Lecture 8: Bond Portfolio Management.

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Lecture 8: Bond Portfolio Management.

I. Reading.
   A. BKM, Chapter 16, Sections 16.1 and 16.2.

II. Risks associated with Fixed Income Investments.
   A. Reinvestment Risk.
      1. If an individual has a particular time horizon $T$ and holds an instrument
         with a fixed cash flow received prior to $T$, then the investor faces
         uncertainty about what yields will prevail at the time of the cash flow.
         This uncertainty is known as reinvestment risk.
      2. Example: Suppose an investor has to meet a obligation of $5M in two
         years time. If she buys a two year coupon bond to meet this obligation,
         there is uncertainty about the rate at which the coupons on the bond can be
         invested. This uncertainty is an example of reinvestment risk.

   B. Liquidation Risk.
      1. If an individual has a particular time horizon $T$ and holds an instrument
         which generates cash flows that are received after $T$, then the investor
         faces uncertainty about the price of the instrument at time $T$. This
         uncertainty is known as liquidation risk.
      2. Example: Suppose an investor has to meet a obligation of $5M in two
         years time. If she buys a five year discount bond to meet this obligation,
         there is uncertainty about the price at which this bond will sell in two years
         time. This uncertainty is an example of liquidation risk.
III. Duration.
A. Definition.
1. Macaulay Duration.
   a. Let 1 period = 1 year.
   b. Consider a fixed income instrument \( i \) which generates the following stream of certain cash flows:

   \[
   \begin{array}{cccccccccc}
   0 & \frac{1}{2} & 1 & 1\frac{1}{2} & 2 & 2\frac{1}{2} & \ldots & \frac{N-1}{2} & N \\
   C_i(\frac{1}{2}) & C_i(1) & C_i(1\frac{1}{2}) & C_i(2) & C_i(2\frac{1}{2}) & & & C_i(N-\frac{1}{2}) & C_i(N) \\
   \end{array}
   \]

   Let \( y'(0) \) be the YTM at time 0 on the instrument expressed as an APR with semi-annual compounding.
   c. Macaulay duration is defined as

   \[
   D_i(0) = \frac{1}{2} k_i(\frac{1}{2}) + 1 k_i(1) + 1\frac{1}{2} k_i(1\frac{1}{2}) + \ldots + N k_i(N)
   \]

   where

   \[
   k_i(t) = \frac{C_i(t)}{P_i(0)} \frac{1}{[1 + y'(0)/2]^{2t}}
   \]

   d. Note that the \( k_i(t) \)'s sum to 1:

   \[
   1 = k_i(\frac{1}{2}) + k_i(1) + k_i(1\frac{1}{2}) + \ldots + k_i(N)
   \]

2. “Modified” Duration.
   a. “Modified” duration \( D^*(0) \) is defined to be Macaulay duration divided by \([1+y'(0)/2] \):

   \[
   D^*(0) = \frac{D_i(0)}{[1+y'(0)/2]}
   \]

   b. “Modified” duration is often used in the industry.
3. Example. 2/15/97. The 3 Feb 98 note has a price of 97.1298 and a YTM expressed as an APR with semi-annual compounding of 6%.

a. Its k’s can be calculated:

\[ k^{3 \text{ Feb 98}(\frac{1}{2})} = \frac{1.5}{1.03}/97.1298 = 1.4563/97.1298 = 0.0150. \]
\[ k^{3 \text{ Feb 98}(1)} = \frac{101.5}{1.03^2}/97.1298 = 95.6735/97.1298 = 0.9850. \]

b. Its duration can be calculated:

\[ D^{3 \text{ Feb 98}(0)} = \frac{1}{2} \times k^{3 \text{ Feb 98}(\frac{1}{2})} + 1 \times k^{3 \text{ Feb 98}(1)} = \frac{1}{2} \times 0.0150 + 1 \times 0.9850 = 0.9925 \text{ years}. \]

c. Its modified duration can be calculated:

\[ D^*^{3 \text{ Feb 98}(0)} = \frac{0.9925}{1+0.03} = 0.9636. \]

<table>
<thead>
<tr>
<th>Instrument</th>
<th>YTM(2/15/97)</th>
<th>Duration(2/15/97)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Feb 98 Strip</td>
<td>6%</td>
<td>1.0000</td>
</tr>
<tr>
<td>3 Feb 98 Bond</td>
<td>6%</td>
<td>0.9925</td>
</tr>
<tr>
<td>10 Feb 98 Bond</td>
<td>6%</td>
<td>0.9766</td>
</tr>
<tr>
<td>40% of 3 Feb 98 and 60% of 10 Feb 98</td>
<td>6%</td>
<td>0.9925 x 0.4 + 0.9766 x 0.6 = 0.98296</td>
</tr>
</tbody>
</table>
B. Duration can be interpreted as the effective maturity of the instrument.

1. If the instrument is a zero coupon bond, then its duration is equal to the
time until maturity.
2. For an instrument with cash flows prior to maturity, its duration is less
than the time until maturity.
3. Holding maturity and YTM constant, the larger the earlier payments
relative to the later payments then the shorter the duration.
4. Example: 2/15/97. Suppose the 10 Feb 98 coupon bond has a price of
103.8270 and a YTM expressed as an APR with semi-annual
compounding of 6%.
   a. Its k's can be calculated:
      \[ k^{10 \text{ Feb 98}}(\frac{1}{2}) = \frac{5/1.03}{103.8270} = 0.0468. \]
      \[ k^{10 \text{ Feb 98}}(1) = \frac{105/1.03^2}{103.8270} = 0.9532. \]
   b. Its duration can be calculated:
      \[ D^{10 \text{ Feb 98}}(0) = \frac{1}{2} x k^{10 \text{ Feb 98}}(\frac{1}{2}) + 1 x k^{10 \text{ Feb 98}}(1) \]
      \[ = \frac{1}{2} x 0.0468 + 1 x 0.9532 = 0.9766 \text{ years}. \]
   c. So the 10 Feb 98 bond with larger earlier cash flows relative to
      later cash flows has a shorter duration than the 3 Feb 98 bond.

C. Duration of a Portfolio.

1. If a portfolio C has \( \omega^{A,C} \) invested in asset A at time 0 and \( \omega^{B,C} \) invested in
   asset B at time 0 and the YTMs of A and B are the same, the duration of C
   is given by:
   \[ D^C(0) = \omega^{A,C} D^A(0) + \omega^{B,C} D^B(0). \]

2. Example (cont): 2/15/97. Suppose Tom forms a portfolio on 2/15/97 with
   40% in 3 Feb 98 coupon bonds and the rest in 10 Feb 98 coupon bonds.
   a. The portfolio’s duration can be calculated:
      \[ D^P(0) = \omega^{3 \text{ Feb 98,P}} D^{3 \text{ Feb 98}}(0) + \omega^{10 \text{ Feb 98,P}} D^{10 \text{ Feb 98}}(0) \]
      \[ = 0.4 \times 0.9925 + (1-0.4) \times 0.9766 = 0.98296. \]
D. Duration as a measure of yield sensitivity.

1. The price of bond $i$ described above is given by:

$$P_i(0) = \frac{C_i(\frac{1}{2})}{[1+y_i(0)/2]^1} + \frac{C_i(1)}{[1+y_i(0)/2]^2} + \frac{C_i(1\frac{1}{2})}{[1+y_i(0)/2]^3} + \ldots + \frac{C_i(N)}{[1+y_i(0)/2]^{2N}}.$$  

2. To assess the impact of the price of bond $i$ to a shift in $y_i(0)$, differentiate the above expression with respect to $y_i(0)$:

$$\frac{d}{dy_i(0)} P_i(0) = -\frac{1}{2} \frac{C_i(\frac{1}{2})}{[1+y_i(0)/2]^2} - 1 \frac{C_i(1)}{[1+y_i(0)/2]^3} - 1\frac{1}{2} \frac{C_i(1\frac{1}{2})}{[1+y_i(0)/2]^4} - \ldots - N \frac{C_i(N)}{[1+y_i(0)/2]^{2N+1}}.$$  

3. Thus, bond $i$’s modified Macaulay duration is related to the sensitivity of its price to shifts in $y_i(0)$ as follows:

$$\frac{d}{dy_i(0)} P_i(0) = -P_i(0) D^{i^i}(0).$$  

4. Thus, bond $i$’s modified Macaulay duration measures its price sensitivity to a change in its yield (where price sensitivity is measured by the percentage change in the price).

5. In particular, a small change in $y_i(0)$, $\Delta y_i(0)$, causes a change in $P_i(0)$, $\Delta P_i(0)$, which satisfies:

$$\Delta P_i(0) \approx -P_i(0) D^{i^i}(0) \Delta y_i(0).$$
6. Example (cont): 2/15/97. The 3 Feb 98 note has a price of 97.1298 and a YTM expressed as an APR with semi-annual compounding of 6%.
   a. Suppose the YTM on the 3 Feb 98 bond increases by 1% to 7%.
      The bond’s price becomes

\[ P_{3\ Feb\ 98}(0) = 1.5/ (1+0.07/2) + 101.5/(1+0.7/2)^2 = 96.2006. \]

which means a price decline of 0.9292.

b. Suppose the YTM on the 3 Feb 98 bond decreases by 1% to 5%.
   The bond’s price becomes

\[ P_{3\ Feb\ 98}(0) = 1.5/ (1+0.05/2) + 101.5/(1+0.05/2)^2 = 98.0725. \]

which implies a price increase of 0.9427.

c. Using the formula involving modified duration above:

YTM increase: \( \Delta P_{3\ Feb\ 98}(0) = -97.1298 \times 0.9636 \times 0.01 = -0.9359. \)
YTM decrease: \( \Delta P_{3\ Feb\ 98}(0) = -97.1298 \times 0.9636 \times 0.01 = 0.9359. \)

d. So the duration based approximation overstates the price decline when the YTM increases and understates the price increase when the YTM decreases. It can be shown that this is generally true.
Duration and Price Sensitivity to Yield Shifts:
30 Yr Strip Price at y=6% is normalized to 100

Duration and Price Sensitivity to Yield Shifts:
10 Yr Strip Price at y=6% is normalized to 100
3 Feb 98  

<table>
<thead>
<tr>
<th>YTM</th>
<th>P(0)</th>
<th>Actual ΔP(0)</th>
<th>P(0)</th>
<th>D*(0)</th>
<th>ΔYTM</th>
<th>Approx ΔP(0) = -P(0)D*(0)ΔYTM</th>
</tr>
</thead>
<tbody>
<tr>
<td>7%</td>
<td>96.2006</td>
<td>-0.9292</td>
<td>97.1298</td>
<td>0.9636</td>
<td>+0.01</td>
<td>-0.9359</td>
</tr>
<tr>
<td>6%</td>
<td>97.1298</td>
<td>0</td>
<td></td>
<td>0</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>5%</td>
<td>98.0725</td>
<td>+0.9427</td>
<td></td>
<td></td>
<td>-0.01</td>
<td>+0.9359</td>
</tr>
</tbody>
</table>

IV. Immunization.
A. Dedication Strategies.
1. A dedication strategy ensures that a stream of liabilities can be met from available assets by holding a portfolio of fixed income assets whose cash flows exactly match the stream of fixed outflows.
2. Note that both the asset portfolio and the liability stream have the same current value and duration.
3. Example: It is 2/15/97 and QX must pay $5M on 8/15/97 and $10M on 2/15/98. A dedication strategy would involve buying Aug 97 U.S. strips with a face value of $5M and Feb 98 U.S. strips with a face value of $10M.

B. Target Date Immunization.
1. Assumptions.
   a. the yield curve is flat at y (APR with semiannual compounding).
   b. any shift in the curve keeps it flat.
2. Target date immunization ensures that a stream of fixed outflows can be met from available assets by holding a portfolio of fixed income assets:
   a. with the same current value as the liability stream; and,
   b. with the same modified duration.
3. Note,
   a. given the flat yield curve, all bonds have the same YTM, y.
   b. for this reason, a small shift in the yield curve will have the same effect on the value of the immunizing assets and on the value of the liabilities (using the definition of duration).
   c. and so, the assets will still be sufficient to meet the stream of fixed outflows.
   d. further, it is easy to work out the current value of the liability stream using y.
   e. equating the modified durations of the assets and the liabilities is the same as equating the durations.
4. Example: Firm GF is required to make a $5M payment in 1 year and a $4M payment in 3 years. The yield curve is flat at 10% APR with semi-annual compounding. Firm GF wants to form a portfolio using 1-year and 4-year U.S. strips to fund the payments. How much of each strip must the portfolio contain for it to still be able to fund the payments after a shift in the yield curve?
   a. The value of the liabilities is given by:

\[
L(0) = \frac{5M}{[1+0.1/2]^2} + \frac{4M}{[1+0.1/2]^6} = 4.5351M + 2.9849M = 7.5200M.
\]

b. The duration of the liabilities is given by:

\[
D^L(0) = 1 \times \frac{4.5351}{7.5200} + 3 \times \frac{2.9849}{7.5200} = 1.7938 \text{ years.}
\]

c. Let \( A^1(0) \) be the portfolio’s dollar investment in the 1-year strips and \( A^4(0) \) be the portfolio’s dollar investment in the 4-year strips.

d. The dollar value of the portfolio must equal the value of the liabilities. So \( A^1(0) + A^4(0) = 7.5200M. \)

e. The duration of the portfolio equals

\[
D^P(0) = \omega^{1,p} D^1(0) +(1-\omega^{1,p}) D^4(0)
\]

\[
\text{where } \omega^{1,p} = A^1(0)/7.5200M. \text{ The duration of the 1-year strip is 1 and the duration of the 4-year strip is 4.}
\]

f. Setting the duration of the portfolio equal to the duration of the liabilities gives:

\[
1.7938 = \omega^{1,p} D^1(0) +(1-\omega^{1,p}) D^4(0) = \omega^{1,p} 1 +(1-\omega^{1,p}) 4 = \omega^{1,p} = 0.7354.
\]

g. Thus,

\[
A^1(0) = 0.7354 \times 7.5200M = 5.5302M; \text{ and, } A^4(0) = 7.5200M - 5.5302M = 1.9898M.
\]
5. Generalizations.
a. The assumption of a flat yield curve has undesirable properties.
b. When the yield curve is allowed to take more general shapes, target-date immunization is still possible, but modified Macaulay duration is not appropriate for measuring the impact of a yield curve shift on price. Need to use a more general duration measure.

C. Comparison.
1. Dedication strategies are a particular type of target date immunization.
2. The one advantage of a dedication strategy is that there is no need to rebalance through time. Almost all other immunization strategies involve reimmunizing over time.
Lecture 8: Derivatives: Definitions and Payoffs.

I. Readings.
II. Preliminaries.
III. Options.
IV. Forward and Futures Contracts.
V. Features of Derivatives.
VI. Motives for Holding Derivatives.
Lecture 8: Derivatives: Definitions and Payoffs.

I. Readings.
   A. Options:
      1. BKM, Chapter 20, Sections 20.1 - 20.4.
   B. Forward and Futures Contracts.
      1. BKM, Chapter 22, Sections 22.1 - 22.3.

II. Preliminaries.
   A. Payoff Diagrams.
      1. Let $S(t)$ be the value of one unit of an asset at time $t$.
      2. Payoff diagram graphs the value of a portfolio at time $T$ as a function of $S(T)$.
      3. Long 1 unit of the asset is represented by a $45^\circ$ line through the origin.
      4. Short 1 unit of the asset is represented by a $-45^\circ$ line through the origin.
      5. Long a discount bond with a face value of $50 maturing at $T$ is represented by a horizontal line at $50$: its value at $T$ is $50 irrespective of the value of the asset.
      6. Short a discount bond with a face value of $50 maturing at $T$ is represented by a horizontal line at -$50.
III. Options.
   A. Call Option.
      1. Definition.
         a. A call option gives its holder the right (but not the obligation) to buy the option’s underlying asset at a specified price.
         b. The specified price is known as the exercise or strike price.
         c. A European option can only be exercised at the expiration date of the option. An American option can be exercised any time prior to the expiration of the option.
      2. Payoff at the Expiration Date to the Holder of a European Option.
         a. Let $T$ be the expiration date, $X=$50 be the strike price and $S(t)$ be the value of the option’s underlying asset at time $t$.
         b. Since the holder of the option is not obligated to buy the asset for $50, she will only do so when the payoff from doing so is greater than zero.
            (1) The price paid by the holder prior to $T$ for the option is irrelevant to the decision to exercise since it is a sunk cost.
            (2) If $[S(T)-50] > 0$, the holder wants to exercise the option as it allows her to buy for $50 an asset worth more than $50.
            (3) If $[S(T)-50] < 0$, the holder does not want to exercise the option since she would be paying $50 for an asset she could buy in the market for less than $50. So her payoff is zero.
            (4) Thus, the holder’s payoff from the call option is max $\{S(T)-50, 0\}$.
            (5) Since the payoff from the option is non-negative, no arbitrage tells you that the call option has a non-negative value at any time $t$, $C_{50}(t)$, prior to $T$.
            (6) At the expiration date, $C_{50}(T) = \max \{S(T)-50, 0\}$.
      3. Writer of the Option.
         a. If the option holder exercises the option, the option writer must sell the asset to the holder for $50. Her payoff is -$[S(T)-50]$.
         b. If the option holder does not exercise the option, the writer is not obliged to do anything. Her payoff is 0.
         c. So the payoff to the option writer is just the negative of the payoff to the option holder.
         d. In return, the option writer receives the option’s value when she first writes the call option and sells it.
         e. Suppose the call is written at time 0 and both the writer and the option buyer keep their positions until expiration. The cash flows of each given below in Table 1.
         f. Can see that the sum of the writer’s and the holder’s payoff is zero.
B. Put Option.

1. Definition.
   a. A put option gives its holder the right (but not the obligation) to sell the option’s underlying asset at a specified price (also called the exercise or strike price).
   b. European and American have the same connotations for puts as for calls.

2. Payoff at the Expiration Date to the Holder of the Option.
   a. Again, let $T$ be the expiration date, $X=50$ be the strike price and $S(t)$ be the value of the option’s underlying asset at time $t$.
   b. Since the holder of the option is not obligated to sell the asset for $50$, she will only do so when the payoff from doing so is greater than zero.
      (1) The price paid by the holder prior to $T$ for the option is irrelevant to the decision to exercise since it is a sunk cost.
      (2) If $[50-S(T)] > 0$, the holder wants to exercise the option since it allows her to sell for $50$ an asset worth less than $50$.
      (3) If $[50-S(T)] < 0$, the holder does not want to exercise the option since she would be receiving $50$ for an asset she could sell in the market for more than $50$. So her payoff is zero.
      (4) Thus, the holder’s payoff from the put option is $\max \{50-S(T), 0\}$.
      (5) Since the payoff from the option is non-negative, no arbitrage tells you that the put option has a non-negative value at any time $t$, $P_{50}(t)$, prior to $T$.
      (6) At the expiration date, $P_{50}(T) = \max \{50-S(T), 0\}$.
      (7) Note that holding a put is not the same as writing a call.

3. Writer of the Option.
   a. Suppose the put is written at time $0$ and both the writer and the option buyer keep their positions until expiration. The cash flows of each given below in Table 1.
   b. Can see that the sum of the writer’s and the holder’s payoff is zero.
C. Markets for Options.
   1. Chicago Board Options Exchange (CBOE).
      a. Most options on individual stocks are traded on the CBOE which was started in 1973.
      b. The CBOE is both a primary and secondary market for options.
   2. Which options have volume?
      a. Presently, there is very little trading or interest in options on individual stocks.
      b. Interest in options on indices, both exchange traded and over the counter is booming. Part of the reason is the growth of index funds.

D. Standardization: Publically traded options are standardized in several respects.
   1. Size of Contract.
      a. Traded options on stocks are usually for 100 shares of the stock, although prices are quoted on a per share basis.
   2. Maturities.
      a. Options expire on a three month cycle.
      b. The convention is that the option expires on the Saturday following the third Friday of the month.
   3. Exercise Prices.
      a. These are in multiples of $2\frac{1}{2}, $5 or $10 depending on the prevailing stock price.

E. Examples.
   1. Call and put options on Microsoft stock from the Bloomberg screen on 4/15/97.
IV. Forward and Futures Contracts.

A. Forward Contracts.

1. Definition.
   a. A forward contract on an asset is an agreement between the buyer and seller to exchange cash for the asset at a predetermined price (the forward price) at a predetermined date (the settlement date).
   b. The asset underlying a futures contract is often referred to as the “underlying” and its current price is referred to as the “spot” price.
   c. The buyer of the forward contract agrees today to buy the asset on the settlement date at the forward price. The seller agrees today to sell the asset at that price on that date.
   d. No money changes hands until the settlement date. In fact, the forward price is set so that neither party needs to be paid any money today to enter into the agreement.

2. Payoff Diagram.
   a. Let T be the settlement date, $S(t)$ be the price of the asset at time $t$ and $F_T(0)$ be the forward price agreed to at time 0 for delivery at time $T$.
   b. At time $T$, can see that the buyer of the forward contract gets $S(T)-F_T(0)$; the seller gets $F_T(0)-S(T)$.
   c. Can see that the payoff to both the buyer and seller at time $T$ could be negative.
   d. Can see that the sum of the payoffs to the buyer and the seller equal zero.
   e. The cash flows to the buyer and the seller at time 0 and at time $T$ are given in Table 1.
   f. Can see that the forward price at the settlement date must equal the spot price: $F_T(T)=S(T)$.
   a. The most organized forward markets are currency markets.
   b. Example: WSJ. On Monday, 4/14/97, the British pound was quoted as $1.6200/£ for immediate delivery. For delivery 6-months forward, the quote was $1.6159/£. When a buyer and seller entered into this contract on 4/14/97, the buyer of the pounds agrees to pay the seller $1.6159 in 6 months (10/97) for each £ and the £’s are delivered at that time. This may differ from the spot exchange rate that prevails in 10/97. No money changes hands on 4/14/97.
   c. The forward market in FX is a telephone dealer market that exists as an adjunct to the spot market.
   d. It is only open to banks and other institutional players.
   e. All transactions are customized as to size, currency and delivery terms.
   f. The buyer and seller bear each others credit risk. So if one defaults the other has to bear the loss.
B. Futures Contracts vs Forward Contracts.
1. Futures contracts are sold on metals, agricultural commodities, oil, livestock and financial securities. Financial futures exist on stocks, bonds, T-bills, notes, Federal funds, Libor loans and municipal bonds.
2. Daily Resettlement.
   a. Forward and futures contracts are essentially the same except for the daily resettlement feature of futures contracts.
   b. With a forward contract, no money changes hands until the settlement date.
   c. With a futures contract:
      (1) on the day that the futures contract is bought or sold: no money is paid or received.
      (2) each day after the contract is bought or sold:
         (a) a positive change in the futures price from the previous day has to be paid by the seller to the buyer of the futures contract.
         (b) a negative change has to be paid by the buyer to the seller.
      (3) the futures price on the settlement day is set equal to the spot price on that day.
3. Why futures and forward contacts are essentially the same?
   a. Suppose that for a given settlement date, the forward and futures prices are the same.
   b. Consider an investor who buys either contract on this day and holds it til the settlement day.
   c. Her total cash flow is the same under both contracts.
      (1) It is the spot price on the settlement day less the futures/forward price on the day the position is opened.
      (2) This explains why futures and forward contracts are essentially the same.
   d. However, the timing is different depending on the contract.
      (1) With the forward contact, the cash flow occurs on the settlement date: there are no intermediate cash flows.
      (2) With the futures contract, cash flows occur over the period that the position is open due to daily resettlement
         (a) When the position is left open til the settlement date, the sum of these cash flows equals the spot price on the settlement day less the futures/forward price on the day the position is opened.
      (3) This timing difference explains why futures and forward prices are typically slightly different.
4. Example: Futures contract on the S&P 500 from the Bloomberg screen on
5. Other differences: Future vs Forward Contract.
   a. Standardization.
      (1) Forward contracts are tailored to the needs of the parties involved.
      (2) Futures contracts traded on the CME (the "Merc") have standardized unit sizes and standardized maturity dates.
   b. Exchange traded.
      (1) Forward contracts are usually traded in dealer markets.
      (2) Merc futures contracts are traded only on the Merc. There is no trading off the Merc (by law). This is different from the situation with stocks: IBM, an NYSE-listed company often trades away from the NYSE.
   c. Credit risk is borne by a clearing house.
      (1) In a forward market, you must be concerned with the credit worthiness of your counterparty.
      (2) In a futures market, the clearing house (an affiliate of the exchange) interposes itself between the two parties. The clearing house sells to the buyer and buys from the seller.
         (a) The buyer's contract is with the clearing house; the seller's contract is with the clearing house.
         (b) The clearing house does not bear any risk other than the credit risk: it is long one contract and short one contract.
      (3) How does the clearing house manage the credit risk?
         (a) All exchange members are required to post funds with the clearing house.
         (b) An exchange member is responsible for its customers' transactions.
   d. Margin.
      (1) In a forward contract, no money changes hands until settlement.
      (2) In a futures transaction, the buyer and seller are each required to post a small amount of the face value of the contract with their brokers. In turn, their brokers are required to post margin with the clearing house.
6. Comment: the standardization, exchange trading facility and reduced credit risk serve to increase liquidity in the futures market, and make it accessible to many smaller investors who could not directly participate in the forward market.
V. Features of Derivatives.
   A. Zero Net Supply.
      1. Stocks and bonds are said to be in positive net supply. They are claims on real assets. On balance the economy is long IBM stock, for example.
      2. Derivatives (forwards, futures and options) are in zero net supply. For everyone who is long a forward contract, the opposite side is short. When you add up the long and short positions, they sum to zero.
   B. Zero Sum Game.
      1. As has been noted, if the long side of a derivative contract gains, the short side loses. One side's profit is the other's loss. The sum of the profits to both sides is zero.
VI. Motives for Holding Derivatives.

A. Speculation.
1. Example (cont). Today is 4/14/97. The buyer of the £'s for delivery on 10/97 may simply be betting that by the 10/97, the £ will have increased in value. Suppose the buyer agrees to buy £10000 on 10/97 (at a price of $16159). If she has no other exposure to £'s, she gains if the spot price of the pound is high on 10/97; she loses if the spot price is low.
2. Example. Nicholas Leeson at Barings.

B. Hedging.
1. Example (cont). The buyer of the £'s may owe £10000 in October but is receiving a fixed number of $ ($16500) at that time. By entering into the forward contract on 4/14/97, she removes the exchange rate risk that she would otherwise face.
2. Example. Can combine the underlying asset with puts and calls to create a riskless position.

C. Risk Management.
1. Can use derivatives to manage the risks of a portfolio.
2. Portfolio Insurance.
   a. Can combine the underlying asset with a long position in puts on the asset to put a floor on the value of a position. This is known as portfolio insurance.
   b. Example. Suppose you have 100 times the value of the S&P 500 index invested in the market at the start of 1997. When the S&P 500 hits 800 on 2/12/97, you decide you want to bound your losses so you buy a European put on 100 times the value of the S&P 500 with an exercise price of $800 and an expiry date of 12/97. You have implemented portfolio insurance.
Lecture 8

Foundations of Finance

<table>
<thead>
<tr>
<th>Position opened at time 0</th>
<th>Time 0</th>
<th>Time T</th>
<th>S(T)&lt;50</th>
<th>S(T)≥50</th>
</tr>
</thead>
<tbody>
<tr>
<td>Long 1 Stock</td>
<td>-S(0)</td>
<td>S(T)</td>
<td>S(T)</td>
<td>S(T)</td>
</tr>
<tr>
<td>Short 1 Stock</td>
<td>S(0)</td>
<td>-S(T)</td>
<td>-S(T)</td>
<td>-S(T)</td>
</tr>
<tr>
<td>Long 1 Discount Bond maturing at T with face value of $50</td>
<td>-50 d_r(0)</td>
<td>50</td>
<td>50</td>
<td>50</td>
</tr>
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<td>Short 1 Discount Bond maturing at T with face value of $50</td>
<td>50 d_r(0)</td>
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<td>-50</td>
</tr>
<tr>
<td>Long 1 European Call Option on the Stock expiring at T with an exercise price of $50</td>
<td>-C_{50,T}(0)</td>
<td>0</td>
<td>S(T)-50</td>
<td>max [S(T)-50,0]</td>
</tr>
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<td>Short 1 European Call Option on the Stock expiring at T with an exercise price of $50</td>
<td>C_{50,T}(0)</td>
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<tr>
<td>Long 1 European Put Option on the Stock expiring at T with an exercise price of $50</td>
<td>-P_{50,T}(0)</td>
<td>50-S(T)</td>
<td>0</td>
<td>max [50-S(T),0]</td>
</tr>
<tr>
<td>Short 1 European Put Option on the Stock expiring at T with an exercise price of $50</td>
<td>P_{50,T}(0)</td>
<td>-[50-S(T)]</td>
<td>0</td>
<td>-max [50-S(T),0]</td>
</tr>
<tr>
<td>Long 1 Forward Contract on the Stock with delivery at T at a forward price F_T(0)</td>
<td>0</td>
<td>S(T)-F_T(0)</td>
<td>S(T)-F_T(0)</td>
<td>S(T)-F_T(0)</td>
</tr>
<tr>
<td>Short 1 Forward Contract on the Stock with delivery at T at a forward price F_T(0)</td>
<td>0</td>
<td>F_T(0)-S(T)</td>
<td>F_T(0)-S(T)</td>
<td>F_T(0)-S(T)</td>
</tr>
</tbody>
</table>
Short 1 Stock:
Payoff at T

\[ -S(T) \]

\[ S(T) \]
Long 1 Discount Bond maturing at \( T \) with $50 face value:
Payoff at \( T \)
Short 1 Discount Bond maturing at T with $50 face value:
Payoff at T
I. Call Option.
   1. Definition.
      a. A call option gives its holder the right (but not the obligation) to buy the option’s underlying asset at a specified price.
      b. The specified price is known as the exercise or strike price.
      c. A European option can only be exercised at the expiration date of the option. An American option can be exercised any time prior to the expiration of the option.

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</table>

Long 1 European Call on Stock expiring at T with $50 exercise price: Payoff at T
Lecture 8

2. Writer of the Call Option.
   a. The payoff to the option writer is just the negative of the payoff to the option holder.
   b. In return, the option writer receives the option’s value when she first writes the call option and sells it.
   c. Can see that the sum of the writer’s and the holder’s payoff is zero.

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</tr>
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<td>Short 1 European Call Option on the Stock expiring at T with an exercise price of $50</td>
<td>C_{50,T}(0)</td>
<td>0</td>
</tr>
<tr>
<td>Sum</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>
II. Put Option.

1. Definition.
   a. A put option gives its holder the right (but not the obligation) to sell the option’s underlying asset at a specified price (also called the exercise or strike price).
   b. European and American have the same connotations for puts as for calls.

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<td>Long 1 European Put Option on the Stock expiring at T with an exercise price of $50</td>
<td>-P_{50,T}(0)</td>
<td>50-S(T)</td>
</tr>
</tbody>
</table>

2. Writer of the Put Option.
   a. The payoff to the option writer is just the negative of the payoff to
the option holder.

b. In return, the option writer receives the option’s value when she first writes the put option and sells it.

c. Can see that the sum of the writer’s and the holder’s payoff is zero.

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<td>$50-S(T)$</td>
</tr>
<tr>
<td>Short 1 European Put Option on the Stock expiring at T with an exercise price of $50</td>
<td>$P_{50,T}(0)$</td>
<td>$-[50-S(T)]$</td>
</tr>
<tr>
<td>Sum</td>
<td>$0$</td>
<td>$0$</td>
</tr>
</tbody>
</table>
III. Forward Contracts.

1. Definition.
   a. A forward contract on an asset is an agreement between the buyer and seller to exchange cash for the asset at a predetermined price (the forward price) at a predetermined date (the settlement date).
   b. The asset underlying a futures contract is often referred to as the “underlying” and its current price is referred to as the “spot” price.
   c. The buyer of the forward contract agrees today to buy the asset on the settlement date at the forward price.
   d. The seller agrees today to sell the asset at that price on that date.

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</tr>
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<tr>
<td>Long 1 Forward Contract on the Stock with delivery at T at a forward price $F_T(0)$</td>
<td>$S(T)&lt;F_T(0)$</td>
<td>$S(T)&gt;F_T(0)$</td>
</tr>
<tr>
<td></td>
<td>$S(T)-F_T(0)$</td>
<td>$S(T)-F_T(0)$</td>
</tr>
</tbody>
</table>
e. No money changes hands until the settlement date. In fact, the forward price is set so that neither party needs to be paid any money today to enter into the agreement.

f. Can see that the sum of the buyer’s and the seller’s payoff is zero.

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<td>S(T)&lt;F_T(0)</td>
<td>S(T)\geq F_T(0)</td>
</tr>
<tr>
<td>Long 1 Forward Contract on the Stock with delivery at T at a forward price F_T(0)</td>
<td>0</td>
<td>S(T)-F_T(0)</td>
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<tr>
<td>Short 1 Forward Contract on the Stock with delivery at T at a forward price F_T(0)</td>
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<td>F_T(0)-S(T)</td>
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