Problem Set 8 Solution: Bond Portfolio Management and Derivatives Definitions and Payoffs

I. Duration and Interest Rate Sensitivity:

A. Since the yield curve expressed as an APR with semiannual compounding is flat at 5.5%, $y = 0.055$. Thus,

$$P_{4\frac{3}{4}} \text{ Feb 96 (Aug 94)} = \frac{4\frac{3}{4}}{2}/1.0275 + \frac{4\frac{3}{4}}{2}/(1.0275)^2 + \frac{100+4\frac{3}{4}}{2}/(1.0275)^3$$

$$= 2.3114 + 2.2496 + 94.3731 = 98.9341.$$

B. Macaulay Duration:

$$D_{4\frac{3}{4}} \text{ Feb 96 (Aug 94)} = 0.5 \times \frac{2.3114}{98.9341} + 1 \times \frac{2.2496}{98.9341} + 1.5 \times \frac{94.3731}{98.9341}$$

$$= 0.5 \times 0.023363 + 1 \times 0.022738 + 1.5 \times 0.953899 = 1.46527.$$

“modified duration”:

$$D_{4\frac{3}{4}} \text{ Feb 96 (Aug 94)} = D_{4\frac{3}{4}} \text{ Feb 96 (Aug 94)} / (1+y/2) = 1.46527/1.0275 = 1.42605.$$

C. 1. Now, $y = 0.06$. Thus, the new price is:

$$P_{4\frac{3}{4}} \text{ Feb 96 (Aug 94)} = \frac{4\frac{3}{4}}{2}/1.03 + \frac{4\frac{3}{4}}{2}/(1.03)^2 + \frac{100+4\frac{3}{4}}{2}/(1.03)^3$$

$$= 2.3058 + 2.2387 + 93.6876 = 98.2321.$$

2. Now, $\Delta y = 0.005$. So

$$\Delta P_{4\frac{3}{4}} \text{ Feb 96 (Aug 94)} = \Delta y \{-P_{4\frac{3}{4}} \text{ Feb 96 (Aug 94)} \times D_{4\frac{3}{4}} \text{ Feb 96 (Aug 94)} /(1+y/2)\}$$

$$= 0.005 \{-98.9341 \times 1.46527 /1.0275\} = 0.005 \times -141.085 = -0.7054.$$

Thus,

$$P_{4\frac{3}{4}} \text{ Feb 96 (Aug 94)} = P_{4\frac{3}{4}} \text{ Feb 96 (Aug 94)} + \Delta P_{4\frac{3}{4}} \text{ Feb 96 (Aug 94)} = 98.9341 - 0.7054 = 98.2287.$$

3. The price decline implied by duration is greater than the actual price decline.

D. 1. Now, $y = 0.05$. Thus, the new price is:

$$P_{4\frac{3}{4}} \text{ Feb 96 (Aug 94)} = \frac{4\frac{3}{4}}{2}/1.025 + \frac{4\frac{3}{4}}{2}/(1.025)^2 + \frac{100+4\frac{3}{4}}{2}/(1.025)^3$$

$$= 2.3171 + 2.2606 + 95.0654 = 99.6430.$$

2. Now, $\Delta y = -0.005$. So

$$\Delta P_{4\frac{3}{4}} \text{ Feb 96 (Aug 94)} = \Delta y \{-P_{4\frac{3}{4}} \text{ Feb 96 (Aug 94)} \times D_{4\frac{3}{4}} \text{ Feb 96 (Aug 94)} /(1+y/2)\}$$

$$= -0.005 \{-98.9341 \times 1.46527 /1.0275\} = -0.005 \times -141.085 = 0.7054.$$

Thus,

$$P_{4\frac{3}{4}} \text{ Feb 96 (Aug 94)} = P_{4\frac{3}{4}} \text{ Feb 96 (Aug 94)} + \Delta P_{4\frac{3}{4}} \text{ Feb 96 (Aug 94)} = 98.9341 + 0.7054 = 99.6395.$$
3. The price increase implied by duration is less than the actual price increase.

II. Immunization:
   A. Now, \( y = 0.06 \). Thus,

\[
L(\text{Aug 94}) = \frac{5M}{(1.03)^2} + \frac{5M}{(1.03)^4} + \frac{5M}{(1.03)^6} + \frac{5M}{(1.03)^8} + \frac{5M}{(1.03)^{10}} = 4.7130M + 4.4424M + 4.1874M + 3.9470M + 3.7205M = 21.0103M.
\]

   B.  
   1.  
   \[
   D_L(\text{Aug 94}) = 1 \times \left(\frac{4.7130}{21.0103}\right) + 2 \times \left(\frac{4.4424}{21.0103}\right) + 3 \times \left(\frac{4.1874}{21.0103}\right) + 4 \times \left(\frac{3.9470}{21.0103}\right) + 5 \times \left(\frac{3.7205}{21.0103}\right) = 2.8820.
   \]

   2. First need to determine the price of the note:

\[
P_{6\text{e Feb 96 (Aug 94)}} = \frac{6\text{e Feb 1996}}{2}/1.03 + \frac{6\text{e Feb 1996}}{2}/1.03^2 + 100 + \frac{6\text{e Feb 1996}}{2}/1.03^3  = 3.2160 + 3.1223 + 94.5456 = 100.8839.
\]

Then can determine the note’s duration:

\[
D_{6\text{e Feb 96 (Aug 94)}} = 0.5 \times \left(\frac{3.2160}{100.8839}\right) + 1 \times \left(\frac{3.1223}{100.8839}\right) + 1.5 \times \left(\frac{94.5456}{100.8839}\right) = 1.4527.
\]

3. The duration of a discount bond is equal to its maturity. Thus,

\[
D_{\text{Aug 94 strip (Aug 94)}} = 10.
\]

   C. To immunize the liability, need to match asset value to the liability value. So let \( \omega_{\text{note}} \) be the fraction of the $21.0103M invested in the note. Also need to match the duration of the assets to the duration of the liabilities:

\[
D_A(\text{Aug 94}) = D_L(\text{Aug 94}) = 2.8820.
\]

But

\[
D_A(\text{Aug 94}) = (1 - \omega_{\text{note}}) D_{\text{Aug 94 strip (Aug 94)}} + \omega_{\text{note}} D_{6\text{e Feb 96 (Aug 94)}}.
\]

So

\[
2.8820 = (1 - \omega_{\text{note}}) 10 + \omega_{\text{note}} 1.4527
\]

which implies

\[
\omega_{\text{note}} = \frac{(10 - 2.8820)/(10 - 1.4527)}{0.8328}.
\]

So XYZ should invest 0.8328 x $21.0103M = $17.4969M in the note and $3.5134M in the strip.

III. Options:
A. BKM, Chapter 20, Question 6, part a.
B. BKM, Chapter 20, Question 23.