VALUING PRIVATE FIRMS

Problem 1

a. & b.

Average beta = 1.35666667
Average debt/equity = 0.13653333
Unlevered beta = 1.24764572
Total beta = 2.49529144
Total levered beta = 2.71306233
Cost of equity = 20.92%
Cost of capital = 19.08%

After year 5, I use a total beta of 2.00 (Market beta of 1/0.5)
Cost of equity = 6% + 2*5.5% = 17%
Cost of capital = 17% (.88) + 8.75% (1-.36)(.12) = 15.63%

<table>
<thead>
<tr>
<th>Year</th>
<th>Current</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>Terminal Year</th>
</tr>
</thead>
<tbody>
<tr>
<td>Revenues</td>
<td>$50.00</td>
<td>$65.00</td>
<td>$84.50</td>
<td>$109.85</td>
<td>$142.81</td>
<td>$185.65</td>
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<tr>
<td>EBIT</td>
<td>$9.50</td>
<td>$12.35</td>
<td>$16.06</td>
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<td>EBIT (1-t)</td>
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<td>$10.28</td>
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<tr>
<td>- Net cap ex</td>
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<tr>
<td>- Chg in WC</td>
<td>$1.50</td>
<td>$1.95</td>
<td>$2.54</td>
<td>$3.30</td>
<td>$4.28</td>
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<tr>
<td>FCF</td>
<td>$1.85</td>
<td>$2.41</td>
<td>$3.13</td>
<td>$4.07</td>
<td>$5.30</td>
<td>$9.04</td>
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</tr>
</tbody>
</table>

Terminal value | $93.86 |

PV | $1.56 | $1.70 | $1.86 | $2.03 | $41.40 |

Firm Value | $48.54 |
- Debt | $5.83 |
Value of Equity | $42.72 |

c. For an initial public offering, you would use a market beta

<table>
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<tr>
<th>Year</th>
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<td>$4.28</td>
<td>$1.11</td>
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<td>$4.07</td>
<td>$5.30</td>
<td>$9.04</td>
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<tr>
<td>Terminal value</td>
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<td>PV</td>
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<td>$2.03</td>
<td>$80.99</td>
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<tr>
<td>Firm Value</td>
<td>$88.13</td>
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</table>
- Debt $10.58
- Value of Equity $77.55

Average beta = 1.35666667
Average debt/equity = 0.13653333
Unlevered beta = 1.24764572
Total beta = 1.24764572
Total levered beta = 1.35653116

High Growth Stable Growth

<table>
<thead>
<tr>
<th>Beta</th>
<th>1.36</th>
<th>1.00</th>
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<tbody>
<tr>
<td>Cost of equity</td>
<td>13.46%</td>
<td>11.50%</td>
</tr>
<tr>
<td>Cost of capital</td>
<td>12.52%</td>
<td>10.79%</td>
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</table>

**Problem 2**

*a. Using the Silber regression*

Silber regression = \((100 - \exp(4.33+0.036\times\ln(\text{Revenues}) - 0.142\times\ln(\text{Block size}\times100) +0.174\times\text{Earnings Dummy})) /100\)

Revenues = 200 million
Block Size = 100% = 1
Earnings Dummy = 1 (positive earnings)
Discount = \((100-\exp(4.33+0.036\times\ln(200)-0.142 \times\ln(1*100) +0.174*1))/100\)

= 43.13%

Value of firm = 250 (1 - .4313) = $142 million

*b. Adjusting the base discount*

Base discount for firm with revenues of $ 10 million = 25%
Adjustment for revenues = \(((100-\exp(4.33+0.036\times\ln(10))-0.142\times\ln(1*100)+0.174*1))/100-(100-\exp(4.33+0.036\times\ln(200)-0.142\times\ln(1*100)+0.174*1))/100\) = 5.81%
Adjusted discount = 25% - 5.815 = 19.19%
Value of firm = 250 (1 - .1919) = $ 202 million

**Problem 3**

*a. Value of Business*

Assuming that the business will be sold to a diversified buyer, we use the market beta:

Cost of equity = 7% + 1.1*(5.5%) = 13.05%
Cost of capital = 13.05% (.5) + 8% (1-.4) (.5) = 8.925%
To estimate cashflows, we consider only the portion of the operating income that is not due to the current owner:

\[
\begin{align*}
\text{EBIT} & = 60000 \\
\text{EBIT (1-t)} & = 36000 \\
- \text{Net Cap ex} & = 10000 \\
\text{FCFF} & = 26000 \\
\end{align*}
\]

Present value of $26,000 growing at 5% a year for 10 years = $203,486

Present value of Salvage value of $ 500,000 in 10 years = $212,664

Value of Business = $ 416,150

b. If the chef offers to stay on,

<table>
<thead>
<tr>
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<tbody>
<tr>
<td>EBIT</td>
<td>105000</td>
<td>110250</td>
<td>115762.5</td>
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<tr>
<td>EBIT without chef</td>
<td>63000</td>
<td>66150</td>
<td>69457.5</td>
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<tr>
<td>Additional income</td>
<td>42000</td>
<td>44100</td>
<td>46305</td>
</tr>
<tr>
<td>After-tax income</td>
<td>25200</td>
<td>26460</td>
<td>27783</td>
</tr>
<tr>
<td>PV</td>
<td>23135</td>
<td>22301</td>
<td>21497</td>
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</table>

Additional value of cashflows = $ 66,934

Value of Business = 416150 + 66934 = $ 483,084.

**Problem 4**

a. Cost of capital

Unlevered beta = 1.20

Estimated market value of equity = $ 10 million * 3 = $ 30 million

Debt/Equity = 10/30

Levered beta = 1.20 (1 + (1-.40)(10/30)) = 1.44

Cost of equity = 6% + 1.44 (5.5%) = 13.94%

Cost of debt = 1/10 (I am assuming that the debt is recent)

Cost of capital = 13.94% (30/40) + 10% (1-.4) (10/40) = 11.94%

When we get to the terminal year, I would lower the beta to 1

Cost of equity = 11.5%

Cost of capital = 11.5% (30/40) + 10% (1-.4) (10/40) = 10.13%

(You could also adjust the cost of debt down)
b. & c. Firm Value & Equity value per share

Reinvestment rate after year 5 = g/ ROC = 5%/15% = 33.33%

Cap ex in year 6 = (3.14 * .3333) + Depreciation in year 5 (1.05)

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<tbody>
<tr>
<td>Revenues</td>
<td>$20.00</td>
<td>$24.00</td>
<td>$28.80</td>
<td>$34.56</td>
<td>$41.47</td>
<td>$49.77</td>
<td>$52.25</td>
</tr>
<tr>
<td>EBIT</td>
<td>$2.00</td>
<td>$2.40</td>
<td>$2.88</td>
<td>$3.46</td>
<td>$4.15</td>
<td>$4.98</td>
<td>$5.23</td>
</tr>
<tr>
<td>EBIT (1-t)</td>
<td>$1.44</td>
<td>$1.73</td>
<td>$2.07</td>
<td>$2.49</td>
<td>$2.99</td>
<td>$3.14</td>
<td></td>
</tr>
<tr>
<td>+ Depreciation</td>
<td>$0.50</td>
<td>$0.60</td>
<td>$0.72</td>
<td>$0.86</td>
<td>$1.04</td>
<td>$1.24</td>
<td>$1.31</td>
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<tr>
<td>- Cap Ex</td>
<td>$1.00</td>
<td>$1.20</td>
<td>$1.44</td>
<td>$1.73</td>
<td>$2.07</td>
<td>$2.49</td>
<td>$2.35</td>
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<tr>
<td>FCFF</td>
<td>$0.84</td>
<td>$1.01</td>
<td>$1.21</td>
<td>$1.45</td>
<td>$1.74</td>
<td>$2.09</td>
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</table>

Terminal value = 2.09/ (.1013 - .05) = $40.78

PV (@11.94%) | $0.75 | $0.80 | $0.86 | $0.92 | $24.20 |
Value of firm = $27.54
- Debt $10.00
Value of Equity = $17.54
Value per share = $17.54

Problem 5

I would make two adjustments. First, I would use the total beta, rather than the market beta:

First 5 years

Total Beta = 1.44/0.6 = 2.40
Cost of equity = 6% + 2.4 (5.5%) = 19.20%
Cost of debt = 10% (1-.4) = 6%
Cost of capital = 19.20% (30/40) + 6% (10/40) = 15.90%

Terminal year

Total Beta = 1/0.6 = 1.67
Cost of equity = 6% + 1.67 (5.5%) = 15.17%
Cost of capital = 15.17% (30/40) + 6% (10/40) = 12.88%

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<td>$1.04</td>
<td>$1.24</td>
<td></td>
</tr>
</tbody>
</table>
- Cap Ex    | $1.00 | $1.20 | $1.44 | $1.73 | $2.07 | $2.49 
FCFF       | $0.84 | $1.01 | $1.21 | $1.45 | $1.74 |
Terminal value |       |       |       |       |       | $26.54 
PV         | $0.72 | $0.75 | $0.78 | $0.80 |       | $13.52 
Value of firm = $16.58 
- Debt $10.00 
Value of Equity = $6.58 
I would apply a liquidity discount:

Illiquidity Discount = 0.14 – 0.015 (ln(Revenues)) = .0951

Value of Equity = 6.58 (1-.0951) = $5.96 million