1. BKM Chapter 1, Problems 1, 2, 3, 12, 13; See Solutions Manual.

2. a) \( C \times APVF(0.16,3) = 6,000 \), implies \( C = 2,671.55 \), therefore:

<table>
<thead>
<tr>
<th>Year</th>
<th>Beginning Balance</th>
<th>Total Payment</th>
<th>Interest Paid</th>
<th>Principal Paid</th>
<th>Ending Balance</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>6,000.00</td>
<td>2,671.55</td>
<td>960.00</td>
<td>1,711.55</td>
<td>4,288.45</td>
</tr>
<tr>
<td>2</td>
<td>4,288.45</td>
<td>2,671.55</td>
<td>686.15</td>
<td>1,985.39</td>
<td>2,303.06</td>
</tr>
<tr>
<td>3</td>
<td>2,303.06</td>
<td>2,671.55</td>
<td>368.49</td>
<td>2,303.06</td>
<td>0.00</td>
</tr>
</tbody>
</table>

b) The loan is amortized with equal principal payments of $2,000:

<table>
<thead>
<tr>
<th>Year</th>
<th>Beginning Balance</th>
<th>Total Payment</th>
<th>Interest Paid</th>
<th>Principal Paid</th>
<th>Ending Balance</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>6,000.00</td>
<td>2,960.00</td>
<td>960.00</td>
<td>2,000.00</td>
<td>4,000.00</td>
</tr>
<tr>
<td>2</td>
<td>4,000.00</td>
<td>2,640.00</td>
<td>640.00</td>
<td>2,000.00</td>
<td>2,000.00</td>
</tr>
<tr>
<td>3</td>
<td>2,000.00</td>
<td>2,320.00</td>
<td>320.00</td>
<td>2,000.00</td>
<td>0.00</td>
</tr>
</tbody>
</table>

c) \( C \times APVF(0.01,120) = 100,000 \), implies \( C = 1,434.71 \).

Balloon payment = \( C \times APVF(0.01,84) = 81,274.07 \)

3. You take a mortgage of $200,000. The terms of the mortgage are the following:

Twenty year mortgage, repaid in equal annual installments, the first installment to be paid one year from now. The mortgage rate is 15%, compounded annually. How much of the first payment is repayment of principal?

\[ C \times APVF(0.15,20) = 200,000.00, \text{ implies } C = 31,952.29 \]

In the first payment, the amount of principal, PRPL₁, is:

\[ PRPL₁ = C - 200,000 \times 0.15 = 1,952.29 \]

How much of the tenth payment is repayment of principal?

For the tenth payment, you can construct an amortization table. Then, we know that After the 9th payment the balance is \( C \times APVF(0.15,11) \), so

\[ C - C \times APVF(0.15,11) \times 0.15 = C \times (1 - 5.2337 \times 0.15) = 6,867.93 \]
But it is worthwhile to see another way to solve this. Note that the amount of principal paid in period $n$, $PRPL_n$, is simply given by

$$PRPL_n = PRPL_1 (1 + r)^{n-1}$$

Why? Because if the last payment is made at the end of period $t$, the outstanding balance at the end of period $n-1$ is

$$C \times APVF(r, t-n+1) = C \left(1 - \frac{1}{(1+r)^{t-n+1}}\right)$$

So,

$$PRPL_n = C - \left\{ C \times APVF(r, t-n+1) \right\} r = C \left(1 - \frac{1}{(1+r)^{t-n+1}}\right)$$

This means that, in particular,

$$PRPL_1 = C / (1+r)^t$$

And therefore, $PRPL_n = PRPL_1 (1 + r)^{n-1}$

Hence, the amount of principal in the tenth payment is

$$PRPL_{10} = PRPL_1 (1 + 0.15)^9 = \$6,867.93$$

which represents 21.5% of the tenth annual payment.

4. You are trying to sell a $10 Million generator. The customer will require $9 Million of financing, which your company will provide at 11% (compounded annually). You propose a five-year loan with equal end-of-year payments. What is the payment size?

$$C \times APVF(0.11,5) = \$9MM, \text{ implies } C = \$2.435MM$$

The customer replies that quarterly payments over five years might be better. What is the quarterly payment?

11% compounded annually is equivalent to an APR of 10.57% compounded quarterly.

$$C \times APVF(0.0264,20) = \$9MM, \text{ implies } C = \$0.585MM$$

Finally the customer suggests an unusual payment schedule. She wants to pay three annual payments. The first payment (due one year from today) is in amount $y$. The second and third payments are $2y$ and $3y$. What is the first payment?

$$PV(y, 11\%, 1) + PV(2y, 11\%, 2) + PV(3y, 11\%, 3)$$

$$= y \times PV(1, 11\%, 1) + 2y \times PV(1, 11\%, 2) + 3y \times PV(1, 11\%, 3)$$

$$= y \times 0.901 + 2y \times 0.812 + 3y \times 0.731$$

$$= 4.718y$$

$$= 9$$

Hence, $y = 1.908MM$
5. You have borrowed $10,000 from your employer-subsidized credit union at 8%. The loan will be repaid in 10 equal annual payments. You intend to invest the proceeds of the loan in an off-shore bank account paying 9%. All loan payments will be made from the off-shore account. What will be the balance of your off-shore account after the last payment?

\[ C \times \text{APVF}(0.08,10) = 10,000, \text{ implies } C = 1,490.3 \]

The cash flows to/from the off-shore account are: a deposit of $10,000 at time 0, and then withdrawals of $1,490.3 in periods 1 through 10.

So, the FV of these cash flows at 9% is

\[ 10,000 \times \text{FVIF}(0.09,10) – 1,490.3 \times \text{AFVF}(0.09,10) = 23,673.63 – 22,641.94 = 1031.7 \]

6. You currently have $10,000 invested in a bank that pays 8% compounded annually. This rate is expected to be constant for the foreseeable future. You are considering switching some of your funds to a $1,000-par annual bond that carries a 10% coupon rate, matures in five years and is selling in the market for $1,039. (If you buy the bond, you'll receive the first coupon payment one year from now.) The coupon payments can be reinvested in the bank and your investment horizon is at least five years. Should you buy the bond?

If you buy the bond and reinvest the proceeds, then at the end of five years you will have

\[ 100 \times \text{AFVF}(0.08,5) + 1,000 = 1586.67 \]

If you keep the 1,039 in the bank, you will have

\[ 1,039 \times \text{FVIF}(0.08,5) = 1526.63 \]

Clearly, it is better to buy the bond.
Answers to Suggested Problems

These answers show you the calculations only, and you should be able to follow these without additional explanations. If you are having difficulties to understand the solution after reviewing again the Lecture Notes and RWJ, then make sure you approach the TAs or me.

S1. (d) 5.49/12 = 0.4575%
PV = 1,000 + 1,000 APVF( [1.004575]^{12} - 1, 9) = 1000 + 6,912.58 = 7,912.58

S2. (d) PV = (75/0.06) / (1.06)^4 = 990.1171

S3. (c) PV = 10,000 APVF(0.08,7) / (1.06)^3 = [10,000 (1-0.08^{-7}) / 0.08 ] / (1.06)^3
= 52,063.7 / (1.06)^3 = 43,713.69

S4. (a) e^{p_{20}} = 3. So, p_{20} = \ln(3), \rho = \ln(3)/20 = 0.05493. Then, r = e^{\rho} - 1 = 0.0565.

S5. (c)

S6. Price = C \frac{1 - \frac{1}{(1 + r)^i}}{r} + \frac{Par}{(1 + r)^i} = Par \times r \frac{1 - \frac{1}{(1 + r)^i}}{r} + \frac{Par}{(1 + r)^i}

= Par - \frac{Par}{(1 + r)^i} + \frac{Par}{(1 + r)^i} = Par

When \ r > \ C/Par, then \ C < Par \times r, and the present value of coupons is lower than above, so \ Price < Par.

An Application of this result

In the Lecture Notes on Money Market and Debt Instruments, we see that for Long Term Treasuries, the Yield to Maturity (YTM) is given by the Ask Yield/2, where the Ask Yield is reported in the WSJ. Since the YTM is our effective rate \ r, YTM=r, which in this context is a semi-annual (s.a.) rate with coupons paid semi-annually, we have that Ask Yield = 2r. Also from the Lecture Notes we see that the Coupon Rate is defined as 2C/Par, with coupons paid semi-annually. Therefore, from what we derived above in this problem, we know that:

if Ask Yield >Coupon Rate (i.e., s.a. YTM > s.a. Coupon Rate) then Price<Par; i.e., the bond is selling at a discount relative to par.

if Ask Yield <Coupon Rate (i.e., s.a. YTM < s.a. Coupon Rate) then Price>Par; i.e., the bond is selling at a premium relative to par.
Example: Consider the WSJ quotes

<table>
<thead>
<tr>
<th>Rate</th>
<th>Bid</th>
<th>Asked</th>
<th>Ask Yld.</th>
</tr>
</thead>
<tbody>
<tr>
<td>4 1/2</td>
<td>99:27</td>
<td>99:28</td>
<td>4.57</td>
</tr>
<tr>
<td>5 1/4</td>
<td>101:06</td>
<td>101:08</td>
<td>4.59</td>
</tr>
</tbody>
</table>

For the 4 1/2 Coupon Rate note, Ask Yield > Coupon Rate and indeed Price<Par. For 5 1/4 Coupon Rate, Ask Yield <Coupon Rate and indeed Price>Par.

This shows that the result in this problem is not a pure mathematical exercise, but it is useful to understanding price quotes in the bond market.

S7. a. 150,000 = APVF(7 % / 12, 360) z. So z = 997.95
b. 150,000 = APVF(r % / 12, 360)1200. So r = 8.94%

S8. a. 10 = APVF(10%, 10) p. So p = 1.628/year
b. First, figure out how much Ruthenia owes. (Construct the amortization schedule.)

<table>
<thead>
<tr>
<th></th>
<th>Payment 1</th>
<th>Payment 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Balance at beg of yr.</td>
<td>10.000</td>
<td>9.372</td>
</tr>
<tr>
<td>10% interest</td>
<td>1.000</td>
<td>0.937</td>
</tr>
<tr>
<td>Amortization</td>
<td>0.628</td>
<td>0.691</td>
</tr>
<tr>
<td>Payment</td>
<td>1.628</td>
<td>1.628</td>
</tr>
<tr>
<td>Balance at end of yr.</td>
<td>9.372</td>
<td>8.681</td>
</tr>
</tbody>
</table>

The first two payments will only cover 9% interest. They are 9%(8.681)=0.781. After that, it’s a regular loan, with payment size p, 8.681=APVF(9%,10) p. p=1.353.

S9. a. APVF(10%, 30) 15,000 = 141,404
b. What do they currently owe? (Construct the amortization schedule.)

<table>
<thead>
<tr>
<th></th>
<th>Payment 1</th>
<th>Payment 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Balance at beg of yr.</td>
<td>141,404</td>
<td>140,544</td>
</tr>
<tr>
<td>Int</td>
<td>14,140</td>
<td>14,054</td>
</tr>
<tr>
<td>Amort</td>
<td>860</td>
<td>946</td>
</tr>
<tr>
<td>Payment</td>
<td>15,000</td>
<td>15,000</td>
</tr>
<tr>
<td>Balance at end of yr.</td>
<td>140,544</td>
<td>139,598</td>
</tr>
</tbody>
</table>

Their new payment will be p, where 139,598 = APVF(8%, 30)p: p=12,400.
c. The EAR of 7.5% compounded monthly is 7.76%<8%, so the monthly mortgage is cheaper.

S10. The EAR of 12% compounded quarterly is 12.55%. The monthly compounded rate that has an EAR of 12.55% is 11.88%.
The amount in the bank will be AFVF(11.88%/12, 48) 1,000= 61,069.
S11. The time line is:

\[
\begin{array}{cccccccc}
0 & \cdots & 20 & 21 & \cdots & 32 \\
2000 & \cdots & 2020 & 2021 & \cdots & 2032 \\
10,000 & \cdots & 10,000 & & & \\
\end{array}
\]

a. At time 20, the PV of the annuity obligations is \( \text{APVF}(9\%, 12) \times 10,000 = 71,607 \). To fund this obligation at time 0 requires \( \text{PVIF}(9\%, 20) \times 71,607 = 12,777 \).

b. If the annuity price is \( X \), then
\[
X - 5\%X - \text{PVIF}(9\%, 3) \times (5\%X) = 12,777
\]
i.e., after subtracting off the profit and the PV of the commission, we must have enough to fund the obligation.
\[
X - 5\%X - 5\%X/(1.09)^3 = 12,777
\]
\[
0.9114X = 12,777, \text{ i.e., } X = 14,019
\]

S12. First, we need an EAR. What annually-compounded rate is equivalent to 5.90% compounded semiannually? \( FVIF(5.90\%/2, 2) = FV(r\%, 1) \Rightarrow r = 5.99\% \). Then, the required future value is \( FV = AFVF(5.99\%, 20) \times 75 = 2,755.96 \).