} = \frac{\text{Net Income}_t - \text{Interest Income from Cash}_t (1 - \text{tax rate})}{\text{Book Value of Equity}_{t-1} - \text{Cash}_{t-1}}
\]

\[
\text{Return on Equity (ROE)} = \frac{\text{Net Income}_t}{\text{Book Value of Equity}_{t-1}}
\]

The problem with accounting measures on both dimensions is well documented, with accounting choices on restructuring charges, amortization and capitalization all making a difference in the final numbers.\(^7\)

The final issue that we have to consider is the difference between marginal and average returns. Note that the return on investment that we use to compute the growth from new investments should be the return earned on those investments alone, i.e., a marginal return. The return on existing assets is an average return on a portfolio of investments already made. While we often use the same value for both numbers in valuation, they can be different, in fact, very different in practice.

**Efficiency Growth**

For many mature firms with limited investment opportunities, the potential for growth from new investments is limited. These firms cannot maintain a high reinvestment rate and deliver a high return on capital with that reinvestment. However, they can still grow at healthy rates if they can improve the returns that they earn on existing assets. Conversely, declines in returns on existing assets can translate into drops in earnings growth rates. Stated again in terms of different measures of earnings, efficiency growth can be written in table 2.5, as follows:

<table>
<thead>
<tr>
<th>Measure of return on existing assets</th>
<th>Efficiency growth</th>
</tr>
</thead>
</table>

\(^7\) To get a sense of the problems with using accounting numbers, and how best to correct for them, see: Damodaran, A., 2007, Return on capital, Return on Invested Capital and Return on Equity: Measurement and Implications, Working Paper, SSRN.
When valuing companies, efficiency growth is pure gravy in terms of value created, since the growth comes with no concurrent cost. Unlike growth from new investments, where the positive effects of growth have to be offset against the negative effect of more investment, improving the return on capital on existing assets increases the growth rate without adversely affecting the cash flows. It should as come as no surprise, then, that analysts who want to increase the value of a company draw on the efficiency argument to justify much higher growth rates than those estimated using fundamentals.

While the potential for efficiency growth is always there, we should put some common sense constraints on how much we can draw on this growth.

1. There is more potential for efficiency growth at mature firms, with poor returns on capital (equity), than there is at firms that are performing well, for two reasons. First, improving the return on capital is a much more feasible option for a firm that generates a return on capital that is well below the sector average than at a firm that already outperforms the sector. Second, the effect of an improvement in returns on growth is much greater when the return on capital is low than when it is high. A firm that improves its return on capital from 5% to 6% will report a 20% growth rate from efficiency in that period, whereas a firm that improves its return on capital from 25% to 26% will generate a 4% growth rate from efficiency in that period.

2. You can draw on increased efficiency to justify growth only for finite periods. After all, a firm cannot be infinitely inefficient. Once the inefficiencies, no matter how significant, are fixed, the firm will have to revert back to its sustainable growth rate, based upon new investments. In discounted cash flow valuation, this has a practical consequence: you can draw on both efficiency and new investments to justify growth during the high growth period, but only on new investments to justify growth forever (in the terminal value computation).
In closing, growth in a specific firm can come from new investments or improved efficiency, but it has to be earned either way. None of us has the power to endow companies with higher growth rates, just because we like the managers or want to make it value increase.

*Illustration 2.5: Estimating Growth - 3M*

It makes sense to start with an estimate of historical growth in earnings at 3M and figure 2.7 presents different estimates of past earnings growth for 3M, given different definitions of earnings and different time periods.

*Figure 2.7: Historical growth rates in earnings – 3M*

Note the wildly divergent numbers that we get for past growth. In September 2008, analysts were estimating growth in earnings per share at 3M of between 8%-9% a year for the next five years.

Looking at the fundamentals, it seems unlikely that 3M, given its high existing return on capital and equity can generate much in terms of efficiency growth. It is, however, reinvesting in new assets and this reinvestment, in conjunction with high returns on capital on new investments will generate growth. Table 2.6 summarizes
growth in dividends, non-cash net income and after-tax operating income, using the reinvestment and return characteristics that we have estimated for 3M:

*Table 2.6: Dividends, Net Income and After-tax Operating Income – Fundamentals*

<table>
<thead>
<tr>
<th></th>
<th>In last financial year - 2007</th>
<th>Expected for next 5 years</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Reinvestment</td>
<td>Return</td>
</tr>
<tr>
<td>Dividends</td>
<td>66.31%</td>
<td>33.93%</td>
</tr>
<tr>
<td>Non-cash net income</td>
<td>-2.27%</td>
<td>47.65%</td>
</tr>
<tr>
<td>After-tax Operating Income</td>
<td>36.37%</td>
<td>25.31%</td>
</tr>
</tbody>
</table>

Note that we have stayed fairly close to last year’s estimates of the reinvestment rate and return on capital for 3M but changed the equity reinvestment rate substantially for the next few years to reflect 3M’s longer history (rather than just 2007).

**Terminal Value**

Publicly traded firms do not have finite lives. Given that we cannot estimate cash flows forever, we generally impose closure in valuation models by stopping our estimation of cash flows sometime in the future and then computing a terminal value that reflects all cash flows beyond that point. There are three approaches generally used to estimate the terminal value. The most common approach, which is to apply a multiple to earnings in the terminal year to arrive at the terminal value, is inconsistent with intrinsic valuation. Since these multiples are usually obtained by looking at what comparable firms are trading at in the market today, this is a relative valuation, rather than a discounted cash flow valuation. The two more legitimate ways of estimating terminal value are to estimate a liquidation value for the assets of the firm, assuming that the assets are sold in the terminal year, and the other is to estimate a going concern or a terminal value.

1. **Liquidation Value**

If we assume that the business will be ended in the terminal year and that its assets will be liquidated at that time, we can estimate the proceeds from the liquidation. This liquidation value still has to be estimated, using a combination of market-based
numbers (for assets that have ready markets) and cashflow-based estimates. For firms that have finite lives and marketable assets (like real estate), this represents a fairly conservative way of estimating terminal value. For other firms, estimating liquidation value becomes more difficult to do, either because the assets are not separable (brand name value in a consumer product company) or because there is no market for the individual assets. One approach is to use the estimated book value of the assets as a starting point, and to estimate the liquidation value, based upon the book value.

2. Going Concern or Terminal value

If we treat the firm as a going concern at the end of the estimation period, we can estimate the value of that concern by assuming that cash flows will grow at a constant rate forever afterwards. This perpetual growth model draws on a simple present value equation to arrive at terminal value:

\[
\text{Terminal Value}_{n} = \frac{\text{Cashflow in year } n+1}{(\text{Discount rate} - \text{Perpetual growth rate})}
\]

Our definitions of cash flow and growth rate have to be consistent with whether we are valuing dividends, cash flows to equity or cash flows to the firm; the discount rate will be the cost of equity for the first two and the cost of capital for the last. The perpetual growth model is a powerful one, but it can be easily misused. In fact, analysts often use it as a piggy bank that they go to whenever they feel that the value that they have derived for an asset is too low or high. Small changes in the inputs can alter the terminal value dramatically. Consequently, there are three key constraints that should be imposed on its estimation:

a. **Cap the growth rate**: Small changes in the stable growth rate can change the terminal value significantly and the effect gets larger as the growth rate approaches the discount rate used in the estimation. The fact that a stable growth rate is constant forever, however, puts strong constraints on how high it can be. Since no firm can grow forever at a rate higher than the growth rate of the economy in which it operates, the constant growth rate cannot be greater than the overall growth rate of the economy. So, what is the maximum stable growth rate that you can use in a valuation? The answer will depend on whether the valuation is being done in real or nominal terms, and if the latter, the currency used to estimate cash flows. With the former, you would use the real growth rate in the economy
as your constraint, whereas with the latter, you would add expected inflation in the currency to the real growth. Setting the stable growth rate to be less than or equal to the growth rate of the economy is not only the consistent thing to do but it also ensures that the growth rate will be less than the discount rate. This is because of the relationship between the riskless rate that goes into the discount rate and the growth rate of the economy. Note that the riskless rate can be written as:

Nominal riskless rate = Real riskless rate + Expected inflation rate

In the long term, the real riskless rate will converge on the real growth rate of the economy and the nominal riskless rate will approach the nominal growth rate of the economy. In fact, a simple rule of thumb on the stable growth rate is that it should not exceed the riskless rate used in the valuation.

b. Use mature company risk characteristics: As firms move from high growth to stable growth, we need to give them the characteristics of stable growth firms. A firm in stable growth is different from that same firm in high growth on a number of dimensions. In general, you would expect stable growth firms to be less risky and use more debt. In practice, we should move betas for even high risk firms towards one in stable growth and give them debt ratios, more consistent with larger, more stable cashflows.

c. Reinvestment and Excess Return Assumptions: Stable growth firms tend to reinvest less than high growth firms and it is critical that we both capture the effects of lower growth on reinvestment and that we ensure that the firm reinvests enough to sustain its stable growth rate in the terminal phase. Given the relationship between growth, reinvestment rate and returns that we established in the section on expected growth rates, we can estimate the reinvestment rate that is consistent with expected growth in table 2.7:

<table>
<thead>
<tr>
<th>Model</th>
<th>Reinvestment Rate in stable growth</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dividend</td>
<td>Stable Growth rate Return on Equity in stable growth</td>
</tr>
<tr>
<td>FCFE</td>
<td>Stable Growth rate Non-cash Return on Equity in stable growth</td>
</tr>
<tr>
<td>FCFF</td>
<td>Stable growth rate Return on capital in stable phase</td>
</tr>
</tbody>
</table>
Linking the reinvestment rate and retention ratio to the stable growth rate also makes the valuation less sensitive to assumptions about stable growth. While increasing the stable growth rate, holding all else constant, can dramatically increase value, changing the reinvestment rate as the growth rate changes will create an offsetting effect.

\[ \text{Terminal Value} = \frac{\text{EBIT}_{n+1}(1-t)(1 - \text{Reinvestment Rate})}{\text{Cost of Capital}_n - (\text{Reinvestment Rate} \times \text{Return on Capital})} \]

The gains from increasing the growth rate will be partially or completely offset by the loss in cash flows because of the higher reinvestment rate. Whether value increases or decreases as the stable growth increases will entirely depend upon what you assume about excess returns. If the return on capital is higher than the cost of capital in the stable growth period, increasing the stable growth rate will increase value. If the return on capital is equal to the stable growth rate, increasing the stable growth rate will have no effect on value. Substituting in the stable growth rate as a function of the reinvestment rate, from above, you get:

\[ \text{Terminal Value} = \frac{\text{EBIT}_{n+1}(1-t)(1 - \text{Reinvestment Rate})}{\text{Cost of Capital}_n - (\text{Reinvestment Rate} \times \text{Return on Capital})} \]

Setting the return on capital equal to the cost of capital, you arrive at:

\[ \text{Terminal Value}_{\text{ROC} = \text{WACC}} = \frac{\text{EBIT}_{n+1}(1-t)}{\text{Cost of Capital}_n} \]

You could establish the same propositions with equity income and cash flows and show that the terminal value of equity is a function of the difference between the return on equity and cost of equity.

\[ \text{Terminal Value of Equity} = \frac{\text{Net Income}_{n+1} \left(1 - \frac{g_n}{\text{ROE}_n} \right)}{(\text{Cost of Equity}_n - g_n)} \]

\[ \text{Terminal Value}_{\text{ROE} = \text{Cost of Equity}} = \frac{\text{Net Income}_{n+1}}{\text{Cost of Equity}_n} \]

In closing, the key assumption in the terminal value computation is not what growth rate you use in the valuation, but what excess returns accompany that growth rate. If you assume no excess returns, the growth rate becomes irrelevant. There are some
valuation experts who believe that this is the only sustainable assumption, since no firm can maintain competitive advantages forever. In practice, though, there may be some wiggle room, insofar as the firm may become a stable growth firm before its excess returns go to zero. If that is the case and the competitive advantages of the firm are strong and sustainable (even if they do not last forever), we may be able to give the firm some excess returns in perpetuity. As a simple rule of thumb again, these excess returns forever should be modest (<4-5%) and will affect the terminal value.

Illustration 2.6: High Growth versus Terminal Value Assumptions: 3M

In table 2.8, we list our assumptions about 3M in both the high growth phase and in steady state:

<table>
<thead>
<tr>
<th></th>
<th>High Growth</th>
<th>Stable Growth</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length of High Growth Period =</td>
<td>Next 5 years</td>
<td>After year 5</td>
</tr>
<tr>
<td>Growth Rate =</td>
<td>7.50%</td>
<td>3.00%</td>
</tr>
<tr>
<td>Debt Ratio used in Cost of Capital Calculation=</td>
<td>8.48%</td>
<td>20.00%</td>
</tr>
<tr>
<td>Beta used for stock =</td>
<td>1.36</td>
<td>1.00</td>
</tr>
<tr>
<td>Riskfree rate =</td>
<td>3.72%</td>
<td>3.72%</td>
</tr>
<tr>
<td>Risk Premium =</td>
<td>4.00%</td>
<td>4.00%</td>
</tr>
<tr>
<td>Cost of Debt =</td>
<td>4.47%</td>
<td>4.47%</td>
</tr>
<tr>
<td>Tax Rate =</td>
<td>35.00%</td>
<td>35.00%</td>
</tr>
<tr>
<td>Cost of capital</td>
<td>8.63%</td>
<td>6.76%</td>
</tr>
<tr>
<td>Return on Capital =</td>
<td>25.00%</td>
<td>6.76%</td>
</tr>
<tr>
<td>Reinvestment Rate =</td>
<td>30.00%</td>
<td>44.40%</td>
</tr>
</tbody>
</table>

Note that as the growth declines after year 5, the beta is adjusted towards one and the debt ratio is raised to the industry average of 20% to reflect the overall stability of the company. Since the cost of debt is relatively low, we leave it unchanged, resulting in a drop in the cost of capital to 6.76%. We do change the reinvestment rate in stable growth to reflect the assumption that there will be no excess returns in stable growth (return on capital = cost of capital). Using the predicted stable growth rate of 3% and the return on capital of 6.76% (equal to cost of capital), we derive a reinvestment rate of 44.4%:
Reinvestment Rate in stable growth = \frac{\text{Expected Growth}}{\text{Stable ROC}} = \frac{3.00\%}{6.76\%} = 44.40\%

**Tying up loose ends**

We have covered the four inputs that go into discounted cash flow valuation models – cash flows, discount rates, growth rates and the terminal value. The present value we arrive at, when we discount the cash flows at the risk-adjusted rates should yield an estimate of value, but getting from that number to what we would be willing to pay per share for equity does require use to consider a few other factors.

a. **Cash and Marketable Securities:** Most companies have cash balances that are not insignificant in magnitude. Is this cash balance already incorporated into the present value? The answer depends upon how we estimated cash flows. If the cash flows are based on operating income (free cash flow to the firm) or non-cash net income, we have not valued cash yet and it should be added on to the present value. If, on the other hand, we estimate cash flows from the cumulative net income or use the dividend discount model, cash already has been implicitly valued; the income from cash is part of the final cash flow and the discount rate presumably has been adjusted to reflect the presence of cash.

b. **Cross Holdings in other companies:** Companies sometimes invest in other firms, and these cross holdings can generally be categorized as either minority or majority holdings. With the former, the holdings are usually less than 50%, and the income from the holdings are reported in the income statement below the operating income line. If we use free cash flow to the firm to value the operating assets, we have not valued these minority holdings yet, and they have to be valued explicitly and added to present value. With majority holdings, which generally exceed 50%, firms usually consolidate the entire subsidiary in their financials, and report 100% of the operating income and assets of the subsidiary. To reflect the portion of the subsidiary that does not belong to them, they report the book value of that portion as minority interest in a balance sheet. If we compute cash flows from consolidated financial statements, we have to subtract out the estimated market value of the minority interest.

c. **Potential liabilities (not treated as debt):** Since we are interested in the value of equity in the firm, we have to consider any potential liabilities that we may face that reduce
that value. Thus, items like under funded pension obligations and health care obligations may not meet the threshold to be categorized as debt for cost of capital purposes but should be considered when valuing equity. In other words, we would subtract out the values of these and other claims (such as potential costs from lawsuits against the firm) on equity from firm value to arrive at equity value.

d. **Employee Options**: Having arrived at the value of equity in the firm, there is one final estimate that we have to make, especially if the firm has made it a practice to grant options to managers. Since many of these options will be still outstanding, we have to consider them as another (and different) claim on equity. While analysts often use short cuts (such as adjusting the number of shares for dilution) to deal with these options, the right approach is to value the options (using an option pricing model), reduce the value of equity by the option value and then divide by the actual number of shares outstanding.

Table 2.9 summarizes the loose ends and how to deal with them in the different models.

**Table 2.9: Dealing with loose ends in valuation**

<table>
<thead>
<tr>
<th>Loose End</th>
<th>Dividend Discount Model</th>
<th>FCFE Model</th>
<th>FCFF Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cash and Marketable Securities</td>
<td>Ignore, since net income includes interest income from cash.</td>
<td>Ignore, if FCFE is computed using total net income. Add, if FCFE is computed using non-cash net income</td>
<td>Add. Operating income does not include income from cash.</td>
</tr>
<tr>
<td>Cross Holdings</td>
<td>Ignore, since net income includes income from cross holdings.</td>
<td>Ignore, since net income includes income from cross holdings.</td>
<td>Add market value of minority holdings and subtract market value of minority interests.</td>
</tr>
<tr>
<td>Other Liabilities</td>
<td>Ignore. The assumption is that the firm is considering costs when setting dividends.</td>
<td>Subtract out expected litigation costs.</td>
<td>Subtract out under funded pension obligations, health care obligations and expected litigation costs.</td>
</tr>
<tr>
<td>Employee options</td>
<td>Ignore.</td>
<td>Subtract out value of equity options outstanding</td>
<td>Subtract out value of equity options outstanding</td>
</tr>
</tbody>
</table>
Illustration 2.7: A valuation of 3M

In the earlier illustrations, we estimated the inputs for 3M, ranging from existing cash flows (in illustration 2.1) to cost of capital in (illustration 2.4) to the terminal value computation (in illustration 2.6). We first use the expected growth rate of 7.5% and reinvestment rate of 30% that we estimated for the first 5 years to obtain the expected FCFF each year in table 2.10:

Table 2.10: Expected FCFF to 3M – Next 5 years

<table>
<thead>
<tr>
<th>After-tax Operating Income (growing at 7.5% a year)</th>
<th>Current</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$3,586</td>
<td>$3,854</td>
<td>$4,144</td>
<td>$4,454</td>
<td>$4,788</td>
<td>$5,147</td>
<td></td>
</tr>
<tr>
<td>- Reinvestment (30% of income)</td>
<td>$1,156</td>
<td>$1,243</td>
<td>$1,336</td>
<td>$1,437</td>
<td>$1,544</td>
<td></td>
</tr>
<tr>
<td>= FCFF</td>
<td>$2,698</td>
<td>$2,900</td>
<td>$3,118</td>
<td>$3,352</td>
<td>$3,603</td>
<td></td>
</tr>
</tbody>
</table>

At the end of the fifth year, we assume that 3M becomes a stable growth firm with a growth rate of 3% a year forever. Staying consistent with the parameters (44.4% reinvestment rate and 6.76% cost of capital) that we estimated for 3M in stable growth in illustration 2.6, we derive the FCFF in year 6 and the terminal value for the firm.

Expected after-tax operating income in year 6 = 5,147 (1.03) = $5,302 million
Reinvestment rate in year 6 (44.4% of income) = $2,375 million
FCFF in year 6 = $2,947 million
Terminal value at end of year 5 = $2,947/ (.0676-.03) = $78,464 million

Using the cost of capital of 8.63% for the first 5 years, we discount back the cashflows for the next 5 years and the terminal value to arrive at an estimate of value for the operating assets of $60,719 million.

Value of operating asset= PV of FCFF in years 1-5 + PV of terminal value

\[
\frac{2698}{1.0863} + \frac{2900}{1.0863^2} + \frac{3118}{1.0863^3} + \frac{3352}{1.0863^4} + (\frac{3603 + 78464}{1.0863^5}) = $64,036 million
\]

Adding on the -existing cash balance of $2,475 million and the value of existing minority cross holdings in other firms of $778 million results in an overall value for 3M of $63,963 million:

Value of operating asset = $64,036 million

+ Cash & Marketable securities = $ 2,475 million
Value of 3M as a firm = $67,289 million

Subtracting out the debt outstanding in the firm yields the value of the equity in 3M:

Value of equity = Value of firm – Value of outstanding debt

= $67,289 million - $5,297 million = $61,992 million

Finally, we estimated a value of $1,216 million for the equity options that have been granted over time to the managers at 3M and are still outstanding:

Value of equity in common stock = Value of equity – Value of option overhang

= $61,992 million - $1,216 million = $60,776 million

Dividing by the actual number of shares outstanding results in a value per share of $86.95, slightly higher than the stock price prevailing in early September 2008 of $80 a share.

**Variations on DCF Valuation**

The discounted cash flow model, described so far in this chapter, is still the standard approach for estimating intrinsic value. However, there are variants on that approach that also have to the same objective. In this section, we begin with a model where we adjust the cash flows for risk, rather than the discount rate, and then move on the adjusted present value model (where the effect of debt on value is separated from the operating assets) and excess return models (where value is derived from earning excess returns on new investments).

**Certainty Adjusted Cashflow Models**

While most analysts adjust the discount rate for risk in DCF valuation, there are some who prefer to adjust the expected cash flows for risk. In the process, they are replacing the uncertain expected cash flows with the certainty equivalent cashflows, using a risk adjustment process akin to the one used to adjust discount rates.

---

8 There were 58.82 million options outstanding at the end of 2007, with a weighted average strike price of $66.83 and 5.5 years left to expiration. We valued these options using a Black-Scholes option pricing model.


**Misunderstanding Risk Adjustment**

At the outset of this section, it should be emphasized that many analysts misunderstand what risk adjusting the cash flows requires them to do. There are some who consider the cash flows of an asset under a variety of scenarios, ranging from best case to catastrophic, assign probabilities to each one, take an expected value of the cash flows and consider it risk adjusted. While it is true that bad outcomes have been weighted in to arrive at this cash flow, it is still an expected cash flow and is not risk adjusted. To see why, assume that you were given a choice between two alternatives. In the first one, you are offered $95 with certainty and in the second, you will receive $100 with probability 90% and only $50 the rest of the time. The expected values of both alternatives is $95 but risk averse investors would pick the first investment with guaranteed cash flows over the second one.

**Ways of computing certainty equivalent cashflows**

The practical question that we will address in this section is how best to convert uncertain expected cash flows into guaranteed certainty equivalents. While we do not disagree with the notion that it should be a function of risk aversion, the estimation challenges remain daunting.

**Risk Adjustments based upon Utility Models**

The first (and oldest) approach to computing certainty equivalents is rooted in the utility functions for individuals. If we can specify the utility function of wealth for an individual, we are well set to convert risky cash flows to certainty equivalents for that individual. For instance, an individual with a log utility function would have demanded a certainty equivalent of $79.43 for the risky gamble presented in the last section (90% chance of $100 and 10% chance of $50):

Utility from gamble = \(0.90 \ln(100) + 0.10 \ln(50) = 4.5359\)

Certainty Equivalent = \(\exp^{4.5359} = $93.30\)

The certainty equivalent of $93.30 delivers the same utility as the uncertain gamble with an expected value of $95. This process can be repeated for more complicated assets, and each expected cash flow can be converted into a certainty equivalent.
One quirk of using utility models to estimate certainty equivalents is that the certainty equivalent of a positive expected cash flow can be negative. Consider, for instance, an investment where you can make $2000 with probability 50% and lose $1500 with probability 50%. The expected value of this investment is $250 but the certainty equivalent may very well be negative, with the effect depending upon the utility function assumed.\(^9\)

There are two problems with using this approach in practice. The first is that specifying a utility function for an individual or analyst is very difficult, if not impossible, to do with any degree of precision. In fact, most utility functions that are well behaved (mathematically) do not seem to explain actual behavior very well. The second is that, even if we were able to specify a utility function, this approach requires us to lay out all of the scenarios that can unfold for an asset (with corresponding probabilities) for every time period. Not surprisingly, certainty equivalents from utility functions have been largely restricted to analyzing simple gambles in classrooms.

**Risk and Return Models**

A more practical approach to converting uncertain cash flows into certainty equivalents is offered by risk and return models. In fact, we would use the same approach to estimating risk premiums that we employ while computing risk adjusted discount rates but we would use the premiums to estimate certainty equivalents instead.

Certainty Equivalent Cash flow = Expected Cash flow/ (1 + Risk Premium in Risk-adjusted Discount Rate)

In the 3M valuation, for instance, note that the cost of capital of 8.63% is a risk-adjusted discount rate, based upon its market risk exposure and current market conditions; the risk free rate used was 3.72%. Instead of discounting the expected cash flow of $2,698 million in year 1 at 8.63%, we would decompose the discount rate into a risk free rate of 3.72% and a compounded risk premium of 4.73%\.\(^{10}\)

\(^9\) The certainty equivalent will be negative in this example for some utility functions for wealth. Intuitively, this would indicate that an investor with this utility function would actually pay to avoid being exposed to this gamble (even though it has a positive expected value).

\(^{10}\) A more common approximation used by many analysts is the difference between the risk adjusted discount rate and the risk free rate. In this case, that would have yielded a risk premium of 4.91% (8.63% - 3.72% = 4.91%)
Risk Premium = \frac{(1 + \text{Risk adjusted Discount Rate})}{(1 + \text{Riskfree Rate})} - 1 = \frac{(1.0863)}{(1.0372)} - 1 = .0473

Using this risk premium, we can compute the certainty equivalent cash flow for 3M in year 1:

Certainty Equivalent Cash flow in year 1 = $ 2.698 million/1.0473 = $ 2,576 million

The present value of this certainty equivalent cash flow can then be computed at the riskfree rate:

Present value of certainty equivalent cash flow = $2576/1.0372 = $2.484 million

This process would be repeated for all of the expected cash flows.

\[ \text{CE (CF}_t\text{)} = \alpha_t \text{E(CF}_t\text{)} = \frac{(1 + r_f)_t}{(1 + r)^t} \text{E(CF}_t\text{)} \]

This adjustment has two effects. The first is that expected cash flows with higher uncertainty associated with them have lower certainty equivalents than more predictable cash flows at the same point in time. The second is that the effect of uncertainty compounds over time, making the certainty equivalents of uncertain cash flows further into the future lower than uncertain cash flows that will occur sooner.

\textit{Cashflow Haircuts}

A far more common approach to adjusting cash flows for uncertainty is to “haircut” the uncertain cash flows subjectively. Thus, an analyst, faced with uncertainty, will replace uncertain cash flows with conservative or lowball estimates. This is a weapon commonly employed by analysts, who are forced to use the same discount rate for projects of different risk levels, and want to even the playing field. They will haircut the cash flows of riskier projects to make them lower, thus hoping to compensate for the failure to adjust the discount rate for the additional risk.

In a variant of this approach, there are some investors who will consider only those cashflows on an asset that are predictable and ignore risky or speculative cash flows when valuing the asset. When Warren Buffet expresses his disdain for the CAPM and other risk and return models, and claims to use the riskfree rate as the discount rate, we suspect that he can get away with doing so because of a combination of the types of
companies he chooses to invest in and his inherent conservatism when it comes to estimating the cash flows.

While cash flow haircuts retain their intuitive appeal, we should be wary of their usage. After all, gut feelings about risk can vary widely across analysts looking at the same asset; more risk averse analysts will tend to haircut the cashflows on the same asset more than less risk averse analysts. Furthermore, the distinction we drew between diversifiable and market risk when developing risk and return models can be completely lost when analysts are making intuitive adjustments for risk. In other words, the cash flows may be adjusted downwards for risk that will be eliminated in a portfolio. The absence of transparency about the risk adjustment can also lead to the double counting of risk, especially when the analysis passes through multiple layers of analysis. To provide an illustration, after the first analyst looking at a risky investment decides to use conservative estimates of the cash flows, the analysis may pass to a second stage, where his superior may decide to make an additional risk adjustment to the already risk adjusted cash flows.

**Risk Adjusted Discount Rate or Certainty Equivalent Cash Flow**

Adjusting the discount rate for risk or replacing uncertain expected cash flows with certainty equivalents are alternative approaches to adjusting for risk, but do they yield different values, and if so, which one is more precise? The answer lies in how we compute certainty equivalents. If we use the risk premiums from risk and return models to compute certainty equivalents, the values obtained from the two approaches will be the same. After all, adjusting the cash flow, using the certainty equivalent, and then discounting the cash flow at the riskfree rate is equivalent to discounting the cash flow at a risk adjusted discount rate. To see this, consider an asset with a single cash flow in one year and assume that \( r \) is the risk-adjusted cash flow, \( r_f \) is the riskfree rate and \( RP \) is the compounded risk premium computed as described earlier in this section.

\[
\text{Certainty Equivalent Value} = \frac{CE}{(1+r_f)} = \frac{E(CF)}{(1+RP)(1+r_f)} = \frac{E(CF)}{(1+r_f)(1+r_f)} = \frac{E(CF)}{(1+r)}
\]
This analysis can be extended to multiple time periods and will still hold.\textsuperscript{11} Note, though, that if the approximation for the risk premium, computed as the difference between the risk-adjusted return and the risk free rate, had been used, this equivalence will no longer hold. In that case, the certainty equivalent approach will give lower values for any risky asset and the difference will increase with the size of the risk premium.

Are there other scenarios where the two approaches will yield different values for the same risky asset? The first is when the risk free rates and risk premiums change from time period to time period; the risk-adjusted discount rate will also then change from period to period. There are some who argue that the certainty equivalent approach yields more precise estimates of value in this case. The other is when the certainty equivalents are computed from utility functions or subjectively, whereas the risk-adjusted discount rate comes from a risk and return model. The two approaches can yield different estimates of value for a risky asset. Finally, the two approaches deal with negative cash flows differently. The risk-adjusted discount rate discounts negative cash flows at a higher rate and the present value becomes less negative as the risk increases. If certainty equivalents are computed from utility functions, they can yield certainty equivalents that are negative and become more negative as you increase risk, a finding that is more consistent with intuition.

The biggest dangers arise when analysts use an amalgam of approaches, where the cash flows are adjusted partially for risk, usually subjectively and the discount rate is also adjusted for risk. It is easy to double count risk in these cases and the risk adjustment to value often becomes difficult to decipher.

**Adjusted Present Value Models**

In the *adjusted present value (APV) approach*, we separate the effects on value of debt financing from the value of the assets of a business. In contrast to the conventional approach, where the effects of debt financing are captured in the discount rate, the APV approach attempts to estimate the expected dollar value of debt benefits and costs separately from the value of the operating assets.

\textsuperscript{11} The proposition that risk adjusted discount rates and certainty equivalents yield identical net present values is shown in Stapleton, R.C., 1971.
**Basis for APV Approach**

In the APV approach, we begin with the value of the firm without debt. As we add debt to the firm, we consider the net effect on value by considering both the benefits and the costs of borrowing. In general, using debt to fund a firm’s operations creates tax benefits (because interest expenses are tax deductible) on the plus side and increases bankruptcy risk (and expected bankruptcy costs) on the minus side. The value of a firm can be written as follows:

\[
\text{Value of business} = \text{Value of business with 100\% equity financing} + \text{Present value of Expected Tax Benefits of Debt} - \text{Expected Bankruptcy Costs}
\]

The first attempt to isolate the effect of tax benefits from borrowing was in Miller and Modigliani (1963), where they valued the present value of the tax savings in debt as a perpetuity using the cost of debt as the discount rate. The adjusted present value approach, in its current form, was first presented in Myers (1974) in the context of examining the interrelationship between investment and financing decisions.

Implicitly, the adjusted present value approach is built on the presumption that it is easier and more precise to compute the valuation impact of debt in absolute terms rather than in proportional terms. Firms, it is argued, do not state target debt as a ratio of market value (as implied by the cost of capital approach) but in dollar value terms.

**Measuring Adjusted Present Value**

In the adjusted present value approach, we estimate the value of the firm in three steps. We begin by estimating the value of the firm with no leverage. We then consider the present value of the interest tax savings generated by borrowing a given amount of money. Finally, we evaluate the effect of borrowing the amount on the probability that the firm will go bankrupt, and the expected cost of bankruptcy.

The first step in this approach is the estimation of the value of the unlevered firm. This can be accomplished by valuing the firm as if it had no debt, i.e., by discounting the expected free cash flow to the firm at the unlevered cost of equity. In the special case where cash flows grow at a constant rate in perpetuity, the value of the firm is easily computed.

\[
\text{Value of Unlevered Firm} = \frac{\text{FCFF}_0 (1 + g)}{\rho_u - g}
\]
where FCFF₀ is the current after-tax operating cash flow to the firm, ρᵤ is the unlevered cost of equity and g is the expected growth rate. In the more general case, we can value the firm using any set of growth assumptions we believe are reasonable for the firm. The inputs needed for this valuation are the expected cashflows, growth rates and the unlevered cost of equity.

The second step in this approach is the calculation of the expected tax benefit from a given level of debt. This tax benefit is a function of the tax rate of the firm and is discounted to reflect the riskiness of this cash flow.

\[
\text{Value of Tax Benefits} = \sum_{i=1}^{\infty} \frac{\text{Tax Rate}_i \times \text{Interest Rate}_i \times \text{Debt}_i}{(1+r)^i}
\]

There are three estimation questions that we have to address here. The first is what tax rate to use in computing the tax benefit and whether that rate can change over time. The second is the dollar debt to use in computing the tax savings and whether that amount can vary across time. The final issue relates to what discount rate to use to compute the present value of the tax benefits. In the early iterations of APV, the tax rate and dollar debt were viewed as constants (resulting in tax savings as a perpetuity) and the pre-tax cost of debt was used as the discount rate leading to a simplification of the tax benefit value:

\[
\text{Value of Tax Benefits} = \frac{(\text{Tax Rate})(\text{Cost of Debt})(\text{Debt})}{\text{Cost of Debt}} = t_c D
\]

Subsequent adaptations of the approach allowed for variations in both the tax rate and the dollar debt level, and raised questions about whether it was appropriate to use the cost of debt as the discount rate. Fernandez (2004) argued that the value of tax benefits should be computed as the difference between the value of the levered firm, with the interest tax savings, and the value of the same firm without leverage. Consequently, he arrives at a much higher value for the tax savings than the conventional approach, by a multiple of the unlevered firm’s cost of equity to the cost of debt. Cooper and Nyborg (2006) argue that Fernandez is wrong and that the value of the tax shield is the present value of the interest tax savings, discounted back at the cost of debt.
The third step is to evaluate the effect of the given level of debt on the default risk of the firm and on expected bankruptcy costs. In theory, at least, this requires the estimation of the probability of default with the additional debt and the direct and indirect cost of bankruptcy. If \( \pi_a \) is the probability of default after the additional debt and BC is the present value of the bankruptcy cost, the present value of expected bankruptcy cost can be estimated.

\[
\text{PV of Expected Bankruptcy cost} = (\text{Probability of Bankruptcy}) \times (\text{PV of Bankruptcy Cost}) = \pi_a BC
\]

This step of the adjusted present value approach poses the most significant estimation problem, since neither the probability of bankruptcy nor the bankruptcy cost can be estimated directly. There are two basic ways in which the probability of bankruptcy can be estimated indirectly. One is to estimate a bond rating, as we did in the cost of capital approach, at each level of debt and use the empirical estimates of default probabilities for each rating. The other is to use a statistical approach to estimate the probability of default, based upon the firm’s observable characteristics, at each level of debt. The bankruptcy cost can be estimated, albeit with considerable error, from studies that have looked at the magnitude of this cost in actual bankruptcies. Research that has looked at the direct cost of bankruptcy concludes that they are small\(^{12}\), relative to firm value. In fact, the costs of distress stretch far beyond the conventional costs of bankruptcy and liquidation. The perception of distress can do serious damage to a firm’s operations, as employees, customers, suppliers and lenders react. Firms that are viewed as distressed lose customers (and sales), have higher employee turnover and have to accept much tighter restrictions from suppliers than healthy firms. These indirect bankruptcy costs can be catastrophic for many firms and essentially make the perception of distress into a reality. The magnitude of these costs has been examined in studies and can range from 10-25% of firm value.\(^{13}\)

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\(^{12}\) Warner, J.N., 1977, studies railroad bankruptcies, and concludes that the direct cost of bankruptcy was only 5% on the day before bankruptcy. In fact, it is even lower when assessed five years ahead of the bankruptcy.

\(^{13}\) For an examination of the theory behind indirect bankruptcy costs, see Opler, T. and S. Titman, 1994. For an estimate on how large these indirect bankruptcy costs are in the real world, see Andrade, G. and S. Kaplan, 1998. They look at highly levered transactions that subsequently became distressed and conclude that the magnitude of these costs ranges from 10% to 23% of firm value.
Illustration 2.8: APV Valuation: 3M

**Cost of Capital versus APV Valuation**

In an APV valuation, the value of a levered firm is obtained by adding the net effect of debt to the unlevered firm value.

\[
\text{Value of Levered Firm} = \frac{\text{FCFF}_u (1 + g)}{\rho_u - g} + t_c D - \pi_s \Delta C
\]

The tax savings from debt are discounted back at the cost of debt. In the cost of capital approach, the effects of leverage show up in the cost of capital, with the tax benefit incorporated in the after-tax cost of debt and the bankruptcy costs in both the levered beta and the pre-tax cost of debt. Inselbag and Kaufold (1997) provide examples where they get identical values using the APV and Cost of Capital approaches, but only because they infer the costs of equity to use in the latter.

Will the approaches yield the same value? Not necessarily. The first reason for the differences is that the models consider bankruptcy costs very differently, with the adjusted present value approach providing more flexibility in allowing you to consider indirect bankruptcy costs. To the extent that these costs do not show up or show up inadequately in the pre-tax cost of debt, the APV approach will yield a more conservative estimate of value. The second reason is that the conventional APV approach considers the tax benefit from a fixed dollar debt value, usually based upon existing debt. The cost of capital and compressed APV approaches estimate the tax benefit from a debt ratio that may require the firm to borrow increasing amounts in the future. For instance, assuming a market debt to capital ratio of 30% in perpetuity for a growing firm will require it to borrow more in the future and the tax benefit from expected future borrowings is incorporated into value today. Finally, the discount rate used to compute the present value of tax benefits is the pre-tax cost of debt in the conventional APV approach and the unlevered cost of equity in the compressed APV and the cost of capital approaches. As we noted earlier, the compressed APV approach yields equivalent values to the cost of capital approach, if we allow dollar debt to reflect changing firm value (and debt ratio assumptions) and ignore the effect of indirect bankruptcy costs. The conventional APV approach yields a higher value than either of the other two approaches because it views the tax savings from debt as less risky and assigns a higher value to it.
Which approach will yield more reasonable estimates of value? The dollar debt assumption in the APV approach is a more conservative one but the fundamental flaw with the APV model lies in the difficulties associated with estimating expected bankruptcy costs. As long as that cost cannot be estimated, the APV approach will continue to be used in half-baked form where the present value of tax benefits will be added to the unlevered firm value to arrive at total firm value.

**Excess Return Models**

The model that we have presented in this section, where expected cash flows are discounted back at a risk-adjusted discount rate is the most commonly used discounted cash flow approach but there are variants. In the excess return valuation approach, we separate the cash flows into excess return cash flows and normal return cash flows. Earning the risk-adjusted required return (cost of capital or equity) is considered a normal return cash flow but any cash flows above or below this number are categorized as excess returns; excess returns can therefore be either positive or negative. With the *excess return valuation* framework, the value of a business can be written as the sum of two components:

\[
\text{Value of business} = \text{Capital Invested in firm today} + \text{Present value of excess return cash flows from both existing and future projects}
\]

If we make the assumption that the accounting measure of capital invested (book value of capital) is a good measure of capital invested in assets today, this approach implies that firms that earn positive excess return cash flows will trade at market values higher than their book values and that the reverse will be true for firms that earn negative excess return cash flows.

**Basis for Models**

Excess return models have their roots in capital budgeting and the net present value rule. In effect, an investment adds value to a business only if it has positive net present value, no matter how profitable it may seem on the surface. This would also imply that earnings and cash flow growth have value only when it is accompanied by excess returns, i.e., returns on equity (capital) that exceed the cost of equity (capital).
Excess return models take this conclusion to the logical next step and compute the value of a firm as a function of expected excess returns.

While there are numerous versions of excess return models, we will consider one widely used variant, which is economic value added (EVA) in this section. The economic value added (EVA) is a measure of the surplus value created by an investment or a portfolio of investments. It is computed as the product of the "excess return" made on an investment or investments and the capital invested in that investment or investments.

Economic Value Added = (Return on Capital Invested – Cost of Capital) (Capital Invested) = After-tax operating income – (Cost of Capital) (Capital Invested)

Economic value added is a simple extension of the net present value rule. The net present value of the project is the present value of the economic value added by that project over its life.\(^{14}\)

\[ NPV = \sum_{t=1}^{n} \frac{EVA_t}{(1 + k_c)^t} \]

where EVA\(_t\) is the economic value added by the project in year \(t\) and the project has a life of \(n\) years and \(k_c\) is the cost of capital.

This connection between economic value added and NPV allows us to link the value of a firm to the economic value added by that firm. To see this, let us begin with a simple formulation of firm value in terms of the value of assets in place and expected future growth.

Firm Value = Value of Assets in Place + Value of Expected Future Growth

Note that in a discounted cash flow model, the values of both assets in place and expected future growth can be written in terms of the net present value created by each component.

\[ \text{Firm Value} = \text{Capital Invested}_{\text{Assets in Place}} + \text{NPV}_{\text{Assets in Place}} + \sum_{t=1}^{\infty} \text{NPV}_{\text{Future Projects, } t} \]

Substituting the economic value added version of net present value into this equation, we get:

\[ \text{NPV} = \sum_{t=1}^{n} \frac{EVA_t}{(1 + k_c)^t} \]

14 This is true, though, only if the expected present value of the cash flows from depreciation is assumed to be equal to the present value of the return of the capital invested in the project. A proof of this equality can be found in Damodaran, A, 1999.
Firm Value = \text{Capital Invested}_{\text{Assets in Place}} + \sum_{t=1}^{\infty} \frac{\text{EVA}_{t, \text{Assets in Place}}}{(1 + k_c)^t} + \sum_{t=1}^{\infty} \frac{\text{EVA}_{t, \text{Future Projects}}}{(1 + k_c)^t}

Thus, the value of a firm can be written as the sum of three components, the capital invested in assets in place, the present value of the economic value added by these assets and the expected present value of the economic value that will be added by future investments. Note that the reasoning used for firm value can be applied just as easily to equity value, leading to the following equation, stated in terms of equity excess returns:

\text{Equity Value} = \text{Equity Invested}_{\text{Assets in Place}} + \sum_{t=1}^{\infty} \frac{\text{Equity EVA}_{t, \text{Assets in Place}}}{(1 + k_c)^t} + \sum_{t=1}^{\infty} \frac{\text{Equity EVA}_{t, \text{Future Projects}}}{(1 + k_c)^t}

\text{Equity EVA} = (\text{Return on equity} – \text{Cost of Equity}) \times \text{Equity Invested}_{\text{Assets in Place}}

Note that \(k_c\) is the cost of equity.

**Measuring Economic Value Added**

The definition of EVA outlines three basic inputs we need for its computation - the return on capital earned on investments, the cost of capital for those investments and the capital invested in them. We talked about the last first inputs in the context of conventional DCF models, and everything that we said in that context applies to measuring EVA as well.

The last input – capital invested in existing assets – is a key input to excess return models, since it represents the base on which the excess returns are computed. One obvious measure is the market value of the firm, but market value includes capital invested not just in assets in place but in expected future growth\(^{15}\). Since we want to evaluate the quality of assets in place, we need a measure of the capital invested in these assets. Given the difficulty of estimating the value of assets in place, it is not surprising that we turn to the book value of capital as a proxy for the capital invested in assets in place. The book value, however, is a number that reflects not just the accounting choices made in the current period, but also accounting decisions made over time on how to depreciate assets, value inventory and deal with acquisitions. The older the firm, the more extensive the adjustments that have to be made to book value of capital to get to a

\(^{15}\) As an illustration, computing the return on capital at Microsoft Google using the market value of the firm, instead of book value, results in a return on capital of about 13%. It would be a mistake to view this as a sign of poor investments on the part of the firm's managers.
reasonable estimate of the market value of capital invested in assets in place. Since this requires that we know and take into account every accounting decision over time, there are cases where the book value of capital is too flawed to be fixable. Here, it is best to estimate the capital invested from the ground up, starting with the assets owned by the firm, estimating the market value of these assets and cumulating this market value.

**Equivalence of Excess Return and DCF Valuation Models**

It is relatively simple to show that the discounted cash flow value of a firm should match the value that you obtain from an excess return model, if you are consistent in your assumptions about growth and reinvestment. In particular, excess return models are built around a link between reinvestment and growth; in other words, a firm can generate higher earnings in the future only by reinvesting in new assets or using existing assets more efficiently. Discounted cash flow models often do not make this linkage explicit, even though you can argue that they should. Thus, analysts will often estimate growth rates and reinvestment as separate inputs and not make explicit links between the two.

The model values can diverge because of differences in assumptions and ease of estimation. Penman and Sourgiannis (1998) compared the dividend discount model to excess return models and concluded that the valuation errors in a discounted cash flow model, with a ten-year horizon, significantly exceeded the errors in an excess return model. They attributed the difference to GAAP accrual earnings being more informative than either cash flows or dividends. Francis, Olson and Oswald (1999) concurred with Penman and also found that excess return models outperform dividend discount models. Courteau, Kao and Richardson (2001) argue that the superiority of excess return models in these studies can be attributed entirely to differences in the terminal value calculation and that using a terminal price estimated by Value Line (instead of estimating one) results in dividend discount models outperforming excess return models.

**What do intrinsic valuation models tell us?**

All of the approaches described in this chapter try to estimate the intrinsic value of an asset or a business. However, it is important that we understand exactly what we are doing in the process. We are estimating what an asset or business is worth, given its cash flows and the risk in those cash flows. To the extent that the value is dependent upon the
assumptions we make about cash flows, growth and risk, it represents what we think the intrinsic value is at any point in time.

So, what if the intrinsic value that we derive is very different from the market price? There are several possible explanations. One is that we have made erroneous or unrealistic assumptions about a company’s future growth potential or riskiness. A second and related explanation is that we have made incorrect assessments of risk premiums for the entire market. A third is that the market is, in fact, making a mistake in its assessment of value.

Even in the last scenario, where our assessment of value is right and the market price is wrong, there is no guarantee that we can make money of our valuations. For that to occur, markets have to correct their mistakes and that may not happen in the near future. In fact, we can buy stocks that we believe are under valued and find them become more under valued over time. That is why a long time horizon is almost a pre-requisite for using intrinsic valuation models. Giving the market more time (say 3 to 5 years) to fix its mistakes provides better odds than hoping that it will happen in the next quarter or the next six months.

Conclusion

The intrinsic value of a company reflects its fundamentals. The primary tool for estimating intrinsic value is the discounted cash flow model. We started by looking at the contrast between valuing the equity in a business and valuing the entire business, and then moved on to the four inputs that we need for the model. The cash flows to equity investors can be defined strictly as dividends, more expansively as dividends augmented with stock buybacks and most generally as free cash flows to equity (potential dividends). The cash flow to the firm is the cumulative cash flow to both equity investors and lenders, and thus is a pre-debt cash flow. The discount rates we apply have to be consistent with the cash flow definition, with the cost of equity used to discount cash flows to equity and the cost of capital to discount cash flows to the firm. When estimating growth, we noted the limitations of historical growth numbers and outside estimates, and the importance of linking growth to fundamentals. Finally, we applied closure to the models by assuming that cash flows will settle into stable growth, sometime in the future,
but imposed constraints on what this growth rate can be and the characteristics of stable growth companies.

We closed the chapter by looking at three variations on the discounted cash flow model. In the certainty equivalent approach, we adjusted the cash flows for risk and discounted back at the riskfree rate. In the adjusted present value approach, we separated debt from the operating assets of the firm, and valued its effects independently of the firm. In the excess return model, we zeroed in on the fact that it is not growth per se that creates value but growth with excess returns. However, we noted that the models agree at the core, though there are minor differences in assumptions.