Problem Set Questions

Problem Set 1: Time Value of Money and Equity Markets.
I-III can be started after Lecture 1.
IV-VI can be started after Lecture 2.
VII can be started after Lecture 3.
VIII and IX can be started after Lecture 5.

I.  Present Value with Multiple Cash Flows: You have just joined the investment banking firm of Godel, Esher, and Bock. They’ve offered you two different salary packages. You can have $40,000 at the end of each year for the next two years, or you can have $20,000 at the end of each year for the next two years along with a $30,000 signing bonus today. If the interest rate available to you is 16% compounded quarterly, which package do you prefer?

II.  Calculating EAR and continuous compounding: Find the EAR in each of the cases below.

<table>
<thead>
<tr>
<th>Stated Rate (APR)</th>
<th>Number of Times Compounded</th>
</tr>
</thead>
<tbody>
<tr>
<td>4%</td>
<td>Semiannually</td>
</tr>
<tr>
<td>6%</td>
<td>Quarterly</td>
</tr>
<tr>
<td>18%</td>
<td>Daily</td>
</tr>
<tr>
<td>22%</td>
<td>Infinite</td>
</tr>
</tbody>
</table>

III.  EAR vs APR: Jiffy’s Pawn Shop charges a rate of 20% per month on loans. Pawnbrokers are legally required to report an APR. What rate should Jiffy report? What is the EAR being charged by Jiffy?

IV.  Calculating Annuity Payments: Your friend Tom is celebrating his 35th birthday today and wants to start saving for his anticipated retirement at age 65. He wants to be able to withdraw $10,000 from his saving account on each birthday for 10 years following his retirement; the first withdrawal will be on his 66th birthday. Tom intends to invest his money in the local savings and loan, which offers 8% interest per year (EAR). He wants to make equal, annual payments on each birthday in a new savings account he will establish for his retirement fund.

A. If Tom starts making these deposits on his 36th birthday and continues to make deposits until he is 65 (the last deposit will be on his 65th birthday), what amount must he deposit annually to be able to make the desired withdrawals on retirement?
B. Suppose Tom has just inherited a large sum of money. Rather than making equal payments, he has decided to make one lump-sum payment on his 36th birthday to cover his retirement needs. What amount would he have to deposit?

V. **Loan Amortization:** Consider a 20 year $90000 mortgage loan with monthly payments. Assume an APR of 9% compounded monthly. The $90000 is lent today and the first payment is in one month's time.
   A. What is the size of the monthly payments?
   B. What is the balance of the loan outstanding in 10 years from now (just after the 120th payment)?
   C. What interest accrues on the loan during the month just after the 119th payment?

VI. **Deferred Annuities:** Consider a single premium deferred annuity (SPDA) which costs $28765.5 and promises yearly payments of $20000 every year beginning 21 years from now. If the advertised EAR for the SPDA is 9%, how many payments must the SPDA make? (use trial and error if you cannot solve it with algebra)

VII. **The Limit-order Book of the NYSE Specialist:** BKM, Chapter 3, Question 7, parts a. and b.

VIII. **Buying Stock on Margin:** BKM, Chapter 3, Question 15 (for part b. assume that interest on the broker’s loan does not accrue until the end of the year).

IX. **Short-selling when the Stock Pays a Dividend:** BKM, Chapter 3, Question 18 (assume the interest rate on any required margin for the short position is zero).
Problem Set 2: Portfolio Management and the CAPM
I.A-I.C and II.A can be started after Lecture 6.
I.D-I.F and II.B-II.E can be started after Lecture 7.
I.G and III can be started after Lecture 8.
IV and V can be started after Lecture 11.

I. Expected Return, Return Standard Deviation, Covariance and Portfolios:

<table>
<thead>
<tr>
<th>State</th>
<th>Probability</th>
<th>Asset A</th>
<th>Asset B</th>
<th>Riskless Asset</th>
</tr>
</thead>
<tbody>
<tr>
<td>Boom</td>
<td>0.25</td>
<td>24%</td>
<td>14%</td>
<td>7%</td>
</tr>
<tr>
<td>Normal Growth</td>
<td>0.5</td>
<td>18%</td>
<td>9%</td>
<td>7%</td>
</tr>
<tr>
<td>Recession</td>
<td>0.25</td>
<td>2%</td>
<td>5%</td>
<td>7%</td>
</tr>
</tbody>
</table>

A. What is the expected return on each asset?
B. What is the standard deviation of return on each asset?
C. What is the correlation and covariance between the returns on
   1. assets A and B?
   2. asset A and the riskless asset?
   3. asset B and the riskless asset?
D. What is the expected return and standard deviation of return of a portfolio consisting of $\omega\%$ invested in asset A and $(1-\omega)\%$ in the riskless asset when $\omega\%$ is
   1. -20%?
   2. 60%?
   3. 120%?
E. What is the expected return and standard deviation of return of a portfolio consisting of $\omega\%$ invested in asset B and $(1-\omega)\%$ in the riskless asset when $\omega\%$ is
   1. -20%?
   2. 60%?
   3. 120%?
F. If a risk-averse investor has to decide whether to hold either asset A with the riskless asset or asset B with the riskless asset, which asset would the investor prefer to hold in combination with the riskless asset? Explain why? Do you need more information about the investor’s preferences to answer the question?
G. What is the expected return and standard deviation of return of a portfolio consisting of $\omega\%$ invested in asset A and $(1-\omega)\%$ in asset B when $\omega\%$ is
   1. -20%?
   2. 80%?
   3. 120%?
II. Using Dividend Yield Information: Suppose the following data is to be used by Ms Q (a risk-averse investor) to form a portfolio that consists of the small firm fund and T-bills.

\[
\begin{align*}
E[R_{\text{Small}}(t)] & = 1.369 \\
\sigma[R_{\text{Small}}(t)] & = 8.779 \\
E[DP(\text{start } t)] & = 4.446 \\
\sigma[DP(\text{start } t)] & = 1.513 \\
\sigma[DP(\text{start } t), R_{\text{Small}}(t)] & = 1.967
\end{align*}
\]

where \(DP(\text{start } t)\) is the dividend yield on the S&P 500 known at the start of month \(t\).
\(R_{\text{Small}}(t)\) is the return on the small firm fund in month \(t\).

A. What is the intercept and slope coefficients from a regression of \(R_{\text{Small}}(t)\) (dependent variable) on \(DP(\text{start } t)\)?

Note that the standard deviation of the residual from the regression of \(R_{\text{Small}}(t)\) on \(DP(\text{start } t)\) is 8.682.

B. Suppose it is the end of March 1997, Ms Q does not know \(DP\) and the return on T-bills for April is 0.3%.
1. What is the expected April return on the small firm fund?
2. Will Ms Q short sell the small firm fund?
3. Will Ms Q buy the small firm fund on margin?
4. Will Ms Q buy a positive amount of both assets?

C. Suppose it is the end of March 1997, Ms Q knows that \(DP\) is 2 and the return on T-bills for April is 0.3%.
1. What is the expected April return on the small firm fund?
2. Will Ms Q short sell the small firm fund?
3. Will Ms Q buy the small firm fund on margin?
4. Will Ms Q buy a positive amount of both assets?

D. Suppose it is the end of October 1997, Ms Q does not know \(DP\) and the return on T-bills for November is 0.4%.
1. What is the expected November return on the small firm fund?
2. Will Ms Q short sell the small firm fund?
3. Will Ms Q buy the small firm fund on margin?
4. Will Ms Q buy a positive amount of both assets?

E. Suppose it is the end of October 1997, Ms Q knows that \(DP\) is 5 and the return on T-bills for November is 0.4%.
1. What is the expected November return on the small firm fund?
2. Will Ms Q short sell the small firm fund?
3. Will Ms Q buy the small firm fund on margin?
4. Will Ms Q buy a positive amount of both assets?

III. The Two Risky Asset Case: A pension fund manager is considering three mutual funds.
Problem Set Questions

The first is a stock fund (S), the second is a long-term government and corporate bond fund (B), and the third is a T-bill money market fund that yields a rate of 9%. The probability distribution of the risky funds can be characterized as follows:

<table>
<thead>
<tr>
<th></th>
<th>Expected Return</th>
<th>Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stock Fund (S)</td>
<td>22%</td>
<td>32%</td>
</tr>
<tr>
<td>Bond Fund (B)</td>
<td>13%</td>
<td>23%</td>
</tr>
</tbody>
</table>

The correlation between the fund returns is 0.15.

A. Tabulate and draw the investment opportunity set of the two risky funds. Use investment proportions for the stock fund of zero to 100% in 20% increments.

B. Draw a tangent from the riskfree rate to the opportunity set. What does your graph show for the expected return and standard deviation of the optimal portfolio?

C. Solve numerically for the proportions of each asset, and for the expected return and standard deviation of the optimal risky portfolio.

D. What is the reward-to-variability ratio of the best feasible capital allocation line (CAL)?

E. You require that your portfolio yield an expected return of 15% and that it is efficient on the best feasible CAL.
   1. What is the standard deviation of your portfolio?
   2. What is the proportion invested in the T-bill fund and each of the two risky funds?

F. If you were to use only the two risky funds, and still require an expected return of 15%, what must be the investment proportions of your portfolio? Compare its standard deviation to that of the optimized portfolio in part E. What do you conclude?

IV. SML and the CAPM:

A. In a CAPM world, what is the beta (with respect to the market portfolio M) of a portfolio with \( \hat{E}[R_{p}] = 20\% \), if \( R_f = 5\% \) and \( \hat{E}[R_M] = 15\% \)?

B. In 1994, the rate of return on short-term government securities (perceived to be riskfree) was about 4%. Suppose the expected rate of return required by the market for a portfolio with a beta measure of 1 is 12%. According to the CAPM (SML):
   1. What is the expected rate of return on the market portfolio?
   2. What would be the expected rate of return on stock with a beta of 0?
   3. Suppose you consider buying a share of stock at $40. The stock is expected to pay $3 dividends next year and you expect it to sell then for $41. The stock’s systematic risk has been evaluated to be \( \beta = -0.5 \). Is the stock over or under priced?
V. **SML vs CML in the CAPM:** Assume that the CAPM holds in the economy. The following data is available about the market portfolio, the riskless rate and two assets, A and B. Remember $\beta_{i,m} = \frac{\sigma[R_i, R_m]}{\sigma[R_m]^2}$.

<table>
<thead>
<tr>
<th>Asset i</th>
<th>$E[R_i]$</th>
<th>$\sigma[R_i]$</th>
<th>$\beta_{i,m}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>m (market)</td>
<td>0.15</td>
<td>0.08</td>
<td></td>
</tr>
<tr>
<td>A</td>
<td>0.096</td>
<td>1.2</td>
<td></td>
</tr>
<tr>
<td>B</td>
<td>0.07</td>
<td>0.6</td>
<td></td>
</tr>
</tbody>
</table>

$R_f = 0.10$.

A. What is $\beta_{i,m}$ for $i$ equal to the market portfolio (i.e., $\beta_{m,m}$)?

B. What is the expected return on asset A (i.e., $E[R_A]$)?

C. What is the expected return on asset B (i.e., $E[R_B]$)?

D. Does asset A plot:
   1. on the SML (security market line)?
   2. on the CML (capital market line)?

E. Does asset B plot:
   1. on the SML?
   2. on the CML?

F. Could any investor hold asset A as her entire portfolio?

G. Could any investor hold asset B as her entire portfolio?

H. What is the correlation of asset A with the market portfolio?

I. What is the correlation of asset B with the market portfolio?

J. Can anything be said about the composition of asset A (i.e., what assets make up asset A)?

K. Can anything be said about the composition of asset B?
Problem Set 3: ICAPM, Market Efficiency and Valuation.

I can be started after Lecture 12.
II can be started after Lecture 14.
III and IV can be attempted after Lecture 15.

I. ICAPM. Let TERM(Jan) be the difference in the yield on a long term hi-grade corporate bond and 1 month T-bill at the end of January. Suppose each individual cares about \{E[R_p(Jan)], \sigma[R_p(Jan)], \sigma[R_p(Jan), TERM(Jan)]\} when forming his/her portfolio p for January. The following additional information is available:

<table>
<thead>
<tr>
<th>i</th>
<th>E[R_i(Jan)]</th>
<th>\beta^*_{i,M}</th>
<th>\beta^*_{i,TERM}</th>
</tr>
</thead>
<tbody>
<tr>
<td>Red</td>
<td>18%</td>
<td>1.4</td>
<td>0.4</td>
</tr>
<tr>
<td>Yellow</td>
<td>1.1</td>
<td>1.2</td>
<td></td>
</tr>
</tbody>
</table>

where \( \beta^*_{i,M} \) and \( \beta^*_{i,TERM} \) are regression coefficients from a regression of \( R_i(Jan) \) on \( R_M(Jan) \) and \( TERM(Jan) \):

\[
R_i = \varphi_{i,0} + \beta^*_{i,M} R_M + \beta^*_{i,TERM} TERM(Jan) + e_i
\]

Also know that \( E[R_M(Jan)] = 14\% \) and \( R_f(Jan) = 8\% \).

A. What is the expected January return for Yellow?
B. What is the risk premium for bearing TERM risk?
C. Is the market on the minimum variance frontier? Why or why not?
D. Give one reason why an individual may care about the covariance of her portfolio return with TERM(Jan).

II. Market Efficiency. Suppose it is possible to use IBM’s annual report after its release date to detect mispricing of IBM’s stock.

A. Does this indicate that the market is weak form inefficient? Why or why not?
B. Does this indicate that the market is semi strong form inefficient? Why or why not?
C. Does this indicate that the market is strong form inefficient? Why or why not?

III. Equity Valuation, the Dividend Discount Model and ROE.

A. MF Corp has an ROE of 16\%, a plowback ratio of 50\% and no debt. The market requires a return on MF Corp of 12\% and the coming year’s earnings are expected to be $2 per share.

1. At what price will the stock sell today?
2. What price do you expect MF shares to sell for in three years?

B. Assume:

\$20 = price of the ZW stock today
8\% = the expected growth rate of ZW dividends

\$0.60 = the annual ZW dividend one year forward
Problem Set Questions

Using only this data and the constant growth dividend discount model, compute the expected long-term total return of ZW stock?

C. The FI Corporation’s dividends per share are expected to grow indefinitely by 5% per year.
   1. If this year’s year-end dividend is $8 and the market requires a 10% return per annum on FI stock, what must the current stock price be according to the DDM?
   2. If the expected earnings per share are $12, what is the implied value of the ROE on future investment opportunities?
   3. How much is the market paying per share for growth opportunities (i.e., for an ROE on future investments that exceeds the market’s required return on equity)?

IV. Equity Valuation, Asset Composition and Leverage. A large computer manufacturer IBX has $1M of riskless debt and assets with a market value of $6M. The assets of IBX have a Beta of 1.3. Assume the CAPM holds. The riskless rate is 10% and the expected return on the market is 18%. IBX has just paid a dividend of $3.76 per share and IBX’s dividend per share is expected to grow at 6.48% per year.
   A. What was price of IBX stock on the exdate?
   B. How many shares of IBX stock were outstanding on the exdate?
Problem Set 4: Fixed Income Valuation.
I.A and II.A can be started after Lecture 18.
I.B-I.E and II.B-II.C can be started after Lecture 19.
III and IV can be started after Lecture 20.

I. **Implied Yield Curve, Forward Rates and No Arbitrage:** Consider the following prices for U.S. treasury notes on 2/15/96.

<table>
<thead>
<tr>
<th>Rate</th>
<th>Maturity</th>
<th>Price</th>
</tr>
</thead>
<tbody>
<tr>
<td>4½</td>
<td>Aug 96</td>
<td>98:11</td>
</tr>
<tr>
<td>5¼</td>
<td>Feb 97</td>
<td>99:01</td>
</tr>
<tr>
<td>5¾</td>
<td>Aug 97</td>
<td>98:23</td>
</tr>
<tr>
<td>6</td>
<td>Feb 98</td>
<td>98:15</td>
</tr>
</tbody>
</table>

A. What is the implied yield curve (expressed in terms of APRs with semiannual compounding)?

B. What are the implied forward rates for the 6 month periods starting in 6 months, in 1 year and in 18 months (expressed as APRs with semiannual compounding)?

C. If there are no arbitrage opportunities, what is the price of a Aug 97 U.S. Treasury strip?

D. Suppose the price of a Feb 97 U.S. Treasury strip is 94. Is there an arbitrage opportunity? If so, describe a strategy which earns an arbitrage profit.

E. Suppose the prices for U.S. Treasury notes on 8/15/96 are given by:

<table>
<thead>
<tr>
<th>Rate</th>
<th>Maturity</th>
<th>Price</th>
</tr>
</thead>
<tbody>
<tr>
<td>5¼</td>
<td>Feb 97</td>
<td>98:11</td>
</tr>
<tr>
<td>5¾</td>
<td>Aug 97</td>
<td>98:21</td>
</tr>
<tr>
<td>6</td>
<td>Feb 98</td>
<td>98:00</td>
</tr>
</tbody>
</table>

1. What is the return from holding the Aug 97 note from 2/15/96 to 8/15/96?
2. What is the return from holding the Aug 96 note from 2/15/96 to 8/15/96?
3. Calculate the implied yield curve (expressed in terms of APRs with semiannual compounding)?
4. Consider 2/15/96's forward rate for the period 8/15/96 to 2/15/97. How does it compare to the 6 month interest rate on 8/15/96? If these two rates differ, discuss why.
II.  *Forward Rates and the Yield Curve.* Today is the 2/15/95. The following information is available.

*Government Bonds and Notes.*

<table>
<thead>
<tr>
<th>Rate</th>
<th>Maturity</th>
<th>Price</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>Feb 97</td>
<td>98</td>
</tr>
<tr>
<td>6</td>
<td>Aug 98</td>
<td>98¼</td>
</tr>
</tbody>
</table>

*U.S. Treasury Strips.*

<table>
<thead>
<tr>
<th>Type</th>
<th>Maturity</th>
<th>Ask Price</th>
</tr>
</thead>
<tbody>
<tr>
<td>ci</td>
<td>Aug 95</td>
<td>97¼</td>
</tr>
<tr>
<td>ci</td>
<td>Feb 96</td>
<td>93¼</td>
</tr>
<tr>
<td>ci</td>
<td>Aug 96</td>
<td>90</td>
</tr>
</tbody>
</table>

A. Based on the above information, what is the yield on a 2 year discount bond expressed as APR with semiannual compounding?
B. What is the forward rate available today for the 1 year period starting in 6 months time? Express the rate as an APR with semiannual compounding.
C. What is today’s forward price for an Aug 96 strip with settlement in six months time?

III.  *Duration and Interest Rate Sensitivity:* Consider the 4¾ Feb 96 U.S. Treasury note on the 8/15/94. Suppose this note’s YTM expressed as an APR with semiannual compounding is 5.5%.

A. What is the price of the note?
B. What is its Macaulay duration and “modified duration”?  
C. Suppose the yield curve shifts up to 6%.
   1. What is the price of the note now?
   2. What is the price implied by Macaulay duration?
   3. Discuss any difference between the two prices.
D. Suppose the yield curve shift down to 5%.
   1. What is the price of the note now?
   2. What is the price implied by Macaulay duration?
   3. Discuss any difference between the two prices.

IV.  *Immunization:* Today is 8/15/94. XYZ is a pension fund that must make five payments of $5M per year for the next 5 years (the first payment is made in 1 year on the 8/15/95). XYZ can invest in U.S. Treasury notes (6% Feb 96) and in U.S. Treasury Aug 04 strips. The yield curve is flat and expected to remain flat. Its current level is 6% expressed as an...
Problem Set Questions

Foundations of Finance

APR with semiannual compounding.
A. What is the current value of XYZ’s liability?
B. What is the Macaulay duration of:
   1. XYZ’s liability?
   2. the note?
   3. the strip?
C. For XYZ’s investments to still be able to satisfy its liabilities after a change in the interest rate, how much should XYZ invest in the notes and how much in the strips?
Problem Set Questions

Problem Set 5: Derivatives

I.A and I.B can be attempted after Lecture 22.
I.C-I.E can be attempted after Lecture 24.
II can be attempted after Lecture 26.

I. Options:
A. BKM, Chapter 20, Question 6, part a.
B. BKM, Chapter 20, Question 23.
C. BKM, Chapter 20, Question 4.
D. BKM, Chapter 20, Question 5.
E. BKM, Chapter 20, Question 9, parts a, b, c and e.

II. Futures and Forward Contracts:
A. BKM, Chapter 22, Question 6.
B. It is now January. The current interest rate is 8% EAR. The June futures price for gold is $346.30, whereas the December futures price is $360. Is there an arbitrage opportunity here? If so, how would you exploit it?
C. The S&P portfolio pays a dividend of 2% annually. Its current value is 1020. The T-bill rate is 5% EAR. Suppose the S&P futures price for delivery in one year is 1060. Construct an arbitrage opportunity. Assume the dividend of 2% is paid in one year hence.
D. BKM, Chapter 23, Question 5. For part c, assume that your portfolio consists of 60% in equities and 40% in T-bills.
E. The following formula converts discount bond discount factors to forward contract discount factors (it was introduced earlier in Lecture Note 15-17):
\[ d_{t,t+\tau}(0) = \frac{d_{t+\tau}(0)}{d_t(0)} \]
Is this formula consistent with the spot futures parity theorem? Explain why or why not. (Hint. Think about \( d_{t,t+\tau}(0) \) as referring to the forward price at time 0 for delivery of a \( \tau \)-discount bond at time \( t \).)
F. BKM, Chapter 23, Question 8.