Problem Set I: Foundations of Finance

Foundations of Finance

Prof. Alex Shapiro

Due in class: B01.2311.10 on or before Tuesday, October 7,
B01.2311.11 on or before Wednesday, October 8,
B01.2311.12 on or before Thursday, October 9.

1. BKM Chapter 1, Problems 1, 2, 3, 12, 13.

2. a) Amortization with Equal Payments:
Prepare an amortization schedule for a three-year loan of $6,000. The interest rate is 16 percent per year and the loan calls for equal annual payments. How much interest is paid in the third year? How much total interest is paid over the life of the loan?

b) Amortization with Equal Principal Payments:
Rework part (a) assuming that the loan agreement calls for a principal reduction of $2,000 every year instead of equal annual payments.

c) Calculating a Balloon Payment:
You have just arranged for a $100,000 mortgage to finance the purchase of a large tract of land. The mortgage has a 12 percent APR, and it calls for monthly payments over the next 10 years. However, the loan has a three-year “balloon payment,” meaning that the loan must be paid off then. How big will the balloon payment be?

3. You take a mortgage of $200,000. The terms of the mortgage are the following:
Twenty year mortgage, repaid in equal annual installments, the first installment to be paid one year from now. The mortgage rate is 15%, compounded annually. How much of the first payment is repayment of principal? How much of the tenth payment is repayment of principal?

4. You are trying to sell a $10 Million generator. The customer will require $9 Million of financing, which your company will provide at 11% (compounded annually).
You propose a five-year loan with equal end-of-year payments. What is the payment size?
The customer replies that quarterly payments over five years might be better. What is the quarterly payment?
Finally the customer suggests an unusual payment schedule. She wants to pay three annual payments. The first payment (due one year from today) is in amount \( y \). The second and third payments are \( 2y \) and \( 3y \). What is the first payment?

5. You have borrowed $10,000 from your employer-subsidized credit union at 8%. The loan will be repaid in 10 equal annual payments. You intend to invest the proceeds of the loan in an off-shore bank account paying 9%. All loan payments will be made from the off-shore account. What will be the balance of your off-shore account after the last payment?

6. You currently have $10,000 invested in a bank that pays 8% compounded annually. This rate is expected to be constant for the foreseeable future. You are considering switching some of your funds to a $1,000-par annual bond that carries a 10% coupon rate, matures in five years and is selling in the market for $1,039. (If you buy the bond, you'll receive the first coupon payment one year from now.) The coupon payments can be reinvested in the bank and your investment horizon is at least five years. Should you buy the bond?
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Additional, SUGGESTED, Problems:

You don't have to submit any of the suggested problems to get full credit for problem set I. However, I recommend you seriously attempt to solve the problems BEFORE peeking at the solutions. It is the only way to really understand the material covered in class. If you don’t have time to consider each problem in depth, then choose a few now, and do the remaining before the Midterm. This strategy will benefit you more than the USELESS strategy of reading the problems, and copying the solutions.

S1. What is the present value of $1,000 payments received at the beginning of each year for the next 10 years? Assume an APR of 5.49% compounded monthly.

(a) $7,069.13
(b) $7,093.62
(c) $7,492.64
(d) $7,912.58
(e) $7,955.26

S2. Five years from now you will begin to receive cash flows of $75 per year. These cash flows will continue forever. If the discount rate is 6%, what is the present value of these cash flows?

(a) $799.68
(b) $894.22
(c) $934.07
(d) $990.12
(e) $1,104.67

S3. Four years from now you will receive the first of seven annual $10,000 payments. The current interest rate is 6%, but by the beginning of year 4, the rate will have risen to 8%. What is the present value of this cash flow stream?

(a) $41,827.45
(b) $42,554.49
(c) $43,713.69
(d) $46,864.49
(e) $55,692.45

S4. What continuous rate of return allows you to triple your money in 20 years?

(a) 5.49%
(b) 5.98%
(c) 6.86%
(d) 6.92%
(e) 8.99%
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S5. You are going to invest $500 at the end of each year for ten years. Given an interest rate you can find the future value of this investment by

I. adding the cash flows together and future valuing the sum by the appropriate future value factor
II. applying the proper future value factor to each cash flow, then adding up these values
III. finding the present value of each cash flow, adding all of the present values together, then finding the future value at the end of year ten of this lump sum
IV. finding the present value of the entire payment stream

(a) II only
(b) III only
(c) II and III only
(d) I, II, and IV only
(e) II, III, and IV only

S6. Show that the Price of a coupon bond paying principal amount Par at the end of the last period, and Coupon Payments, C, every period (at the end of the period), where the coupon payments are r×Par, and r is the one-period effective rate, is just the Par. Therefore, if r > C/Par, we get Price < Par, and if r < C/Par, we get Price > Par.

S7. Tom and Sue are trying to buy a house for which they need to take out a 30-year fixed rate mortgage with monthly payments. They need to borrow $150,000.

a. If the APR is currently 7% (compounded monthly), what is the monthly payment?
b. Tom and Sue are worried that interest rates might rise. If the maximum monthly payment they can afford is $1,200, what is the maximum APR that will still allow them to purchase the house?

S8. The Republic of Ruthenia is in trouble. Two years ago, they borrowed $10 Billion at 10% from a consortium of U.S. banks. The loan was to be repaid in ten equal annual payments.

a. What was the initial payment size?
b. Ruthenia has just made the second payment, but the Finance Minister knows that they won’t be able to meet the original terms of the loan. After a hasty negotiation, the banks agree:
   1) The term of the loan will be extended to twelve years from today.
   2) The interest rate will be reset to 9%
   3) There will be twelve annual payments, with the first due at the end of the year
   4) The first two payments will cover interest only.
   5) The next ten payments will be of equal size.

What will the remaining payments be?
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S9. Two years ago, Jill and Jack bought a house with a 30-year annual payment mortgage at an APR of 10% (compounded annually). They have just made the second payment of $15,000.

a. What was the original size of the mortgage? (How much was initially borrowed?)
b. Since interest rates lower, they want to refinance the balance of the mortgage with a new 30-year annual payment mortgage. If the APR is now 8% (compounded annually), what will their new payment be?
c. At the refinancing, the bank offers Jill and Jack a monthly mortgage at an APR of 7.5% (compounded monthly). Is the annual or monthly mortgage the better deal?

S10. To save for the down payment on a new house, a couple is planning to save $1,000 per month. They will deposit the money at the end of the month into a bank account that pays 12% compounded quarterly. How much will they have at the end of four years?

S11. It is now January 1, 2000. A customer walks into the Methuselah Life Insurance Company wants to buy an annuity that will pay a $10,000 annually as long as he lives. The first payment will begin on January 1, 2021. Methuselah’s actuaries expect that the company will have to make 12 payments. The interest rate is currently 9%, Methuselah assumes that it will stay there for the duration of the contract. If the customer buys the annuity, Methuselah will invest the proceeds immediately.

a. What is the actuarially “fair” price of the annuity (i.e., assuming that the actuary’s expectations are met and that the company makes no profit?).
b. Now suppose that the company prices the annuity so that
   1) it can book an immediate profit of 5% of the annuity’s sale price.
   2) it can also pay the sales agent a commission of 5% of the annuity’s sale price.
      Because the company is concerned about customers who pull their money out of the investment early, the 5% is not actually paid to the agent for three years (January 1, 2003).

S12. You will be making 20 annual contributions of $75 to a bank account that pays 5.90% compounded semiannually. What will be the balance of the bank account at the end of year 20?