Lecture 1: Overview

I. Reading
II. Asset Classes
III. Financial System.
IV. Financial Markets
V. Financial Intermediaries
VI. Issues addressed by Finance Theory
VII. Key Concepts.
Lecture 1: Overview

I. Reading
   A. BKM Chapter 1.
   B. Skim BKM Chapters 2 and 4.

II. Asset Classes
   A. Real Assets
      1. natural resources.
      2. physical capital.
      3. human capital.
   B. Financial Assets (referred to as securities)
      1. Money (as a medium of exchange)
         a. is held to allow the completion of transactions.
      2. Debt
         a. a claim to a predetermined payment stream secured on a set of real
            or financial assets.
         b. maturity is time from issue to expiration.
      3. Equity
         a. residual claim to a set of real or financial assets (usually of a
            corporation) usually coupled with corporate control.
      4. Derivatives
         a. payoff is dependant on the value of some other (usually financial)
            asset.
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C. Illustration.
1. Debt vs. Equity.
   a. Suppose XYZ Co’s assets pay off a random amount CF in 1 year’s time and XYZ has issued debt with a promised payment of $100 in 1 year’s time, and equity.

<table>
<thead>
<tr>
<th>CF</th>
<th>&lt;100</th>
<th>60</th>
<th>80</th>
<th>100</th>
<th>&gt;100</th>
<th>120</th>
<th>140</th>
<th>160</th>
<th>180</th>
<th>200</th>
</tr>
</thead>
<tbody>
<tr>
<td>Debt</td>
<td>CF</td>
<td>60</td>
<td>80</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>Equity</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>CF-100</td>
<td>20</td>
<td>40</td>
<td>60</td>
<td>80</td>
<td>100</td>
</tr>
<tr>
<td>Firm</td>
<td>CF</td>
<td>60</td>
<td>80</td>
<td>100</td>
<td>CF</td>
<td>120</td>
<td>140</td>
<td>160</td>
<td>180</td>
<td>200</td>
</tr>
</tbody>
</table>

   b. If CF is uncertain, XYZ’s equity is riskier than XYZ’s debt.

2. Derivatives.
   a. A call option gives its holder the right (but not the obligation) to buy an asset by paying a prespecified price (the strike price).
   b. Consider a call option on XYZ Co’s equity with a strike price of $40 that can be exercised in 1 year’s time.

<table>
<thead>
<tr>
<th>Equity</th>
<th>&lt;40</th>
<th>0</th>
<th>20</th>
<th>40</th>
<th>&gt;40</th>
<th>60</th>
<th>80</th>
<th>100</th>
</tr>
</thead>
<tbody>
<tr>
<td>Option</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>Equity-40</td>
<td>20</td>
<td>40</td>
<td>60</td>
</tr>
</tbody>
</table>

   c. Notice that the call option’s payoff at the exercise date is never less than 0.

D. Example: IBM Corporation.
1. Real Assets: plant used to build Thinkpads.
2. Claims on the Real Assets:
   a. Equity: IBM stock.
   b. Debt: IBM bonds.
   c. IBM stock is much more volatile than IBM bonds.
3. Derivatives: Claims on IBM stock.
   a. A call option on IBM stock gives the holder the right (but not the obligation) to buy the stock at a given exercise price.
III. Financial System: refers to the collection of institutions by which financial assets are created and traded.

A. Purposes (which allow the financial system to create wealth)
   1. transfer capital from savers (investors) to capital users (usually corporations).
   2. discipline investment decisions by firms.

Example: A firm may want to expand by going public; i.e., by issuing equity to the public in return for cash. Since the public knows that the firm is going to use cash to expand, an investor will only subscribe to the IPO if she thinks expansion is a value enhancing strategy.

3. allow investors to smooth consumption intertemporally.

Example: An MBA student has low income now but high future income. A student loan allows the student to smooth her consumption through time relative to her income through time.

4. facilitate the reduction of riskbearing by repackaging risks.
5. disseminate information.

B. Institutions
   1. Government
   2. Financial Markets: institutions which trade financial assets.
   3. Financial Intermediaries: entities which operate within or outside financial markets to facilitate the trading of financial assets.
IV. Financial Markets

A. Primary vs Secondary Markets
   1. Primary Market: new issues of a security are sold to initial buyers.
   2. Secondary Market: previously issued securities are traded in a secondary market.
   3. Exchange vs Over-the-Counter Market
   4. Exchange: Buyers and sellers of securities meet in one central location to conduct trades.

Examples: 1) NYSE (stocks); 2) Chicago Board of Trade (futures).

5. Over-the Counter Market: Dealers at different locations stand ready to buy and sell securities "over the counter" to anyone that accepts their prices.

Examples: 1) government bonds are traded over the counter through primary and secondary dealers; 2) the National Association of Securities Dealers Automated Quotation System (NASDAQ) is an example of a trading network for stocks.

B. Money vs Capital Markets
   2. Capital: long term debt instruments (>1 year maturity) and equity.
V. Financial Intermediaries
   A. Services Provided
      1. reduce search costs associated with finding saving or investment opportunities.
      2. generate information needed by investors.
      3. provide risk and portfolio management services.
      4. issue financial assets that repackage risks.
      5. take advantage of the economies of scale associated with buying and selling financial assets.
   B. Types
      1. Depository Institutions
         a. Commercial Banks.
         b. Savings and Loan Associations, Mutual Savings Banks.
         c. Credit Unions.
      2. Contractual Savings Institutions
         a. Life Insurance Companies.
         b. Fire and Casualty Insurance Companies.
         c. Pension Funds.
      3. Investment Intermediaries
         a. Brokers.
         b. Mutual Funds.
         c. Money Market Mutual Funds.
         d. Finance Companies.
C. Growth of Mutual Funds. A mutual fund is a firm that manages a pool of money that has been placed with the fund by other people. Money placed with the fund is invested in certain specified types of assets. People buy shares in the fund and their value changes over time with changes in the value of the fund’s assets. All investors in the fund earn the same return over any given interval of time.

1. Example:
   a. Consider a fund with two investors at time 0, Bob and Mary. Bob has $1000 invested and Mary has $500. Their fund has $1500 in assets under management and 1500 shares. Each share is worth $1 so Bob owns 1000 shares and Mary 500 shares. Suppose no withdrawals or contributions occur over the year.
   b. After 1 year, the fund’s portfolio has grown in value to $1800 which is a $300/$1500 = 20% return. Each share is now worth $1800/1500 = $1.2. Both Bob and Mary earn a 20% return from time 0 to time 1. Bob’s 1000 shares are now worth $1200 and Mary’s 500 shares are worth $600.
   c. At time 1, Jill wants to invest $300 in the fund. Each share is worth $1.2 so Jill receives 250 shares and assets under management are $1800 + $300 = $2100.
   d. At time 2, the fund’s portfolio is worth $2310 which is a $210/$2100 = 10% return. Each share is now worth $1.2 \times 1.1 = $1.32 and all three earn a 10% return from their investments in the fund from time 1 to time 2.

2. Over the last 20 years, there has been tremendous growth in:
   a. number of funds.
   b. types of funds.
   c. dollars invested in funds.

3. Index Funds: Particularly high growth has occurred for a type of mutual fund known as an index fund.
   a. An index fund is a mutual fund whose investment goal is to track the return on a particular stock index (for example, the S&P 500 index).
   b. A stock index is a portfolio of stocks formed according to a predetermined rule. For example, the S&P 500 index is a portfolio of 500 stocks chosen so that the index mirrors the U.S. stock market. A stock is chosen (according to the S&P Corporation) for inclusion in the index if its performance is representative of the performance of its industry. The S&P 500 index invests more in a large stock contained in the index than a small stock: it is a value-weighted index.
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VI. Issues addressed by Finance Theory
   A. Financial decision-making by corporations. How do corporations decide whether to undertake an investment project? (Corporate Finance)
   B. Financial decision-making by individuals. How do individuals invest their savings?
   C. Valuation of assets both real and financial. Why do expected returns vary across assets?

Examples: 1) CAPM is a asset pricing model; 2) Black-Scholes model values call options; 3) Cox Ingersoll Ross model values fixed income assets.

Mean Nominal 1 Month Return on U.S. Stock and Bond Portfolios: 7/26–12/95

![Chart showing mean nominal 1 month return on U.S. stock and bond portfolios from 7/26-12/95]
TB - 1 mth Treasury Bills
GB - Long-term U.S. Government Bonds
CB - Corporate Bonds
S&P - S&P 500 Index
SMV - Portfolio of Small Market Value Stocks (Small Caps)
HBM - Portfolio of High Book-to-market Stocks (Value Stocks)

Mean Nominal 5 Year Return on U.S. Stock and Bond Portfolios:
7/26:6/31 - 1/91:12/95
5 Year Returns on U.S. Stocks and Treasury Bills:
7/26/631 - 1/91:12/95

- 1 mth T-bills
- S&P 500

5 Year Returns on U.S. Stocks and Treasury Bills:
7/26/631 - 1/91:12/95

- 1 mth T-bills
- Small Market Value
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VII. Key Concepts.

A. Time value of money
   1. A dollar today is worth more than a dollar later.
   Example: Treasury strips pay a face value of $100 at the maturity date. Strips always trade for less than $100. See for example Wall Street Journal for May 11 Friday.

B. Diversification.
   1. Portfolios of assets and individual assets have similar average returns but the portfolios have much lower return volatility.
   Example: Monthly Returns (in %) from 1/91-12/95.

<table>
<thead>
<tr>
<th>Asset</th>
<th>Average Return</th>
<th>Return Volatility</th>
<th>Category</th>
</tr>
</thead>
<tbody>
<tr>
<td>IBM</td>
<td>0.246</td>
<td>8.004</td>
<td>Individual Stocks</td>
</tr>
<tr>
<td>Apple</td>
<td>0.452</td>
<td>13.050</td>
<td></td>
</tr>
<tr>
<td>Microsoft</td>
<td>3.126</td>
<td>8.203</td>
<td></td>
</tr>
<tr>
<td>Nike</td>
<td>2.619</td>
<td>9.265</td>
<td></td>
</tr>
<tr>
<td>ADM</td>
<td>0.953</td>
<td>6.712</td>
<td></td>
</tr>
<tr>
<td>S&amp;P500</td>
<td>1.326</td>
<td>2.886</td>
<td>Stock Portfolios</td>
</tr>
<tr>
<td>Small Firm</td>
<td>1.912</td>
<td>3.711</td>
<td></td>
</tr>
</tbody>
</table>

C. Risk-adjustment.
   1. Assets offer different average returns because they have different risk levels. The flip side is that investors require different returns on different investments depending on their risk levels.
   2. Need to quantify what we mean by risk.
   Example: Above discussion of differences in average return across risk classes. Stocks offer higher average return than government and corporate debt but risk (however defined) is also higher.
D. No arbitrage.

1. An investment that does not require any cash outflows and generates a strictly positive cash inflow with some probability is known as an arbitrage opportunity.

2. In well functioning markets arbitrage opportunities can not exist since any individual who prefers more to less wants to invest as much as possible in the arbitrage opportunity.

3. The absence of arbitrage implies that any two assets with the same stream of cash flows must have the same price. This implication is known as the law of one price.

Example:

**U.S. Treasury Strips**

<table>
<thead>
<tr>
<th>Maturity</th>
<th>Type</th>
<th>Bid</th>
<th>Asked</th>
<th>Chg</th>
<th>Asked Yield</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aug 05</td>
<td>ci</td>
<td>81:23</td>
<td>81:26</td>
<td>-11</td>
<td>4.77</td>
</tr>
<tr>
<td>Aug 05</td>
<td>bp</td>
<td>81:06</td>
<td>81:10</td>
<td>-11</td>
<td>4.92</td>
</tr>
<tr>
<td>Aug 05</td>
<td>np</td>
<td>81:23</td>
<td>81:26</td>
<td>-10</td>
<td>4.77</td>
</tr>
</tbody>
</table>

4. Even small price differences represent a riskless profit:

Example (cont):

<table>
<thead>
<tr>
<th>Arbtrage Opportunity</th>
<th>5/01</th>
<th>8/05</th>
</tr>
</thead>
<tbody>
<tr>
<td>Buy 1 x Aug 05 bp Strip</td>
<td>-81:10</td>
<td>100</td>
</tr>
<tr>
<td>Sell 1 x Aug 05 ci Strip</td>
<td>81:23</td>
<td>-100</td>
</tr>
<tr>
<td>Net Cash Flow</td>
<td>0:13</td>
<td>0</td>
</tr>
</tbody>
</table>

5. Price differences of assets with identical cash flows must be due to tax and liquidity differences plus transaction costs.
E. Option value.
1. An option gives the holder the right but not the obligation to take a certain action in the future.
2. An option has non-negative value for the holder.

Example:
A firm has two bond issues outstanding. The first is 5% semi-annual coupon bonds maturing in 11/04. The second is 5% semi-annual coupon bonds maturing in 11/04 but the holder has the option to convert the bonds to stock at any time prior to 11/04. For a par value of $100, which bond is more expensive. The second is more expensive since the option to convert is valuable: the second bond cannot be less valuable than the first.

F. Market Efficiency
1. In an efficient market, the price of a security is an unbiased estimate of its value.
2. U.S. stock market is probably efficient with respect to publicly available information. So cannot use publicly available information to earn higher than average risk-adjusted returns on average.
3. Market efficiency is one of the big warnings issued by the course.

Example:
a) Studies that examine the cross-sectional relation between U.S. mutual fund performance and expense ratio find that annual fund return varies inversely 1 for 1 with the expense ratio. So if a fund increases its expense ratio from 0.5% p.a. to 1.5% p.a., its annual performance can be expected to decline by about 1%.
b) Using account data for over 60,000 households from a large discount brokerage firm, Barber and Odean (2000) analyze the common stock investment performance of individual investors from February 1991 through December 1996.
   - On one hand, the gross returns (before accounting for transaction costs) earned by the average household are unremarkable; the average household earned an annualized geometric mean gross return of 17.7 per cent while the value-weighted market index earned 17.1 per cent.
   - On the other hand, the net returns earned by the average household lag reasonable benchmarks by economically and statistically significant amounts; the average household earned an annualized geometric mean net return of 15.3 per cent.
   - The 20 per cent of households that trade most (which average at least 9.6 per cent turnover per month) earned an annualized geometric mean net return of 10.0 per cent.
   - The central message is that trading is hazardous to your wealth.
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Lecture 1: Time Value of Money

I. Reading
II. Time Line
III. Interest Rate: Discrete Compounding
IV. Single Sums: Multiple Periods and Future Values
V. Single Sums: Multiple Periods and Present Values
VI. Equivalent Effective Interest Rates Over Different Compounding Periods.
VII. Alternate Interest Rate Concepts
VIII. Multiple Cash Flows.
IX. Particular Cash Flow Pattern: Annuity
X. Particular Cash Flow Pattern: Perpetuity
XI. Application
Lecture 1: Time Value of Money

I. Reading
   A. RWJ Chapter 4 and 5.

II. Time Line
   A. $1 received today is not the same as a $1 received in one period's time; the timing of a cash flow affects its value.
   B. Hence, when valuing cash flow streams, the timing of the cash flows is crucial: a good idea is to draw a time line.

$100

is not the same as

$100
III. Interest Rate: Discrete Compounding

A. Example:
1. Question: Today is the start of 2001. Suppose I can invest $100 at an effective annual interest rate of 12%. What is my $100 worth at the end of the year?

\[
\begin{align*}
\text{end 00} & \quad \text{end 01} \\
$100 & \quad V_1 \\
\end{align*}
\]

2. Answer: \( V_0 = $100; \) Interest = $100 \times 0.12 = $12
\[ V_1 = V_0 + \text{Interest} = $100 + $12 = $100(1+0.12) = $112. \]

B. Definition:
1. The effective interest rate \( r \) (expressed as a decimal) over any period tells what \( x \) will be worth at the end of the period using the following formula:
\[ x (1+r). \]
2. The effective interest rate \( r \) (expressed as a decimal) over any period from \( t \) to \( (t+1) \) satisfies:
\[ V_{t+1} = V_t (1+r) \]
where \( V_t \) is the value at time \( t \) and \( V_{t+1} \) is the value at time \( t+1 \).

C. Example:
1. Question: Today is the start of 2001. Suppose I can invest $100 at the start of 2002 at an effective annual interest rate of 12%. What is my $100 worth at the end of 2002?

\[
\begin{align*}
\text{end 00} & \quad \text{end 01} & \quad \text{end 02} \\
$100 & \quad V_1 & \quad V_2 \\
\end{align*}
\]

2. Answer: \( V_1 = $100; \) \( V_2 = V_1(1+0.12) = $100(1+0.12) = $112. \)
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IV. Single Sums: Multiple Periods and Future Values
   A. Example (cont):
      1. Question: Today is the start of 2001. Suppose I can invest $100 at an effective annual interest rate of 12%. What is my $100 worth at the end of 3 years?

      $100 $112 $125.44 $140.49

      2. One Answer: Obtain $V_3$ in three steps
         \[
         V_0 = 100 \\
         V_1 = V_0 (1+r) = 100(1+0.12) = 112 \\
         V_2 = V_1 (1+r) = 112(1+0.12) = 125.44 \\
         V_3 = V_2 (1+r) = 125.44(1+0.12) = 140.49
         \]

      3. Another Answer: Can see that the 3 steps could be combined into 1
         \[
         V_0 = 100 \\
         V_3 = V_0 (1+r)^3 = 100(1+0.12)^3 = 140.49
         \]

      4. Notice how the answer is the initial investment $V_0$ times a multiplier that only depends on the effective interest rate and the investment interval. This multiplier is known as the future value interest factor.
B. The future value formula answers the following question
1. If we invest some money at a given effective interest rate, how much money would we have at some future time?
2. If we invest \( V_0 \) today at a given effective interest rate per period of \( r \) (expressed as a decimal), how much money would we have in \( n \) periods time; i.e., what is \( V_n \)?
3. If we invest \( V_t \) in \( t \) periods time at a given effective interest rate per period of \( r \) (expressed as a decimal), how much money would we have after \( n \) periods from investing the money; i.e., what is \( V_{t+n} \)?

C. The general future value formula:
1. \( V_{t+n} = V_t (1+r)^n \) where \([(1+r)^n] = FVIF_{r,n} \) is the future value interest factor.
2. Notice that the future value interest factor does not depend on when the money is invested.

D. Example:
1. Question: Today is the start of 2001. Suppose I can invest $100 at the end of 2001 at an effective annual interest rate of 12%. What is my $100 worth after being invested for 3 years?

   \[
   \begin{array}{c|c}
   \text{end 01} & \text{end 04} \\
   \hline
   \$100 & V_4 \\
   \end{array}
   \]

   \( V_4 = V_1 \times FVIF_{0.12,3} = V_1 (1+0.12)^3 = $100(1+0.12)^3 = $140.49 \)

2. Answer: Use the future value formula. \( V_1 = $100 \)
V. Single Sums: Multiple Periods and Present Values

A. Example cont:

1. Question: Today is the start of 2001. Suppose I can invest at an effective annual interest rate of 12%. How much do I need to invest today to have $140.49 at the end of 3 years?

\begin{align*}
\text{end 00} & \quad \text{end 03} \\
V_0 & \quad \$140.49 \\
\end{align*}

2. Answer: Use future value formula which tells you (from above)

\[ V_3 = V_0 (1+r)^3 = \$100(1+0.12)^3 = \$140.49 \]

which implies

\[ V_0 = \frac{V_3}{(1+r)^3} = \frac{\$140.49}{(1+0.12)^3} = \$100 \]

\begin{align*}
\text{end 00} & \quad \text{end 03} \\
\$100 & \quad \$140.49 \\
\end{align*}

3. Notice how the answer is the final value \( V_3 \) times a multiplier that only depends on the effective interest rate and the investment interval. This multiplier is known as the present value interest factor.
B. The present value formula answers the following question
1. If we can invest money at a given effective interest rate, how much money do we need to invest today to have a given sum at some future time?
2. If we can invest money at a given effective interest rate $r$ (expressed as a decimal), how much money do we need to invest today $V_0$ to have a given sum $V_n$ in $n$ periods time?
3. If we can invest money at a given effective interest rate $r$ (expressed as a decimal), how much money do we need to invest in $t$ periods time $V_t$ to have a given sum $V_{t+n}$ in $(t+n)$ periods from today?

C. The general present value formula:
1. $V_t = V_{t+n} \left[ \frac{1}{(1+r)^n} \right]$ where $\left[ \frac{1}{(1+r)^n} \right] = \frac{1}{(1+r)^n} = \text{PVIF}_{r,n}$ is the present value interest factor.

D. Example:
1. Question: Today is the start of 2001. Suppose I can invest at an effective annual interest rate of 12%. How much do I need to invest at the start of 2002 to have $140.49 at the end of 4 years from today?

   end 01 | end 04
   V_1 | $140.49

   $V_1 = V_4 \times \text{PVIF}_{0.12,3} = V_4/(1+r)^3 = $140.49/(1+0.12)^3 = $100

2. Answer: Use the present value formula:
VI. Equivalent Effective Interest Rates Over Different Compounding Periods.

A. Example:

1. Question: Suppose the effective annual interest rate is 12%. What is the effective 3 year interest rate?
2. Answer: Showed above that

\[
V_3 = V_0 (1+r)^3 = 100(1+0.12)^3 = 100(1+0.4049) = 140.49
\]

which implies using the definition of effective rate that the effective 3-year rate is 40.49%.

3. Note that Effective 3-year rate = 40.49% = 3 x Effective 1-year rate.

B. General relation between the effective rates for compounding periods of different lengths:

1. The effective \( n \)-period rate \( r_n \) (expressed as a decimal) is related to the effective one period rate

\[
(1+r_n) = (1+r)^n.
\]

C. Example

1. Question: Suppose the effective monthly rate is 0.94888%. What is the effective annual rate?

2. Answer: Here one period is a month. \( r = 0.0094888 \). Using the effective rate formula

\[
1+r_{12} = (1+r)^{12} = 1.0094888^{12} = 1.12
\]

and so \( r_{12} = 0.12 \) and the effective annual rate is 12%.
D. Does the effective rate formula apply for $n$ a fraction; e.g., if the effective 1-year rate is known, can the monthly effective rate be calculated by using the formula with $n = 1/12$? The answer is yes.

E. Example

1. Question: Suppose the effective annual rate is 12% What is the effective monthly rate?

\[
1 + r_{1/12} = \left(1 + r\right)^{1/12} = 1.12^{1/12} \approx 1.009488
\]

and so $r_{1/12} = 0.009488$ and the effective monthly rate is 0.9488%.

F. EAR

1. Definition: EAR is the effective annual rate.
VII. Alternate Interest Rate Concepts
A. Nominal Rate or APR
1. Definition: when the compounding period is some fraction of a year \( \frac{1}{m} \), the nominal rate \( i_{\text{nom}} \) (expressed as a decimal) equals \( m r_{1/m} \) where \( r_{1/m} \) is the effective \( \frac{1}{m} \)-year rate (expressed as a decimal).

2. Example: if the compound period is a month \( (m=12) \) and the effective monthly rate is 0.94888\% then \( r_{1/12}=0.0094888 \), \( i_{\text{nom}} = 12 \times 0.0094888=0.11387 \) and the nominal rate is 11.387\%.

3. Fact: the nominal rate only equals the effective annual rate when the compound period is a year.

4. Example (cont): with a compound period of a month, the nominal rate is 11.387\% while the effective annual rate is 12\%.

5. Periodic rate
   a. Definition: when the compounding period is some fraction of a year \( \frac{1}{m} \), the periodic rate (expressed as a decimal) equals the effective \( \frac{1}{m} \)-year rate \( r_{1/m} \).
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B. Continuous Compounding.

1. Definition:
   a. Let \( r \) be the effective interest rate for one period and define \( r' \), the continuous interest rate for a period as follows:
   \[
e^{r'} = (1 + r).
   \]
   b. Thus, the continuous rate can be obtained from the effective rate as follows:
   \[
r' = \ln (1 + r).
   \]

2. Example: The effective annual rate is 12%.
   a. Question: What is the continuously compounded annual rate?
   b. Answer: \( r = 0.12 \). The continuously compounded annual rate (expressed as a decimal) is \( r' = \ln(1+r) = \ln(1.12) = 0.1133 \).

3. Rules for using continuous interest rates.
   a. The continuous interest rate for any fraction of a period \( n \) is given by \( r'_n = nr' \) (c.f. effective interest rates for which this is not true \( r_n \neq nr \)). This additivity is a major advantage of continuous compounding.
   b. To obtain future values, use the following formula:
   \[
   V_{t+n} = V_t \exp \{nr'\}.
   \]
   c. To obtain present values, use the following formula:
   \[
   V_t = V_{t+n} \exp \{-nr'\}.
   \]

4. Example (cont): The effective annual rate is 12%.
   a. Question: What is the continuously compounded semiannual rate?
   b. Answer: The continuously compounded annual rate is 11.33%; i.e., \( r' = 0.1133 \). So the continuously compounded semiannual rate is \( (11.33/2)\% = 5.666\% \) since \( r'_{1/2} = (1/2) r' = 0.5 \times 0.1133 = 0.05666 \).
   c. Question: How much will $500 invested today be worth in 6 months?
   d. Answer: Two approaches to obtaining what $500 will be worth in 6 months:
      (1) Continuous Compounding: \( V_{1/2} = \$500 \exp \{(1/2) r'\} = \$500 \exp \{0.05666\} = \$500 \times 1.0583005 = \$529.15 \).
      (2) Discrete Compounding: \( V_{1/2} = \$500 \left(1+r\right)^{1/2} = \$500 \times 1.12^{1/2} = \$500 \times 1.0583005 = \$529.15 \).
C. Interpretation of the continuously compounded interest rate:

1. It can be shown that \( e^{r'} = \lim_{m \to \infty} [1 + \frac{r'}{m}]^m \).

2. So \( r' \) can be interpreted as the nominal interest rate associated with an infinitesimally small compound period.

D. Formulas.

1. From Nominal Rate to EAR (both expressed as decimals)

\[ r = (1 + \frac{i_{nom}}{m})^m - 1 \]

2. From EAR to Nominal Rate (both expressed as decimals)

\[ i_{nom} = [(1+r)^{1/m} - 1] m \]

Example:

<table>
<thead>
<tr>
<th>Compound Period</th>
<th>EAR</th>
<th>Nominal Rate</th>
<th>EAR</th>
<th>Nominal Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 year</td>
<td>12%</td>
<td>12%</td>
<td>15%</td>
<td>15%</td>
</tr>
<tr>
<td>6 months</td>
<td>12%</td>
<td>11.6601%</td>
<td>15.5625%</td>
<td>15%</td>
</tr>
<tr>
<td>3 months</td>
<td>12%</td>
<td>11.4949%</td>
<td>15.8650%</td>
<td>15%</td>
</tr>
<tr>
<td>1 month</td>
<td>12%</td>
<td>11.3866%</td>
<td>16.0755%</td>
<td>15%</td>
</tr>
<tr>
<td>1 day (365 days=1 year)</td>
<td>12%</td>
<td>11.3346%</td>
<td>16.1798%</td>
<td>15%</td>
</tr>
<tr>
<td>continuous</td>
<td>12%</td>
<td>11.3329%</td>
<td>16.1834%</td>
<td>15%</td>
</tr>
</tbody>
</table>
VIII. Multiple Cash Flows.

A. Example:

1. Question: Today is the start of 2001. i. How much do I need to invest today at an effective annual rate of 10% to meet a $500 obligation in 2 years and a $800 payment in 3 years? ii. How much would I have to invest if I delay my investment date 1 year?

   i.

   

<table>
<thead>
<tr>
<th>end 00</th>
<th>end 01</th>
<th>end 04</th>
</tr>
</thead>
<tbody>
<tr>
<td>$500</td>
<td>$800</td>
<td></td>
</tr>
</tbody>
</table>

2. Answer i.:

   a. using present value formula, amount needed to be invested today to meet the $500 obligation in 2 years:

   

<table>
<thead>
<tr>
<th>end 00</th>
<th>end 01</th>
<th>end 04</th>
</tr>
</thead>
<tbody>
<tr>
<td>$500</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

   \[ V^2_0 = \$500 \times PVIF_{0.1,2} = \$500 \times (1+0.1)^{-2} = \$413.22 \]

   b. using present value formula, amount needed to be invested today to meet the $800 obligation in 3 years:

   

<table>
<thead>
<tr>
<th>end 00</th>
<th>end 01</th>
<th>end 04</th>
</tr>
</thead>
<tbody>
<tr>
<td>$800</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

   \[ V^3_0 = \$800 \times PVIF_{0.1,3} = \$800 \times (1+0.1)^{-3} = \$601.05 \]

   c. so total amount needed to be invested today is

   \[ V_0 = V^2_0 + V^3_0 = \$413.22 + \$601.05 = \$1014.27. \]
3. Question: ii. How much would I have to invest if I delay my investment date 1 year?

<table>
<thead>
<tr>
<th>end00</th>
<th>end 01</th>
<th>end 04</th>
</tr>
</thead>
<tbody>
<tr>
<td>V₁</td>
<td>$500</td>
<td>$800</td>
</tr>
</tbody>
</table>

4. Answer ii.:
   a. Once the stream of cash flows has been converted to a single sum at a certain point in time (here time 0), can use the present and future value formulas to convert the stream to a single sum at any other time.
   b. So total amount that would have to be invested in one year using the future value formula:

<table>
<thead>
<tr>
<th>end 00</th>
<th>end 01</th>
<th>end 04</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1014.27</td>
<td>V₁</td>
<td></td>
</tr>
</tbody>
</table>

V₁ = $1014.27  \( \text{FVIF}_{0.1,1} \) = $1014.27 (1+0.1) = $1115.70
B. Rules

1. Once each of a set of cash flows at different points in time has been converted to a cash flow at the same point in time, those cash flows can be added to get the value of the set of cash flows at that point.

| 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | ... | N-1 | N |
|---|---|---|---|---|---|---|---|---|---|----|     |     |   |
| C1| C2| C3| C4| C5| C6| C7| C8| C9| C10| C_N-1|     | C_N |

\[
V_0^1 = C_1 \text{PVIF}_{r,1} = C_1 (1+r)^{-1}
\]

\[
V_0^2 = C_2 \text{PVIF}_{r,2} = C_2 (1+r)^{-2}
\]

\[
V_0^3 = C_3 \text{PVIF}_{r,3} = C_3 (1+r)^{-3}
\]

\[
V_0^4 = C_4 \text{PVIF}_{r,4} = C_4 (1+r)^{-4}
\]

\[
\vdots
\]

\[
V_0^{N-1} = C_{N-1} \text{PVIF}_{r,N-1} = C_{N-1} (1+r)^{-(N-1)}
\]

\[
V_0^N = C_N \text{PVIF}_{r,N} = C_N (1+r)^{-N}
\]

\[
V_0 = V_0^1 + V_0^2 + V_0^3 + V_0^4 + \ldots + V_0^{N-1} + V_0^N
\]

2. The value of the stream of cash flows at any other point can then be ascertained using the present or future value formulas:

\[
V_n = V_0 \text{FVIF}_{r,n} = V_0 (1+r)^n.
\]

3. So cash flows occurring at different points in time can not be added but cash flows which occur at the same time can be added.
IX. Particular Cash Flow Pattern: Annuity

A. Definition:
1. N equal payments made at equal intervals (which can be more or less than one period).

B. Converting to a single sum:
1. One Approach: convert each cash flow to a single sum at a given point in time using the present or future value formula and then add up these sums to give the annuity’s value at that point in time.
2. Another Approach:
   a. choose the period so that it is equal to the interval between cash flows.
   b. the annuity’s first cash flow occurs at (t+1) and its last at (t+N).
   c. use the present value annuity factor \( PVAF_{r,N} \) which satisfies
      \[
      V_t = C \times PVAF_{r,N}
      \]
      where
      (1) the effective interest rate over a period is \( r \).
      (2) the formula gives the single sum equivalent at the point in time one period before the first cash flow.
      (3) \( PVAF_{r,N} = \frac{1 - (1+r)^{-N}}{r} \).

   d. or use the future value annuity factor \( FVAF_{r,N} \) which satisfies
      \[
      V_{t+N} = C \times FVAF_{r,N}
      \]
      where
      (1) the effective interest rate over a period is \( r \).
      (2) the formula gives the single sum equivalent at the point in time corresponding to the last cash flow.
      (3) \( FVAF_{r,N} = \frac{(1+r)^N - 1}{r} \).

e. notice that
   (1) \( FVAF_{r,N} = PVAF_{r,N} (1+r)^N \); and so
   (2) \( V_{t+N} = V_t (1+r)^N \) which is consistent with the future value formula.
C. Example:

1. Question: Today is the start of 2001. Suppose I receive $1000 at the end of each year for the next 3 years. If I can invest at an effective annual rate of 10%, how much would I have in 4 years time?

<table>
<thead>
<tr>
<th>end 00</th>
<th>end 01</th>
<th>end 02</th>
<th>end 03</th>
<th>end 04</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1000</td>
<td>$1000</td>
<td>$1000</td>
<td>$1000</td>
<td>V_4</td>
</tr>
</tbody>
</table>

2. One Answer: using the future value formula

\[ V_4^1 = 1000 \times FVIF_{0.1,3} = 1000 \times (1+0.1)^3 = 1331 \]
\[ V_4^2 = 1000 \times FVIF_{0.1,2} = 1000 \times (1+0.1)^2 = 1210 \]
\[ V_4^3 = 1000 \times FVIF_{0.1,1} = 1000 \times (1+0.1)^1 = 1100 \]

\[ V_4 = V_4^1 + V_4^2 + V_4^3 = 1331 + 1210 + 1100 = 3641 \]

3. Another Answer: using the future value annuity formula gives \( V_3 \) and then can use the future value formula to get \( V_4 \)

\[ V_3 = C \times FVAF_{0.1,3} = 1000 \times \frac{((1+0.1)^3-1)/0.1}{1} = 3310 \]
\[ V_4 = V_3 \times FVIF_{0.1,1} = V_3 \times (1+0.1) = 3310 \times 1.1 = 3641. \]

4. Another Answer: using the present value annuity formula gives \( V_0 \) and then can use the future value formula to get \( V_4 \)

\[ V_0 = C \times PVAF_{0.1,3} = 1000 \times \frac{1-(1+0.1)^{-3}}{0.1} = 2486.852 \]
\[ V_4 = V_0 \times FVIF_{0.1,4} = V_0 \times (1+0.1)^4 = 2486.852 \times 1.4641 = 3641. \]
D. A More Difficult Example:

1. Question: Suppose I receive $100 at the end of the month and three further $100 payments in three, five and seven months from today. The effective monthly interest rate is 1%. i. What amount could I borrow today using these four $100 payments to repay the loan? ii. What is the APR with monthly compounding? iii. What is the APR with compounding every 2 months?

<table>
<thead>
<tr>
<th></th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>$100</td>
<td>$100</td>
<td>$100</td>
<td>$100</td>
<td>$100</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

r=0.01

\[1 + r = (1+r)^2\] and so \(r = 1.01^2 - 1 = 0.0201\).

2. Answer to i.: Since payments are made every 2 months, the appropriate interest rate to use for the present value annuity factor is the effective 2 month rate 2.01%; i.e., use \(r = 0.0201\). And since payments are made every 2 months, the present value annuity factor gives the single sum equivalent 2 months before the first payment: here one month before today. So

\[V_{-1} = \$100 \times PVAF_{0.0201, 4} = \$100 \times \frac{1 - \frac{1}{1.0201^4}}{0.0201} = \$100 \times 3.8068 = \$380.68.\]

is the amount that could have been borrowed one month earlier. To get the amount that could be borrowed today, use the future value interest factor:

\[V_0 = V_{-1} \times FVIF_{0.01, 1} = \$380.68 \times 1.01 = \$384.49.\]

3. Answer to ii. With monthly compounding the APR is 12\% = 12%

4. Answer to iii. With compounding every 2 months, the APR is 6\% \times 2.01\% = 12.06\%.
E. Amortization:

1. Question: Suppose I borrow $5000 on 1st January 1998 at an APR of 18% compounded monthly. 
   i. If I have a three year loan and I make loan repayments at the end of each month, what is my monthly payment? 
   ii. What is the loan balance outstanding after the first loan payment? 
   iii. How much interest accumulates in the first month of the loan?

2. Effective Interest Rate: APR is 18% so the effective monthly rate is 
   \(\frac{18\%}{12}=1.5\%\). Thus, \(r=0.015\).

3. Answer i.

\[V_0 = \frac{5000}{PVAF_{0.015,36}} = C \times \frac{1-(1+0.015)^{-36}}{0.015} = C \times 27.660684; \text{ and so}\]

\[C = \frac{5000}{27.660684} = 180.762.\]

4. Answer ii.

   a. Loan balance outstanding at time 1 (after the first payment is made):

\[5000 \times 1.015 - C = 5075 - 180.76 = 4894.24.\]

   b. But the loan balance outstanding at time 1 (after the first payment is made) is equal to the single sum equivalent at time 1 of the remaining 35 payments:

\[V^2_{1,36} = \frac{180.76 \times PVAF_{0.015,35}}{0.015} = 180.76 \times \frac{1-(1+0.015)^{-35}}{0.015} = 4894.24.\]

5. Answer iii.

   a. Interest for 1/98 = $5000 x 0.015 = $75.
6. Question (cont): iv. What is the loan balance outstanding after the twelfth loan payment?

7. Answer iv.

a.

<table>
<thead>
<tr>
<th>Time</th>
<th>Interest</th>
<th>Balance prior to Payment</th>
<th>Payment</th>
<th>Balance after Payment</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td></td>
<td></td>
<td></td>
<td>5000</td>
</tr>
<tr>
<td>1</td>
<td>75</td>
<td>5075</td>
<td>180.76198</td>
<td>4894.238</td>
</tr>
<tr>
<td>2</td>
<td>73.41357</td>
<td>4967.6516</td>
<td>180.76198</td>
<td>4786.8896</td>
</tr>
<tr>
<td>3</td>
<td>71.803344</td>
<td>4858.693</td>
<td>180.76198</td>
<td>4677.931</td>
</tr>
<tr>
<td>4</td>
<td>70.168965</td>
<td>4748.0999</td>
<td>180.76198</td>
<td>4567.338</td>
</tr>
<tr>
<td>5</td>
<td>68.51007</td>
<td>4635.848</td>
<td>180.76198</td>
<td>4455.0861</td>
</tr>
<tr>
<td>6</td>
<td>66.826291</td>
<td>4521.9124</td>
<td>180.76198</td>
<td>4341.1504</td>
</tr>
<tr>
<td>7</td>
<td>65.117256</td>
<td>4406.2676</td>
<td>180.76198</td>
<td>4225.5057</td>
</tr>
<tr>
<td>8</td>
<td>63.382585</td>
<td>4288.8882</td>
<td>180.76198</td>
<td>4108.1263</td>
</tr>
<tr>
<td>9</td>
<td>61.621894</td>
<td>4169.7482</td>
<td>180.76198</td>
<td>3988.9862</td>
</tr>
<tr>
<td>10</td>
<td>59.834793</td>
<td>4048.821</td>
<td>180.76198</td>
<td>3868.059</td>
</tr>
<tr>
<td>11</td>
<td>58.020885</td>
<td>3926.0799</td>
<td>180.76198</td>
<td>3745.3179</td>
</tr>
<tr>
<td>12</td>
<td>56.179768</td>
<td>3801.4977</td>
<td>180.76198</td>
<td>3620.7357</td>
</tr>
</tbody>
</table>

b. Alternatively, the loan balance outstanding after 12 payments is the single sum equivalent at time 12 of the last 24 payments:

\[
V_{12}^{13-36} = 180.76 \times PVAF_{0.015,24} = 180.76 \left[ \frac{1-(1+0.015)^{-24}}{0.015} \right] = 3620.736.
\]
8. Question (cont): How much interest accumulates in the twelfth month of the loan?

9. Answer: To determine how much interest accumulates in the 12th month, need to determine the loan balance outstanding after 11 payments. It is the single sum equivalent at time 11 of the last 25 payments:

\[
\begin{array}{cccccccc}
1/1/98 & 2/1/98 & 3/1/98 & 12/1/98 & 1/1/99 & 2/1/99 & 12/1/00 & 1/1/01 \\
0 & 1 & 2 & 11 & 12 & 13 & 35 & 36 \\
\hline \\
\end{array}
\]

\[
V_{11}^{12-36} = \frac{V_{11}^{12-36}}{\text{PVAF}_{0.015,25}} = 180.76 \left[1 - \frac{(1+0.015)^{-25}}{0.015}\right] = 3745.32.
\]

So the amount of interest accumulating during the 12th month is $3745.32 \times 0.015 = $56.180.
X. Particular Cash Flow Pattern: Perpetuity

A. Definition:
1. equal payments made at equal intervals forever.

<table>
<thead>
<tr>
<th>t-1</th>
<th>t</th>
<th>t+1</th>
<th>t+2</th>
<th>t+3</th>
<th>t+4</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>C</td>
<td>C</td>
<td>C</td>
<td>C</td>
<td>C</td>
</tr>
</tbody>
</table>

B. Converting to a single sum:
1. cannot convert each cash flow to a single sum at a given point in time using the present or future value formulas and then add up these sums to give the value of the annuity at that point in time.
2. Approach:
   a. choose the period so that it is equal to the interval between cash flows.
   b. the perpetuity's first cash flow occurs at \( t+1 \).
   c. use the present value perpetuity factor (PVPF) which satisfies

\[
V_t = C \times PVPF_r
\]

and

\[
PVPF_r = \frac{1}{r} = \lim_{N \to \infty} PVAF_{r,N} = \lim_{N \to \infty} \frac{1 - (1+r)^{-N}}{r}.
\]
C. Example.

1. Question: Mr X wants to set aside an amount of money today that will pay his son and his descendants $10000 at the end of each year forever, with the first payment to be made at the end of 2002. If Mr X can invest at an effective annual rate of 10%, how much would he have to invest today (the 31st December 2000)?

<table>
<thead>
<tr>
<th>end 00</th>
<th>end 01</th>
<th>end 02</th>
<th>end 04</th>
</tr>
</thead>
<tbody>
<tr>
<td>V₀</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

2. Answer:
   a. First, calculate how much Mr X needs to have at the end of 2001 using the present value annuity formula; can use this formula because the first payment is at the end of 2001 and the payments are made annually.

   \[ V₁ = 10000 \times PVIF_{0.1} = 10000 / 0.1 = 100000. \]

   b. Second, calculate the amount that Mr X must invest at the end of 2000 using the present value formula:

   \[ V₀ = V₁ \times PVIF_{0.1,1} = 100000 / (1 + 0.1) = 90909. \]
XI. Application

A. U.S. Treasury Notes and Bonds.

1. Introduction.

a. The distinction between notes and bonds is one of original maturity: notes have an original maturity of 1-10 years; bonds have a maturity>10 years.

b. A plain-vanilla bond is characterized by:
   (1) Maturity: when the bond will be repaid.
   (2) Par or face value: the amount that will be repaid at maturity.
   (3) Coupon rate: the rate used in computing the semiannual coupon payments (0.5 x coupon rate x par value gives the semiannual coupon).
   (4) Coupons are either paid on the 15th or at the end of the month.
   (5) The quoted prices are on the basis of $100 par, in dollars + 1/32nds.

c. Example: See WSJ clipping for Govt Bonds and Notes on 2/18/97.

<table>
<thead>
<tr>
<th>Rate</th>
<th>Maturity Mo/Yr</th>
<th>Bid</th>
<th>Asked</th>
<th>Chg</th>
<th>Ask Yld.</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>Aug 99n</td>
<td>104:28</td>
<td>104:30</td>
<td>-1</td>
<td>5.84</td>
</tr>
</tbody>
</table>

(1) The time line for this bond:

\[
\begin{array}{cccccc}
2/15/97 & 8/15/97 & 2/15/98 & 2/15/99 & 8/15/99 \\
0 & 1 & 2 & 4 & 5 \\
4 & 4 & 4 & 4 & +100 \\
\end{array}
\]

(2) Coupons for this note are paid on the 15th of the month.

(3) The asked price is 104+30/32=104.9375.
2. Yield to maturity (YTM).
   a. Definition.
      (1) YTM is the interest rate such that the present value of the remaining cash flows from the note/bond exactly equals the invoice price.
      (2) The “Ask Yld” in the WSJ is the YTM expressed as an APR with semiannual compounding.
   b. Calculation.
      (1) Suppose the bond has just paid a coupon. Then the YTM expressed as an APR with semi-annual compounding satisfies:

\[ V_0 = C \times PVAF_{YTM/2,N} + 100 \times PVIF_{YTM/2,N} \]

where N is the number of coupon payments to maturity and \( V_0 \) is the invoice price today.
      (2) If the bond has not just paid a coupon, the calculation is more complicated.
   c. Example (cont): On 2/18/97, Aug 99 note has just paid a coupon. Thus, can use the formula to get the invoice price which will also equal the quoted price:

\[ V_0 = 4 \times PVAF_{(5.84/2)\%,.5} + 100 \times PVIF_{(5.84/2)\%,.5} = 18.361 + 86.597 = 104.958 \approx 104:30. \]
Lecture 1: Equities: Characteristics and Markets

I. Reading.
II. Terminology.
IV. Trading on the NYSE.
V. Discussion
VI. Decimalization.
Lecture 1: Equities: Characteristics and Markets

I. Reading.
   A. BKM Chapter 3: Sections 3.2-3.5 and 3.8 are the most closely related to the material covered here.
   B. Useful websites
      1. NYSE: http://www.nyse.com
      2. NASDAQ: http://www.nasdaq.com

II. Terminology.
   A. Bid Price:
      1. Price at which an intermediary is ready to purchase the security.
      2. Price received by a seller.
   B.Asked Price:
      1. Price at which an intermediary is ready to sell the security.
      2. Price paid by a buyer.
   C. Spread:
      1. Difference between bid and asked prices.
      2. Bid price is lower than the asked price.
      3. Spread is the intermediary’s profit.

<table>
<thead>
<tr>
<th>Investor</th>
<th>Price</th>
<th>Intermediary</th>
</tr>
</thead>
<tbody>
<tr>
<td>Buy</td>
<td>Asked</td>
<td>Sell</td>
</tr>
<tr>
<td>Sell</td>
<td>Bid</td>
<td>Buy</td>
</tr>
</tbody>
</table>

A. Exchanges.
   1. National:
      a. NYSE: largest.
      b. AMEX.
   2. Regional: several.
   3. Some stocks trade both on the NYSE and on regional exchanges.
   4. Most exchanges have listing requirements that a stock has to satisfy.
   5. Only members of an exchange can trade on the exchange.
   6. Exchange members execute trades for investors and receive commission.

B. Over-the-Counter Market.
   1. NASDAQ
      a. the major over-the-counter market.
      b. utilizes an automated quotations system which computer-links dealers (market makers).
      c. dealers:
         (1) maintain an inventory of selected stocks; and,
         (2) stand ready to buy a certain number of shares of stock at their stated bid prices and ready to sell at their stated asked prices.
            (a) pre-Jan 21, 1997: up to 1000 shares.
            (b) post-Jan 21, 1997: up to 100 shares.
            (3) post-Jan 21, 1997: required to reveal customer limit orders if better than their stated bid and ask prices.
      d. no centralized trading floor.
      e. individuals hire brokers to find the dealer offering the best deal.
   2. Third Market: refers to the trading of exchange-listed securities on the over-the-counter market.

C. Fourth Market.
   1. This market refers to direct trading between investors in exchange-listed stocks without using a broker.

D. Intermarket Trading.
   1. A “Consolidated Tape” reports trades on the NYSE, the AMEX, and the major regional exchanges as well as on NASDAQ stocks.
   2. The Intermarket Trading System links several markets by computer (NYSE, AMEX, Boston, Cincinnati, Midwest, Pacific, Philadelphia, the Chicago Board Options Exchange, and the NASD); brokers and market makers can thus display quotes on all markets and execute cross-market trades.
IV. Trading on the NYSE.

A. Participants: 4 types of members
   1. Commission Brokers: execute orders that the public has placed with brokerage firms.
   2. Floor Brokers: assist commission brokers when there are too many orders flowing into the market.
   3. Floor Traders: trade solely for themselves.
   4. Specialists: maintain a market in one or more listed stocks.

B. Types of Orders:
   1. Market Orders: simple buy or sell orders that are to be executed immediately at current market prices.
   2. Limit Orders: provide liquidity to the market.
      a. A limit buy order says that if the price falls below a certain price then buy the stock.
      b. A limit sell order says that if the price goes above a certain price then sell the stock.
   3. Stop-loss Orders
      a. A stop-loss buy order says that if the price goes above a certain price then buy the stock.
      b. A stop-loss sell order says that if the price falls below a certain price then sell the stock.

Example: The closing price for IBM on Monday 2/6/95 at 74.375. A limit order to buy IBM stock at 74 tells the broker to buy if the price of IBM falls to 74 or below. A stop-loss order to buy IBM stock at 75 tells the broker to buy if the price of IBM rises to 75 or above. A limit order to sell IBM stock at 75 tells the broker to sell if the price of IBM rises to 75 or above. A stop-loss order to sell IBM stock at 74 tells the broker to sell if the price of IBM falls to 74 or below.

C. Role of the Specialist
   1. maintain a “book” of all unfilled limit orders entered by brokers on behalf of customers.
   2. when the highest outstanding limit buy order exceeds the lowest outstanding limit sell order: execute or “cross” the trade.
   3. be willing at any time to buy at her listed bid price and sell at her listed asked price.
   4. maintain a “a fair and orderly market” by trading in the stock personally.
D. Effective Price
1. bid price: the higher of the specialist’s bid price and the highest unfilled limit buy order.
2. asked price: the lower of the specialist’s asked price and the lowest unfilled limit sell order.

E. Example.
1. The specialist’s book for a stock looks as follows:

<table>
<thead>
<tr>
<th>Price</th>
<th>Limit Sell</th>
<th>Limit Buy</th>
<th>Specialist</th>
</tr>
</thead>
<tbody>
<tr>
<td>90.8</td>
<td>100 sh</td>
<td></td>
<td></td>
</tr>
<tr>
<td>90.7</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>90.6</td>
<td>100 sh</td>
<td></td>
<td>asked</td>
</tr>
<tr>
<td>90.5</td>
<td>100 sh</td>
<td>100 sh</td>
<td>bid</td>
</tr>
<tr>
<td>90.4</td>
<td></td>
<td>100 sh</td>
<td></td>
</tr>
<tr>
<td>90.3</td>
<td></td>
<td>100 sh</td>
<td></td>
</tr>
</tbody>
</table>

Will any trades take place, given this book? Answer: Yes. The limit sell for 100 shares at 90.5 will cross with the limit buy for 100 shares at 90.5.

2. The book now looks as follows:

<table>
<thead>
<tr>
<th>Price</th>
<th>Limit Sell</th>
<th>Limit Buy</th>
<th>Specialist</th>
</tr>
</thead>
<tbody>
<tr>
<td>90.8</td>
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<td></td>
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<tr>
<td>90.7</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>90.6</td>
<td>100sh</td>
<td></td>
<td>asked</td>
</tr>
<tr>
<td>90.5</td>
<td></td>
<td></td>
<td>bid</td>
</tr>
<tr>
<td>90.4</td>
<td></td>
<td>100 sh</td>
<td></td>
</tr>
<tr>
<td>90.3</td>
<td></td>
<td>100 sh</td>
<td></td>
</tr>
</tbody>
</table>

a. If a market sell order for 100 shares comes in, at what price will it execute? Answer: The higher of the highest limit buy (90.4) and the specialist’s bid price (90.5). So 90.5.

b. If a market buy order for 100 shares comes in, at what price will it execute? Answer: The lower of the lowest limit sell (90.6) and the specialist’s asked price (90.6). So 90.6.
Lecture 1  Foundations of Finance

F. How the specialist determines her asked and bid prices: 2 examples.

1. Example: The specialist’s obligation to maintain an orderly market.
   a. The specialist’s book for a stock looks as follows:

<table>
<thead>
<tr>
<th>Price</th>
<th>Limit Sell</th>
<th>Limit Buy</th>
<th>Specialist</th>
</tr>
</thead>
<tbody>
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<td>90.8</td>
<td>100 sh</td>
<td></td>
<td></td>
</tr>
<tr>
<td>90.7</td>
<td>100 sh</td>
<td></td>
<td></td>
</tr>
<tr>
<td>90.6</td>
<td>100 sh</td>
<td></td>
<td>asked</td>
</tr>
<tr>
<td>90.5</td>
<td>100 sh</td>
<td></td>
<td>bid</td>
</tr>
<tr>
<td>90.4</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>90.3</td>
<td></td>
<td>100 sh</td>
<td></td>
</tr>
<tr>
<td>90.2</td>
<td></td>
<td>100 sh</td>
<td></td>
</tr>
</tbody>
</table>

Suppose a market sell for 100 shares arrives. It will get executed at 90.5.

b. The book now looks as follows:

<table>
<thead>
<tr>
<th>Price</th>
<th>Limit Sell</th>
<th>Limit Buy</th>
<th>Specialist</th>
</tr>
</thead>
<tbody>
<tr>
<td>90.8</td>
<td>100 sh</td>
<td></td>
<td></td>
</tr>
<tr>
<td>90.7</td>
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<td></td>
</tr>
<tr>
<td>90.6</td>
<td>100 sh</td>
<td></td>
<td></td>
</tr>
<tr>
<td>90.5</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>90.4</td>
<td></td>
<td></td>
<td>bid</td>
</tr>
<tr>
<td>90.3</td>
<td></td>
<td>100 sh</td>
<td></td>
</tr>
<tr>
<td>90.2</td>
<td></td>
<td>100 sh</td>
<td></td>
</tr>
</tbody>
</table>

Suppose another market sell for 200 shares arrives.

c. If the specialist does not offer a better bid, 100 shares of the sell order will get executed at 90.3 and the other 100 at 90.2.
   Although the specialist may be concerned about the possibility of bad news, her responsibility to maintain an orderly market may oblige her to post a bid of 90.4 and take the other side of the trade.
2. Example: The specialist’s ability to profit from observing the order flow.
   a. The specialist’s book for a stock looks as follows:

<table>
<thead>
<tr>
<th>Price</th>
<th>Limit Sell</th>
<th>Limit Buy</th>
<th>Specialist</th>
</tr>
</thead>
<tbody>
<tr>
<td>90.8</td>
<td>100 sh</td>
<td></td>
<td></td>
</tr>
<tr>
<td>90.7</td>
<td>100 sh, 200 sh, 200sh</td>
<td></td>
<td></td>
</tr>
<tr>
<td>90.6</td>
<td>100 sh,100 sh, 200 sh</td>
<td></td>
<td>asked</td>
</tr>
<tr>
<td>90.5</td>
<td>100 sh, 200 sh, 200sh</td>
<td></td>
<td>bid</td>
</tr>
<tr>
<td>90.4</td>
<td>100 sh, 200 sh, 200sh</td>
<td></td>
<td></td>
</tr>
<tr>
<td>90.3</td>
<td>200 sh</td>
<td></td>
<td></td>
</tr>
<tr>
<td>90.2</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

   Suppose limit sell orders for 300 shares at 90.6 and 400 shares at 90.7 are withdrawn. The implication is possible good news about the stock.

   b. The book now looks as follows:

<table>
<thead>
<tr>
<th>Price</th>
<th>Limit Sell</th>
<th>Limit Buy</th>
<th>Specialist</th>
</tr>
</thead>
<tbody>
<tr>
<td>90.8</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>90.6</td>
<td>100 sh</td>
<td></td>
<td>bid</td>
</tr>
<tr>
<td>90.5</td>
<td>100 sh, 200 sh, 200sh</td>
<td></td>
<td></td>
</tr>
<tr>
<td>90.4</td>
<td>100 sh, 200 sh, 200sh</td>
<td></td>
<td></td>
</tr>
<tr>
<td>90.3</td>
<td>200 sh</td>
<td></td>
<td></td>
</tr>
<tr>
<td>90.2</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

   Suppose a market sell for 100 shares arrives.

   c. If the specialist does not offer a better bid, the sell order will get executed at 90.5

   d. However, based on the order flow the specialist may prefer to bid 90.6 and take the other side of the trade. She may think it is a good bet given the large quantity of limit buy orders at 90.5 and 90.4 (limited downside) and the evaporation of limit sell orders at 90.7 and 90.6 (potential for upside gain).
G. Block Trades
1. Definition: If an investor wishes to trade more than 10,000 shares of a NYSE stock or any quantity worth over $200,000, it is known as a block trade.
2. There are a number of ways such a trade can be executed.

H. The SuperDOT System
1. SuperDOT enables exchange members to send orders directly to the specialist over computer lines.
2. Most trades are executed and reported back to the originating member within 2 minutes.
3. The largest market order that can be handled is 30,099 shares and most orders handled are small; so although a large fraction of all orders are handled by SuperDOT, these only account for a small fraction of trading volume.
V. Discussion
A. NYSE vs OTC market
1. NYSE has higher fixed costs (due to the need for a central physical location), lower per-trade costs (since more likely to be trading with another investor and not a market maker) and greater structure (rules and regimentation).
2. The higher fixed costs for the NYSE means that a stock must have an active market to cover these costs; this is why the NYSE has stringent listing requirements.
3. The distinction is becoming blurrier:
   a. NASDAQ has listing requirements like an exchange.
   b. NASDAQ new order handling rules (post Jan 1997) force dealers to display customer limit orders allowing one customer to trade with another customer.

B. Motivations for Trading.
1. Information: You believe that you have information about the asset that is not reflected in price.
2. Liquidity: You have surplus cash to invest (and you will buy securities) or you need to raise cash (and you will sell securities).
3. Rebalancing: The composition of your investment portfolio has changed due to differences in the performance of the assets currently in the portfolio. You want to readjust the composition to reflect your risk preferences.
Lecture 1

VI. Decimalization.

A. Two issues:
   1. Quoting prices in decimals.
   2. Minimum tick size becomes 1 cent rather than 1/8 or 1/16.

B. Dates:
   1. Island: July 2000.

C. Effects:
   1. Spreads narrower particularly for active stocks.
      a. Impatient market orders benefit.
      b. Limit order traders are disadvantaged
      c. NASDAQ dealers are harmed.
      d. Institutional investors are harmed
   2. Specialist will trade more often and more profitably: easier to “step ahead”
      of limit orders to exploit her information about order flow.