Foundations of Finance

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Due in class on or before the day of the Midterm Exam.

1. With $10,000 cash, you wish to sell short XYZ stock that is currently priced at $50 per share.
   a. If the initial margin is 50%, what is the maximum number of shares that you can short?
   b. Assume that you short the maximum and hold the position over a year, at the end of which time you close it out. What is your rate of return assuming:
      Cash balances (including short sale proceeds) invested with your broker earn interest at a rate of 4%.
      On the day before you repurchased your shares, XYZ went ex dividend ($1 per share).
      You repurchase your shares at a share price of $53.

2. Consider the following data for two stocks.

   State of the Economy | Probability | Return on 1 | Return on 2
   ---------------------|-------------|-------------|-------------
   Bad                  | 1/3         | 0.05        | -0.05       
   Average              | 1/3         | 0.05        | 0.18        
   Good                 | 1/3         | 0.15        | 0.18        

   a. What are the expected return and standard deviation on stock 2?
   b. What is the correlation between 1 and 2?
   c. Portfolio $P$ is 1/3 in stock 1 and 2/3 in stock 2. What are the expected return and standard deviation of $P$?
   d. Our combined portfolio ($10,000 net worth) is divided 40% in the T-bills (currently paying 5%) and 60% in the portfolio $P$ described in the last question. What are the expected return and standard deviation of our combined portfolio?

3. BKM Chapter 7, Problems 1-6.

4. BKM Chapter 8, Problems 1-7.
5. Refer to the Lecture Notes on Uncertainty and the Lecture Notes on Optimal Risky Portfolios for the data to use in this problem:
Assuming you have Mean-Variance preferences, suppose you form a portfolio of the Small Firm “asset,” Microsoft, and T-bills, with the T-bill, with \( r_{\text{T-bill}} = 0.323\% \). What are the preferred weights of the two risky assets in the risky portfolio? What are the preferred weights of the risky portfolio \( T \) and the riskless asset in the individual's portfolio? Suppose you want to invest 75\% in the tangency portfolio \( T \) (denoted by \( \star \) in the plots) and 25\% in T-bills. What is the weight of the small firm asset and of Microsoft in your total portfolio?

6. BKM Chapter 10, Problems 1-3, 4(a,b,c,e) [4(d) after covering ch. 9]

7. BKM Chapter 9, Problems 1, 6-12, 17

8. You are given the following information: \( r_f = 9\% \), \( r_M = 12\% \), \( \sigma_M = 3\% \).
Use the Capital Asset Pricing Model to answer the following questions. Assume the market is in equilibrium.
   a. If a security has a covariance with the market of \( 0.00045 \), What is the beta of the security?
   b. What is the expected return on the above security?
   c. If a security has an expected return of 10\%, what is its beta? What is its covariance with the market?
   d. What is the beta of a security with an expected return of 15\%? What is the covariance of this security with the market?

9. The two assets below are assumed to be correctly priced according to the CAPM:
   Asset A: \( r_A = 8\% \) \( \beta_A = .40 \) \( \sigma_A = 6\% \)
   Asset B: \( r_B = 16\% \) \( \beta_B = 1.20 \)
Answer the following questions:
   a. What is the expected return on the market portfolio
   b. What is the Beta of the market portfolio?
   c. What is the total risk of the market portfolio given that the covariance of asset A with the market is equal to \( 0.00225 \)?
   d. What is the expected return on asset C given that it is perfectly correlated with asset A and that its total risk is 11.25\%?
   e. What is the total risk of asset B given that its correlation coefficient with the market portfolio is \( 0.9 \)?
   f. Look at the relationship between the expected return and the total risk of asset A, asset B, asset C and the market portfolio. What can you conclude, particularly in comparison to the relationship between expected return and systematic risk for these same assets?
Problem Set II: Foundations of Finance

Additional, SUGGESTED, Problems

S1. (Future Value/Money Market) A T-bill is listed as:

<table>
<thead>
<tr>
<th>Number of days to maturity</th>
<th>Bid</th>
<th>Ask</th>
</tr>
</thead>
<tbody>
<tr>
<td>150</td>
<td>5.95</td>
<td>5.80</td>
</tr>
</tbody>
</table>

The 150 days is “n”, i.e., it is net of settlement delays. We own this bill with a $100,000 par value. We also have $100,000 in a bank that is currently quoting a rate of 5.95% compounded quarterly. If we don’t need the funds until maturity should we
(a) sell the T-bill and put the proceeds in the bank?
(b) take some of our money out of the bank and buy another T-bill?
(c) do nothing?

S2. (Portfolio theory) Suppose that we have two stocks with the following characteristics:
\[ E_{r1}=10\%, \sigma_{1}=10\%, E_{r2}=20\%, \sigma_{2}=15\%, \rho=0.2. \]
A risk-free return of 5% is available in T-bills. A customer walks into our brokerage firm with all of his $10,000 net worth invested in stock 1.
(a) Is there any portfolio (consisting of stocks 1, 2 and the risk-free security) that we can put him in that will make him better off? What is it? (No short selling of stock is allowed. You don’t need to find the best portfolio, just a better portfolio.)
(b) When he looks us straight in the eye and asks us, “One year from now, am I going to regret following your advice?”, can we assure him “No regrets, guaranteed.”

S3. (PortfolioTheory/Index Models/CAPM) Using 10 past years of annual data, the market model has been estimated for stocks A and B with the following results:
\[
R_A = 0.01 + 0.8 R_M + \epsilon_A \\
R_B = 0.02 + 1.2 R_M + \epsilon_B \\
\sigma_M = 0.20, \sigma(\epsilon_A) = 0.20, \sigma(\epsilon_B) = 0.10
\]
(R’s are excess returns, e.g., \( R_A = r_{A} - r_f \). The return units are annual returns, e.g., “0.20” means 20%)
(a) What does the market model predict for the values of: \( \sigma_A, \sigma_B, \sigma_{A,B}, \rho_{A,B} \)
(b) Suppose we construct a portfolio that has weights: \( w_A = \frac{1}{2}, w_B = \frac{1}{4}, w_f (\text{risk-free}) = \frac{1}{4} \). What is the risk of this portfolio?

Suppose that the current risk-free rate is 5% and that as an estimate for the expected market risk premium \( R_M = r_M - r_f \), we use a long-run historical average of 8%.
(c) Using the market model’s estimate of \( \beta_A \), what does the capital asset pricing model predict \( E_{r_A} \) should be?
(d) According to the market model, what is the implied value of \( E_{r_A} \)?
(e) If you find that the two values of \( E_{r_A} \) agree, comment on the reasons for this agreement.
   If you find that the two values of \( E_{r_A} \) disagree, comment on the reasons for this disagreement.
Problem Set II: Foundations of Finance

S4. (Portfolio Theory/Efficient Diversification) Use the Portfolio Optimizer Program (distributed by e-mail) to analyze the opportunity set with 5 risky assets from those mentioned in the Lecture Notes, when combined with a riskless asset that yields 0.5%. Make a copy of the Portfolio Optimizer Program (POP), so that you can always retrieve the original settings, because you will modify the spreadsheet as you work on this problem.

a) Consider the following monthly correlations, means, and standard deviations of five firms, based on their return performance over several years, and assume these are good estimates for the actual moments describing future returns:

<table>
<thead>
<tr>
<th></th>
<th>IBM</th>
<th>Apple</th>
<th>Microsoft</th>
<th>Nike</th>
<th>ADM</th>
<th>E [r] %</th>
<th>σ[r] %</th>
</tr>
</thead>
<tbody>
<tr>
<td>IBM</td>
<td>1.000</td>
<td>0.226</td>
<td>0.320</td>
<td>0.145</td>
<td>-0.093</td>
<td>0.246</td>
<td>8.004</td>
</tr>
<tr>
<td>Apple</td>
<td>0.226</td>
<td>1.000</td>
<td>0.441</td>
<td>0.321</td>
<td>0.243</td>
<td>0.452</td>
<td>13.050</td>
</tr>
<tr>
<td>Microsoft</td>
<td>0.320</td>
<td>0.441</td>
<td>1.000</td>
<td>0.312</td>
<td>0.161</td>
<td>3.126</td>
<td>8.203</td>
</tr>
<tr>
<td>Nike</td>
<td>0.145</td>
<td>0.321</td>
<td>0.312</td>
<td>1.000</td>
<td>0.281</td>
<td>2.619</td>
<td>9.265</td>
</tr>
<tr>
<td>ADM</td>
<td>-0.093</td>
<td>0.243</td>
<td>0.161</td>
<td>0.281</td>
<td>1.000</td>
<td>0.953</td>
<td>6.712</td>
</tr>
</tbody>
</table>

Working with the POP, enter these data into the spreadsheet to obtain a plot of the MVF similar to that in the Lecture Notes.

Note: The data entries should be entered directly into the POP spreadsheet in columns B,D,J-N, and not by using the “up” and “down” arrows. For example, the correlation between IBM and Apple should be entered as =0.226 at the appropriate cell. IBM’s expected return should be entered as =0.00246, and its standard deviation as 0.008004. (The POP will round the data for display, but you will no longer be able to use the arrows once you enter the data. Ignore any error messages you get while entering the data. Also note that the program is valid for use with daily, weekly, or monthly data, even though it is labeled as “annual returns version”).

b) Assuming your clients are interested to consider only these 5 risky investments, how do you advise them to divide their funds in forming the optimal combination of these risky stocks?

c) How does your advice changes if ADM were uncorrelated with the other four stocks?

d) Being a well respected investment advisor, convince your clients with words that changing your recommendation from that in (b) to that in (c) makes sense.
Suppose you have the following two mutually exclusive projects that you can carry out on the corner of 6th avenue and West 4th Street: Build a day care center or a health spa. Suppose the day care center has the following cash flows: An immediate cash outlay of $5,000 followed by inflows of $2,500 in each of the next 3 years, and zero thereafter. Suppose the health spa has the following cash flows: An immediate outlay of $5,000 followed by inflows of nothing in year one, $1,000 in year 2, $7,100 in year 3, and zero thereafter.

a) If you base your investment decision on whichever project has the highest Net Present Value (NPV), which do you choose when the cost of capital is 15%, and which do you choose if the cost of capital is 5%?

b) People often calculate the Internal Rate of Return (IRR), defined as the cost of capital (i.e. required rate of return) at which the project’s NPV=0. This can lead to incorrect investment decisions, if the IRR is used as an investment criterion, ignoring the actual cost of capital. If you base your investment decision on whichever project has the highest IRR, which do you choose?

c) Suppose you could triple the size of the health spa project and triple its revenues but you cannot change the size of the day care center. Would any of your answers in (a) or (b) change?