Problem Set 5 Solution: ICAPM and Performance evaluation.

I. *ICAPM.* Let TERM(Jan) be the difference in the yield on a long term hi-grade corporate bond and 1 month T-bill at the end of January. Suppose each individual cares about \{E[R_p(Jan)], \sigma[R_p(Jan)], \sigma[R_p(Jan), TERM(Jan)]\} when forming his/her portfolio p for January. The following additional information is available:

<table>
<thead>
<tr>
<th>i</th>
<th>E[R_i(Jan)]</th>
<th>\beta_{i,M}</th>
<th>\beta_{i,TERM}</th>
</tr>
</thead>
<tbody>
<tr>
<td>Red</td>
<td>18%</td>
<td>1.4</td>
<td>0.4</td>
</tr>
<tr>
<td>Yellow</td>
<td>1.1</td>
<td>1.2</td>
<td></td>
</tr>
</tbody>
</table>

where \( \beta_{i,M} \) and \( \beta_{i,TERM} \) are regression coefficients from a regression of \( R_i(Jan) \) on \( R_M(Jan) \) and TERM(Jan):

\[
R_i = \phi_{i,0} + \beta_{i,M} R_M + \beta_{i,TERM} TERM(Jan) + e_i
\]

Also know that \( E[R_M(Jan)] = 14\% \) and \( R_f(Jan) = 8\% \).

A. What is the expected January return for Yellow?

Since each individual cares about \{E[R_p(Jan)], \sigma[R_p(Jan)], \sigma[R_p(Jan), TERM(Jan)]\} when forming his/her portfolio p for January, it follows that any asset satisfies:

\[
E[R_i(Jan)] = R_f(Jan) + \beta_{i,M} \lambda_{M} + \beta_{i,TERM} \lambda_{TERM}.
\]

Also know that \( \lambda_{M} = E[R_M(Jan)] - R_f \).

So

\[
E[R_{Red}(Jan)] = R_f(Jan) + \beta_{Red,M} \lambda_{M} + \beta_{Red,TERM} \lambda_{TERM}
\]

18\% = 8\% + 1.4 \times \{14\%-8\%\} + 0.4 \times \lambda_{TERM}

which implies \( \lambda_{TERM} = 1.6\%/0.4 = 4\% \).

So then the expected return on Yellow can be calculated

\[
E[R_{Yellow}(Jan)] = R_f(Jan) + \beta_{Yellow,M} \lambda_{M} + \beta_{Yellow,TERM} \lambda_{TERM}
\]

= 8\% + 1.1 \times 6\% + 1.2 \times 4\%

= 19.4\%.

B. What is the risk premium for bearing TERM risk?

As calculated above, \( \lambda_{TERM} = 4\% \).

C. Is the market on the minimum variance frontier? Why or why not?

Not necessarily. Because investors care about \( \sigma[R_p(Jan), TERM(Jan)] \), investors do not all want to hold combinations of the riskless asset and the mean-variance tangency portfolio. Investors
who like negative $\sigma[R_p(Jan), \text{TERM}(Jan)]$ may be prepared to hold a risky portfolio with a flatter-sloped Capital Allocation Line than that of the tangency portfolio because that portfolio offers more negative covariance with TERM(Jan) than the tangency portfolio. Thus, the market portfolio is not the tangency portfolio and need not lie on the minimum variance frontier for the N risky assets.

D. Give one reason why an individual may care about the covariance of her portfolio return with TERM(Jan).

TERM(Jan) may be correlated with the individual’s human capital value (at the end of January). Alternatively, TERM(Jan) may be correlated with expected February asset returns (at the start of February).

E. Characterize the portfolios that individuals hold in this economy.
All individuals hold portfolios that are combinations of:
1) the riskless asset.
2) the market portfolio.
3) a hedging portfolio which is highly correlated with TERM(Jan).

II. Hedging Portfolios and the I CAPM. Suppose each individual cares about \{E[R_p(Jan)], $\sigma[R_p(Jan), \text{TERM}(Jan), s_i(Jan)]$\} when forming his/her portfolio $p$ for January. The variable $s_i(Jan)$ is a macroeconomic indicator that correlates positively with the state of the economy at the end of January. Investors care about the state of the economy because it affects their human capital values, though to differing degrees depending on the investor. Let $R_{HML}(Jan)$ be the return on the hedging portfolio that all investors hold in combination with the market portfolio and the riskless asset; $R_{HML}(Jan)$ has a large positive correlation with $s_i(Jan)$. The following additional information is available:

<table>
<thead>
<tr>
<th>i</th>
<th>E[R_i(Jan)]</th>
<th>$\beta^h_{i,M}$</th>
<th>$\beta^h_{i,HML}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Market</td>
<td>10%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>HML</td>
<td>14%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>KS Fund</td>
<td>?</td>
<td>1.1</td>
<td>-0.4</td>
</tr>
<tr>
<td>LM Fund</td>
<td>?</td>
<td>1.1</td>
<td>0.5</td>
</tr>
</tbody>
</table>

where $\beta^h_{i,M}$ and $\beta^h_{i,HML}$ are regression coefficients from a regression of $r_i(Jan)$ on $r_m(Jan)$ and $r_{HML}(Jan)$:

\[
r_i = \phi_{i,0} + \beta^h_{i,M} r_m(Jan) + \beta^h_{i,HML} r_{HML}(Jan) + e_i; \text{ and,}
\]

\[
r_i(Jan) = R_i(Jan)-R_f, \quad r_m(Jan) = R_m(Jan)-R_f, \quad \text{and} \quad r_{HML}(Jan) = R_{HML}(Jan)-R_f.
\]

Also know that $R_f=6\%$. 

2
A. What is the expected January return for KS?

Since $R_{HML}(Jan)$ is the hedging portfolio that individuals hold in combination with the market portfolio and the riskless asset, it follows that any asset satisfies:

\[ E[R_i(\text{Jan})] = R_f(\text{Jan}) + \beta_{h,i,M} \lambda^*_{M} + \beta_{h,i,HML} \lambda^*_{HML}. \]

where $\lambda^*_{M} = E[r_M] = E[R_M] - R_f = 10 - 6 = 4\%$ and $\lambda^*_{HML} = E[r_{HML}] = E[R_{HML}] - R_f = 14 - 6 = 8\%$

So for any asset:

\[ E[R_i(\text{Jan})] = 6 + \beta_{h,i,M} 4 + \beta_{h,i,HML} 8. \]

And

\[ E[R_{KS}(\text{Jan})] = 6 + \beta_{h,KS,M} 4 + \beta_{h,KS,HML} 8 = 6 + 1.1 \times 4 + (-0.4) \times 8 = 7.2\%. \]

B. What is the expected January return for LM?

For any asset:

\[ E[R_i(\text{Jan})] = 6 + \beta_{h,i,M} 4 + \beta_{h,i,HML} 8. \]

And

\[ E[R_{LM}(\text{Jan})] = 6 + \beta_{h,LM,M} 4 + \beta_{h,LM,HML} 8 = 6 + 1.1 \times 4 + 0.5 \times 8 = 14.4\%. \]

C. Explain why these two assets have different expected January returns?

LM has a positive sensitivity to the hedging portfolio $R_{HML}$ while KS has a negative sensitivity. This hedging portfolio is strongly positively correlated with the state variable $s_1(\text{Jan})$ which means that LM is positively correlated with the state variable while KS is negatively correlated with the state variable. Since sufficiently risk averse investors like negative correlation with the state variable, they demand a higher expected return on LM than KS because of LM’s higher correlation with the state variable.

D. Describe an investor in this economy who would choose to hold a combination of the riskless asset and the mean-variance tangency portfolio. Characterize qualitatively this investor’s portfolio in terms of the weight invested in the hedging portfolio.

An investor who does not care about the covariance of her portfolio’s return with the state variable behaves like a mean-variance investor and holds the riskless asset and the mean-variance tangency portfolio. This investor has a human capital value that is insensitive to the state of the economy. This investor holds the tangency portfolio by loading up positively on the hedging portfolio. This investor has a large weight in the hedging portfolio to take advantage of its high expected return. While this high expected return is offset by the portfolio’s large positive correlation with the state variable, this investor does not care about this correlation which make the hedging portfolio attractive to her.

E. Consider a highly risk-averse investor whose job prospects at the end of January are closely linked to the state of the economy at the end of January. Characterize qualitatively this investor’s portfolio in terms of the weight invested in the hedging portfolio.
This investor really cares about the covariance of her portfolio’s return with the state variable and the economy. Consequently, this investor will find the hedging portfolio with its large positive correlation with the state variable particularly unattractive. In a portfolio of the hedging portfolio, the market portfolio and the riskless asset, this investor will have a low, and possibly negative, weight invested in the hedging portfolio.

III. Performance Measurement. The following information is to be used to evaluate the performance of the Bull Fund and the Boom Fund.

<table>
<thead>
<tr>
<th>i</th>
<th>$E[R_i]$</th>
<th>$\sigma[R_i]$</th>
<th>$\sigma[R_i, R_{S&amp;P}]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>S&amp;P</td>
<td>15</td>
<td>20</td>
<td>400</td>
</tr>
<tr>
<td>Bull</td>
<td>17</td>
<td>30</td>
<td>440</td>
</tr>
<tr>
<td>Boom</td>
<td>19</td>
<td>40</td>
<td>460</td>
</tr>
<tr>
<td>Riskfree</td>
<td>5</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

A. Calculate the Sharpe ratio for
1. the S&P 500 index fund.
   \[
   \text{Sharpe}_{S&P} = \frac{E[R_{S&P}]}{\sigma[R_{S&P}]} = \frac{15-5}{20} = 0.5.
   \]
2. the Bull fund.
   \[
   \text{Sharpe}_{Bull} = \frac{E[R_{Bull}]}{\sigma[R_{Bull}]} = \frac{17-5}{30} = 0.4.
   \]
3. the Boom fund.
   \[
   \text{Sharpe}_{Boom} = \frac{E[R_{Boom}]}{\sigma[R_{Boom}]} = \frac{19-5}{40} = 0.35.
   \]

B. Calculate Jensen’s alpha for
1. the S&P 500 index fund.
   \[
   \beta_{S&P,S&P} = \frac{\text{cov}[r_{S&P}(t), r_{S&P}(t)]}{\text{var}[r_{S&P}(t)]} = 1
   \]
   \[
   \alpha_{S&P,S&P} = E[r_{S&P}(t)] - \beta_{S&P,S&P} E[r_{S&P}(t)]
   = (15-5) - 1 \times (15-5) = 0.
   \]
2. the Bull fund.
   \[
   \beta_{Bull,S&P} = \frac{\text{cov}[r_{Bull}(t), r_{S&P}(t)]}{\text{var}[r_{S&P}(t)]} = \frac{440}{20^2} = 1.1
   \]
   \[
   \alpha_{Bull,S&P} = E[r_{Bull}(t)] - \beta_{Bull,S&P} E[r_{S&P}(t)]
   = (17 - 5) - 1.1 \times (15-5) = 1.
   \]
3. the Boom fund.
   \[
   \beta_{Boom,S&P} = \frac{\text{cov}[r_{Boom}(t), r_{S&P}(t)]}{\text{var}[r_{S&P}(t)]} = \frac{460}{20^2} = 1.15
   \]
   \[
   \alpha_{Boom,S&P} = E[r_{Boom}(t)] - \beta_{Boom,S&P} E[r_{S&P}(t)]
   = (19 - 5) - 1.15 \times (15-5) = 2.5.
   \]
C. An investor who only cares about the mean and standard deviation of her portfolio’s return is trying to decide which of these funds to hold in combination with T-bills. Which fund should the investor choose? The investor should choose Bull Fund rather than Boom Fund since \( \text{Sharpe}_{\text{Bull}} > \text{Sharpe}_{\text{Boom}} \). However, the investor would rather hold an S&P 500 index fund instead of either of these funds because \( \text{Sharpe}_{\text{S&P}} \) is higher than \( \text{Sharpe}_{\text{Bull}} \) or \( \text{Sharpe}_{\text{Boom}} \).

D. An investor who only cares about the mean and standard deviation of her portfolio’s return is considering combining Bull with the S&P 500 index fund (the market portfolio) and the riskfree asset. Will Bull’s weight be positive, negative or zero in the investor’s portfolio? Bull’s weight will be positive since \( \alpha_{\text{Bull, S&P}} > 0 \).

E. An investor who only cares about the mean and standard deviation of her portfolio’s return is considering combining the Boom fund with the S&P 500 index fund (the market portfolio) and the riskfree asset. Will Boom’s weight be positive, negative or zero in the investor’s portfolio? Boom’s weight will be positive since \( \alpha_{\text{Boom, S&P}} > 0 \).