Solutions to Quiz 3: Spring 1998

1. The inputs are as follows:
   \[ S = 25 \frac{(1-.6)(1.05)}{.10 - .05} = 315 \] ; (300 if $25 mil is assumed to be EBIT next year)
   \[ K = 500 \]
   \[ r = 7\% \]
   \[ t = 3 \text{ years (Equally Weighted average of 2 and 4 year durations)} \]
   \[ \left[\frac{\sigma^2}{.5^2} (25)^2 + \frac{.5^2}{.40^2} + 2 (.5) (.5) (.25) (.4)\right] = 0.0806 \]

2. **Company B’s reserves will be more valuable.** To value the oil reserves as an option

<table>
<thead>
<tr>
<th>Input</th>
<th>Company A</th>
<th>Company B</th>
</tr>
</thead>
<tbody>
<tr>
<td>S</td>
<td>1000 * (24-6)/(1+y)^2</td>
<td>1000 * (24-4)/(1+y)^2</td>
</tr>
<tr>
<td>K</td>
<td>1000* 10</td>
<td>1000* 12</td>
</tr>
<tr>
<td>t</td>
<td>15</td>
<td>15</td>
</tr>
</tbody>
</table>

If we assume that \( y = 1/15 = 0.06667 \) and \( r = 7\% \), the value of oil reserves as options can be graphed as follows:
In other words, the present value effect on the value of $S$ is dominated by the option effect (which is captured in the variance. That is why the difference gets narrower as the standard deviation in oil prices decreases; the actual standard deviation in oil prices is about 30%)

The next graph shows the value of the options as a function of the dividend yield.
Again, the value of company B’s reserves are greater than company A’s reserves but the difference narrows as the yield increases. [Since there is sufficient ambiguity in the problem and there is an outside chance that company A’s reserves could be worth more, I gave 2 points for those who picked (a) or (d).]

2B. The value of oil is higher (increasing S) and the variance is lower. The net effect can be positive or negative.

3a. The variance is lower but the value of the project will go up. The net effect will almost certainly be positive, but I gave credit to those who said it was uncertain.
3b. The value of the option will increase.

3c. The variance may increase but the value effect will be negative. The net effect again will almost certainly be negative, but I gave full credit for those who picked uncertain.

3d. The interest rate increase will have a positive effect but it will lower the present value of the project, thus lowering value. The net effect is likely to be negative, but I gave full credit for uncertain.
Solution to Quiz 3: Fall 1998

Problem 1

Adjusted pre-tax operating income = $10 million - $1.5 million = $8.50

Adjusted after-tax operating income = $8.5 million (1-.40) = $5.10 (I also gave full credit)

Firm Value = 5.1(1.05)/(.09-.05) = $133.88

Illiquidity Discount = .30 - .04 (ln(100) = 11.58%)

Firm Value after Illiquidity Discount = $118.37 (if you did not use (1+g))

Problem 2

a. Net present value of the project = $30 - $40 = - $10 million

b. Inputs

S = Present Value of Net Revenues = $30 million
K = Cost of televising the Olympics = $40 million
\( t = Time \ until \ Olympics = 2 \ years \)
\( r = Riskless \ rate = 5\% \)

Variance in value = 0.09

\( y = Cost \ of \ delay = 0 \)

\( d_1 = -0.2302 \quad N(d_1) = 0.409 \)

\( d_2 = -0.6545 \quad N(d_2) = 0.2564 \)

Value of the Rights = 30 (0.409) - 40 exp (-0.05)(2) (.2564) = $2.99 ($2.71 mil)

c. Range of Probability that rights are profitable = 0.2564 - 0.4090 with approx N(d))
Solution to Spring 1999 Quiz

Problem 1

VS = AT Operating Margin * (1 - Reinvestment Rate) * (1 + g)/(WACC - g)
Reinvestment Rate = g / ROC = g / (AT Operating Margin * Sales/Capital)
Let the margin for Generic Office be x,
VS for Generic Office = 1.5 = x (1-.05/2x)(1.05)/(.10-.05)
Solve for x,
x = (1.5*.05+(.05/2)*1.05)/1.05
x = 9.64%
HK’s after-tax operating margin = 9.64% (1.05) = 10.13%! I also gave full credit if you added 5% to get 14.64%
HK’s VS ratio =.1013 (1-.05/2*.1013)(1.05)/(.10-.05) = 1.6023

Problem 2

a. Value of Developed Reserves = 100,000 * 300 * (PV of Annuity, 5 years, 9%) = $116,689,538
   Net Value of Developed Reserves = 100,000 *(300-200)*( PV of Annuity, 5 years, 9%) = $38,896,513

b. Value of Undeveloped Reserves (on DCF basis)
   Value of oil in the ground = 40,000 * (300-200) * (PV of Annuity, 9%, 12.5 years) = $29,309,310.89
   ! 40000 barrels a year for 12.5 years. You run out of oil after 12.5 years.
   Value of oil in the ground allowing for development lag = 29,309,311/1.08 = $27,138,250.82
   Fixed cost of development = $50,000,000.00
   DCF Value of Undeveloped reserves = $22,861,749.18
I also gave full credit for a number of variations, including

a. Assuming that costs and prices are in present value dollars, in which case the value of reserves is [500000*(300-200)]/1.08 = 46,296,296
b. Inputs for Option Pricing Model
   y = Annual Production Revenue/Reserves = 40,000/500,000 = 8.00%! Many of you used 1/15. You actually lose more because your production potential is much higher.
$ S = $ PV of reserves discounted back by development lag = $ 27,138,251 \text{ Full credit also for $ 46,296,296 }$

$ K = $ Upfront Cost of Developing Reserves = $ 50,000,000$

$ t = $ Period for which firm has rights = 15

Variance = Expected variance in $ \ln(\text{gold prices}) = 0.09 \text{ Use forward looking variance}$

$ r = $ Riskfree rate = 6.00%

d. The developed reserves will become less valuable, since oil prices are down.
The undeveloped reserves may become more or less valuable depending upon whether the effect of $ S $ or the effect of variance is greater on the option value.
Spring 2000: Quiz 3 solutions

Problem 1
Adjusted EBIT = 1500 - 100 = 1400
Adjusted EBIT (1-t) = 840
- Reinvestment = 336
FCFF = 504

Total Beta = 0.80/0.5 = 1.6
Levered beta = 1.6 (1 + (1-.4)(3/7)) = 2.01142857
Cost of Equity = 6% + 2.01 (4%) = 14.04%
Cost of Capital = 14.04% (.7) + 7% (1-.4)(.3) = 11.09%
Value of private firm = 504 (1.05)/(0.1109-.05) = $ 8,692.51

Problem 2
Present Value of FCFF from developed reserves = 300 (PVA, 10 years, 9.375%) = $ 1,894
Cost of Capital = 12%(25/40) + 5% (15/40) = 9.38%
Value of undeveloped reserves = 4000 - 1894 = $ 2,106

Problem 3
\[ d_1 = -0.15 \quad N(d_1) = 0.4404 \]
\[ d_2 = -0.90 \quad N(d_2) = 0.1841 \]

Value of Equity = 400 (.4404) - 800 exp (-.06*6) (.1841) = $ 73.41
Value of Debt = 400 - 73.41 = $ 326.59
Interest rate on debt = (800/326.59)^(1/6) - 1 = 16.08%
Default spread on debt = 16.08% - 6% = 10.08%
Spring 2001: Quiz 3 solution

Problem 1
Adjusted Operating Income = $100,000! I added back the manager’s salary
Adjusted after-tax Operating Income = $65,000
Adjusted ROC = 13.00%! I divided by book value of capital (200000+300000)
Expected Growth Rate = 3%
Expected Reinvestment rate = 23.08%

Total Unlevered Beta = 2.00%! I used total beta because I am not diversified
Total Levered Beta = 2.33! 2 (1+ (1-.35)(.25))
Cost of equity = 14.30%
Cost of debt = 5.20%
Cost of capital = 12.48%! I used industry averages, not the book value

Value of Bagel shop = $543,249

Problem 2
total equity value: independent firms = 294

To value equity in the combined firm,
Variance in combined firm value = 0.104! Used variance of two asset portfolio
Standard deviation in combined firm = 32.25%!w=50% (based on firm value)

\[ S = Value \ of \ firms = 600 \]
\[ K = Face \ value \ of \ combined \ debt = 500 \]
\[ t = 5 \]
\[ r = 5\% \]
\[ d1 = 0.96 \]
N(d1) = 0.83  
d2 = 0.24  
N(d2) = 0.59

Value of equity = $267.00 = 600*0.83 - 500* e^(-0.05*5) *0.59$

Loss in value of equity = $27.00
Solutions: Spring 2002

Problem 1

<table>
<thead>
<tr>
<th>BigName</th>
<th>NoName</th>
<th>Brand Name</th>
</tr>
</thead>
<tbody>
<tr>
<td>After-tax Margin</td>
<td>12%</td>
<td>6%</td>
</tr>
<tr>
<td>Sales/ Capital</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>ROC</td>
<td>24.00%</td>
<td>12.00%</td>
</tr>
<tr>
<td>Reinvestment rate</td>
<td>16.67%</td>
<td>33.33%</td>
</tr>
<tr>
<td>Value/Sales</td>
<td>2.08</td>
<td>0.832</td>
</tr>
<tr>
<td>Value</td>
<td>10.4</td>
<td>4.16</td>
</tr>
</tbody>
</table>

Problem 2

| Net Income | 2 | ! Remember to subtract after-tax expense = 2.6 - 1(1-.4) |
| ROE | 20.00% |
| Total Beta | 2.75 |
| PBV | 1.94 |
| MV of Equity | 19.4 |

Problem 3

| S | 106.698524 |
| K | 131.698524 |
| t | 12 |
| Variance | 0.0225 |
| Riskless rate | 5% |
| cost of delay | 0.08333333 |

! I think you can also make a reasonable case for 1/8 if you argued that competition would kick in after the 8th year. 
! In fact, you can even make a reasonable case for it being 0. If you entered 0 and want to justify it, give me your reason…
Solution: Fall 2002

Problem 1
First, clean up the market value of equity for the cross holdings
Total market value of equity = 3000
- Value of equity in Coleman Holdings = 500 ! 20% of 2500
- Value of equity in Silicon Tech = 1200 ! 60% of 2000
Value of Equity in Steel business = 1300

Next clean up the debt for the consolidated debt from Silicon
Value of debt = 2000
- Value of Debt in Silicon Tech = 500 ! Silicon Tech's debt is consolidated in balance sheet
Value of debt in Steel business = 1500

Finally, clean up the EBITDA
Consolidated EBITDA = 700
- EBITDA of Silicon Tech = 300
EBITDA of Steel Business = 400

EV of Steel Business = 2800
EV/ EBITDA of Steel Business = 7.00

Problem 2
a. Estimated PS in year 5 = 3.5 ! Net margin = 10%; 1.5 + .2 (10) = 3.5
   Estimated Equity value in year 5 = 1750 ! 3.5 * 500
b. Total beta = 2.25
   Cost of equity = 14.00%
c. Value of equity today (going concern) = $908.90 ! 1750/1.14^5
   Survival adjusted equity value = $363.56 ! 908.90 * .40
Problem 1

a. Unlevered beta = 1
Total beta = 2.5
Cost of equity = 15.00% ! Use total beta because buyer is not diversified
Return on capital = 0.175 ! 280/1600
Reinvestment rate = 0.22857143 !4%/17.5%

EBIT (1-t) $280.00
- Reinvestment $64.00 ! 22.86% of EBIT (1-t)
FCFF $216.00

Value of firm = $2,042.18 ! 224 (1.04)/(.15-.04)

b. Unlevered beta = 1 ! IPO valuation: use market beta
Cost of equity = 9.00%
Return on capital= 0.15 ! Using reestimated return on capital
Reinvestment rate = 0.26666667 ! 3.5%/15%

EBIT (1-t) $240.00 ! Reestimate operating income with 40% tax rate
- Reinvestment $64.00 ! 23.33% of 240
FCFF $176.00

Value of firm = $3,660.80 ! 176 (1.04)/(.09-.04)

c. There should be an illiquidity discount in the private transaction but not on the IPO.

Problem 2

Part a
Value of commercial product = $88.63
Value of R&D = $45.00
Total = $133.63

Market value of firm = $240.00
Value of patent = $106.37

Part b
If the firm develops the patent today, you will replace the option value above with the NPV
NPV = $49.39
Change in value = -$56.98 ! 49.39 - 106.37
New firm value = $183.02 ! 240 - 56.98
Problem 1
a. Intrinsic price to book ratio = 0.833333333 \(= (\text{ROE} - g)/(\text{COE} - g); \text{ROE} = 8\%, \text{Cost of equity} = 5+4\% = 9\% \)
Stock is undervalued slightly.
b. if the market is correct
Price to book ratio = .8 = (.08-.03)/(r-.03)
Solving for r, Cost of equity = 9.25%

Problem 2
Levered beta = 1.56
EV/Sales based upon regression = 2.028 ! 0.30 + .08 (15/100) - 1.2 (1.56) + .12 (20)
Actual EV/Sales ratio = 2.5 ! (200 + 100 -50)/100
Stock is overvalued by 23.27% ! Divided the actual by the expected value

Problem 3
Total beta = 3 ! Sale in a private transaction. Divided beta by correlation.
Cost of equity = 17.00% ! 5\% + 3*4\%
Reinvestment rate = 0.4 ! G/ROC = 4\%/10\%
Status quo value of firm = 6.923076923 ! 1.5 (1-.4)/(.17-.04); I don't need a (1+g) because 1.5 is next year's income
Value of 25\% stake in firm = 1.730769231

With a doubled return on capital
Reinvestment rate = 0.2 ! Doubled return on capital only on new investments. So existing operating income unaffected
Optimal firm value = 9.230769231 ! 1.5 (1-.2)/(.17-.04)
Value of 51\% stake in firm = 4.707692308

For IPO valuation
Market beta = 0.9
Cost of equity = 8.600% ! Since this is an IPO valuation, you have to revalue the company with a market beta.
Status quo value = 19.56521739 ! 1.5 (1-.4)/(.086-.04)
Optimal firm value = 26.08695652 ! 1.5 (1-.2)/(.086-.04)
Value per voting share = 4.891304348
Value per voting share = 5.543478261 !4.89+.2(26.09-19.57)/2

Fall 2003
### Problem 1

<table>
<thead>
<tr>
<th></th>
<th>Gloria Inc.</th>
<th>Generic</th>
<th>Brand Name Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>After-tax Operating Margin</td>
<td>0.15</td>
<td>0.075</td>
<td></td>
</tr>
<tr>
<td>Return on Capital</td>
<td>0.25</td>
<td>0.125</td>
<td></td>
</tr>
<tr>
<td>Reinvestment Rate</td>
<td>0.2</td>
<td>0.4</td>
<td></td>
</tr>
<tr>
<td>EV/Sales</td>
<td>2.4</td>
<td>0.9</td>
<td></td>
</tr>
<tr>
<td>Enterprise Value</td>
<td>2400</td>
<td>900</td>
<td>1500</td>
</tr>
</tbody>
</table>

### Problem 2

Cost of equity = 10.0% ! Use market beta since this is for initial public offering

**Part a. Under existing management**

- EBIT (1-t) = 5
- Return on capital = 0.1
- Reinvestment Rate = 0.4 ! 4/10
- Value of firm = 50 ! No surprise here. If you earn your cost of capital, MV = BV

**Part b. Under new management**

- EBIT (1-t) = $6.50
- Return on capital = 13.00%
- Reinvestment Rate = 30.77% ! 4/13
- Value of firm = $75.00

- Value of control = $5.00 ! (100 - 75) * .20

- Value per non-voting share = $10.00 ! Divide status quo value by total number of shares
- Value per voting share = $12.50 ! Add Value of control/ Number of voting shares
Problem 1
EV/Sales = After-tax operating margin \((1 - \frac{g}{ROC})\) / (Cost of capital - g)
\[1.20 = 0.10 \times (1 - \frac{0.04}{ROC}) / (0.09 - 0.04)\]
Solving for return on capital, we get
Return on capital = 10.00%

![There are other ways to get to the same solution. You could set up firm value solving for return on capital, we get]

3000 = 250 \((1 - \frac{0.04}{ROC}) / (0.09 - 0.04)\)
While technically you do not need a \((1+g)\) in the numerator since I have given you next year's operating income I did give full credit to those who used it.

Problem 2

<table>
<thead>
<tr>
<th>Bank</th>
<th>PBV</th>
<th>ROE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hibernia Bank</td>
<td>1.25</td>
<td>16%</td>
</tr>
<tr>
<td>Bancomer</td>
<td>1.1</td>
<td>14%</td>
</tr>
<tr>
<td>North Fork Bank</td>
<td>0.9</td>
<td>12%</td>
</tr>
</tbody>
</table>

North Fork Bank is the most undervalued bank.....

If we use the regression, PBV Predicted PBV
Hibernia Bank 1.25 1.2 Overvalued
Bancomer 1.1 1.1 Correctly valued
North Fork Bank 0.9 1 Correctly valued

b. Low Price to book, high ROE, low risk, high growth

Problem 3
Corrected income = $300,000.00
After-tax income = $180,000.00
Subtracted out rent (75,000), accounting expense (25,000) and dental salary of 150,000.

Total beta = 2.4 \times 0.80 \times 3
Cost of equity = 13.85%

Value of practice = $1,549,367.09

Fall 2004
### Problem 1

**a.** Tax Rate: The higher the tax rate, the lower the EV/EBITDA multiple should be (not higher)

**b.**

\[
\text{EV/EBITDA} = 2.26 + 0.1513 \times \text{(Tax Rate)} + 0.2156 \times \text{(Return on Capital)} - 0.1335
\]

You cannot change the sign on a regression coefficient if you don't agree with it.

\[
\text{EV/EBITDA} = 7.9059
\]

### Problem 2

**Market value of equity = 1800**

\[
\frac{\text{MV of Equity}}{\text{Forward Earnings}} = 1.2 \Rightarrow 1800/1500
\]

\[
\text{PE} = 1.2 = \frac{\text{Payout Ratio}}{0.10 - 0.04}
\]

\[
\text{Payout ratio} = 7.20\%
\]

\[
\text{Return on equity} = \frac{\text{g}}{1 - \text{Payout ratio}} = 4.31\%
\]

Loser company but dem's the breaks...

### Problem 3

**Total Beta = \frac{1.2}{.4} = 3 ! Market beta/ Square root of R squared**

\[
\text{Cost of equity} = 5\% + 3 \times 4\% = 17.00\%
\]

\[
\text{Reinvestment rate} = 0.25 ! \frac{\text{g}}{\text{ROC}}
\]

\[
\text{Value of firm} = $66.21
\]

\[
\text{Cost of equity to publicly traded firm} = 0.098 \! \text{ ! Investors in publicly traded firm are diversified.}
\]

\[
\text{new return on capital} = 0.18 \! \text{ ! I also gave full credit if you adjusted the growth rate upwards to 4.5%}
\]

\[
\text{New reinvestment rate} = 0.16666667
\]

\[
\text{Value of firm} = 151.4705882 \! \text{ ! Only the growth rate or reinvestment rate will be affected...}
\]

*Value of 51% = 77.25*

Existing operating income remains unchanged.
Problem 1

Enterprise value = Equity + Debt - Cash
EV/Sales = \[ EV/Sales = 1.25 \] \[ 2500 \]

b. Estimated ROC = 0.075 ! Assumed book equity = 1000
EV/Sales = Expected operating margin next year/(1-RIR) (Cost of capital -g) ! The book value of capital was missing on this problem. So, I gave full credit to any book value of capital that was reasonable. The solution assumes that the book value of equity is $1 billion and the book value of capital is $2 billion. Solving for the cost of capital, Cost of capital = 6.60% ! If you assume a 5% return on capital, this number will be lower (5.4%) Did not use any reinvestment: -0.5 point (No ROC would support this) Reinvestment rate wrong (given your ROC): -0.5 point Failed to consider operating margin: -0.5 point Other mechanical errors: -0.5 point each

Problem 2

Market value of equity of parent company = 650 ! 1000*10 - 0.1*500 - 0.75*400 ! Did not net out cash of consolidated sub: -0.5 point
Debt of parent company = 300 ! 500 - 200 ! Got parent EBITDA incorrect: -0.5 to -1 point
Cash of parent company = 50 ! 150 - 100 ! Other computational errors: -0.5 point each
Enterprise value of parent company = 900
EBITDA of parent company = 150 ! 250 - 100
EV/EBITDA for parent company = 6 ! 900/150

Problem 3

C. Best combination: Low P/BV, Low risk, High growth, High ROE ! One point for D and F. They were close but failed one one dimension (D on growth and F on risk)
b. Expected price to book for company A = 0.8 ! Mechanical errors: -0.5 point
Actual price to book ratio = 1.25
Company is overvalued by approximately 36.00%
c. PBV of C = 1.2 = 0.80 + 0.75 X - 0.5 (1) ! Used intrinsic equation: -1 point
Solving for X ! Set up equation in way to make solution impossible: -1 point
ROE for company C would have to be 12% ! Mechanical errors: -0.5 point
The ROE would have to be approximately 12%. It is actually 20%