For problems 3,4,6,7, see solutions manual for BKM.

For problems 1,2,5,8,9, see below:

1. a. Because the initial margin is 50%:
   \[0.5 = \frac{\text{NW}}{\text{Value of stock sold short}} = \frac{10,000}{\text{Value of stock sold short}}\]
   Value of stock sold short = 10,000 / 0.5 = $20,000. So, can short at most 400 shares.

   b. Our balance sheet looks like this:

   \begin{align*}
   \text{Assets} & \quad \text{Liabilities} \\
   \text{Cash} & \quad 30,000 \quad 20,000 (\text{short stock, 400 shares, market value}) \\
   & \quad 10,000 \text{ NW} \\
   \end{align*}

   At the end of the year:

   \begin{align*}
   \text{Assets} & \quad \text{Liabilities} \\
   \text{Cash} & \quad 30,000 \quad 21,200 (\text{short stock, 400 shares, market value at 53}) \\
   \text{Accrued} & \quad \text{Interest} \quad 1,200 \quad 400 (\text{dividends payable on the shorted stock}) \\
   & \quad 9,600 \text{ NW} \\
   \end{align*}

   The rate of return is \(-4\%\).

2. a. \(\bar{E}_2=10.3\%; \ \sigma_2 = 10.8\%\)

   b. The covariance between 1 and 2 is
   \[\frac{1}{3}(5-8.3)(-5-10.3)+(1/3)(5-8.3)(18-10.3)+(1/3)(15-8.3)(18-10.3)=25.56\]
   \[\rho = \frac{25.56}{(4.7 \times 10.8)} = 0.5\]

   c. \(\bar{E}_p = (1/3)8.3 + (2/3)10.3 = 9.6\%\)
   \[\sigma_p^2 = \frac{1}{3}(4.7)^2 + \frac{2}{3}(10.8)^2 + 2(1/3)(2/3)25.56 = 65.65\]
   \[\sigma_p = 8.1\%\]

   d. \(y=0.6\), so \(\bar{E}_C = (0.6 \times 9.6) + (0.4 \times 5) = 7.76\%\), and \(\sigma_C = 0.6 \times 8.1 = 4.86\%\)
5. Refer to the Lecture Notes on Uncertainty and the Lecture Notes on Optimal Risky Portfolios for all the moments.

Suppose you form a portfolio of the Small Firm “asset,” Microsoft, and T-bills, with the T-bill, with \( r_{T-bill} = 0.323\% \). What are the preferred weights of the two risky assets in the risky portfolio?

Graphical analysis (based on the efficient frontier in the notes) suggests that the risky asset portfolio you want to hold has positive weights invested in the small firm asset and in Microsoft (since \( \Phi \) is between + and x on the portfolio possibility curve for the small firm asset and Microsoft).

Can calculate the exact weight of the small firm asset in the tangency portfolio using the moments given in the notes and the following formula (from the notes as well):

\[
W_{Small,T} = \frac{\sigma^2[r_{Msft}] \cdot E[R_{Small}] - \sigma[r_{Small}, r_{Msft}]E[R_{Msft}]}{\sigma^2[r_{Msft}] \cdot E[R_{Small}] - \sigma[r_{Small}, r_{Msft}]E[R_{Msft}] + \sigma^2[r_{Small}] \cdot E[R_{Msft}] - \sigma[r_{Small}, r_{Msft}]E[R_{Small}]}
\]

Now:
- \( E[R_{Small}] = E[r_{Small}] - r_{T-bill} = 1.912 - 0.323 = 1.589 \).
- \( E[R_{Msft}] = E[r_{Msft}] - r_{T-bill} = 3.126 - 0.323 = 2.803 \).
- \( \sigma^2[r_{Small}] = 3.711 \times 3.711 = 13.772 \).
- \( \sigma^2[r_{Msft}] = 8.203 \times 8.203 = 67.289 \).
- \( \sigma[r_{Msft}, r_{Small}] = 12.030 \).

\[
W_{Small,T} = \frac{67.289 \times 1.589 - 12.030 \times 2.803}{67.289 \times 1.589 - 12.030 \times 2.803 + 13.772 \times 2.803 - 12.030 \times 1.589}
\]

\[
= 73.202 / [73.202 + 19.487] = 0.790.
\]

What are the preferred weights of the risky portfolio \( T \) and the riskless asset in the individual's portfolio?

Depends on the tastes and preferences of the particular individual.

Suppose you want to invest 75% in the tangency portfolio \( T \) (denoted by \( \Phi \) in the plots) and 25% in T-bills. What is the weight of the small firm asset and of Microsoft in your total portfolio?

Use the following formula for the total (complete) portfolio \( C \):

\[
w_{i,C} = w_{i,T}w_{T,C}
\]

where
- \( w_{i,C} \) is the weight of risky asset \( i \) in the total portfolio \( C \).
- \( w_{i,T} \) is the weight of risky asset \( i \) in the tangency portfolio \( T \).
- \( w_{T,C} \) is the weight of portfolio \( T \) in the total portfolio \( C \) (another notation we used for this weight is \( y \)).

So, the answer is:
- \( w_{Small,C} = w_{Small,T}w_{T,C} = 0.79 \times 0.75 = 0.5925 \).
- \( w_{Msft,C} = w_{Msft,T}w_{T,C} = 0.21 \times 0.75 = 0.1575 \).
8. a. \( \beta_i = \frac{\sigma_i^{2}}{\sigma_M^{2}} = \frac{0.00045}{(0.03)^2} = 0.50 \)

b. \( E[r_i] = r_f + \beta_i (E[r_M] - r_f) = 0.09 + 0.5 \times 0.03 = 0.105 \) or 10.5%.

c. Rearrange the SML to express \( \beta_i \) as a function of \( E[r_i] \).
\[
\beta_i = \frac{(E[r_i] - r_f)}{(E[r_M] - r_f)} = \frac{0.10 - 0.09}{0.12 - 0.09} = 0.333
\]
\[
\sigma_M = \beta_i \sigma_M^2 = 0.333 \times 0.03^2 = 0.0003
\]

d. \( \beta_i = \frac{(0.15 - 0.09)}{(0.12 - 0.09)} = \frac{0.06}{0.03} = 2 \)
\[
\sigma_M = 2 \times 0.03^2 = 0.0018
\]

9. a. Rearrange the SML to express \( E[r_M] \) as \( E[r_M] = r_f + (E[r_i] - r_f) / \beta_i \).

Therefore,
\[
E[r_M] = r_f + (E[r_A] - r_f) / \beta_A = r_f + (0.08 - r_f)/0.4
\]
\[
E[r_M] = r_f + (E[r_B] - r_f) / \beta_B = r_f + (0.16 - r_f)/1.2
\]

Use the above two equations to solve for \( r_f \):
\[
 r_f + (0.08 - r_f)/0.4 = r_f + (0.16 - r_f)/1.2, \text{ so } r_f = 0.04 \text{ or } 4%.
\]
Hence,
\[
E[r_M] = 0.04 + (0.08 - 0.04)/0.4 = 0.14 \text{ or } 14%.
\]

b. \( \beta_M = 1 \), by definition.

c. \( \sigma_M^2 = \sigma_M / \beta_A = 0.00225 / 0.4 = 0.005625, \sigma_M=0.075 \text{ or } 7.5%.
\]

d. \( \beta_C = \sigma_CM/\sigma_M^2 = (\rho_{CM} \sigma_C \sigma_M) / \sigma_M^2 = (\rho_{CM} \sigma_C) / \sigma_M = (\rho_{AM} \sigma_C) / \sigma_M = ((\sigma_A / \sigma_M) \sigma_C) / \sigma_M = (\sigma_A \sigma_C) / (\sigma_M^2) = (0.00225 \times 0.1125) / (0.06 \times 0.005625) = 0.75
\]
So,
\[
E[r_C] = 0.04 + 0.75 (0.14 - 0.04) = 0.115 \text{ or } 11.50%
\]

e. \( \beta_B = \sigma_BM/\sigma_M^2 = (\rho_{BM} \sigma_B \sigma_M) / \sigma_M^2 = (\rho_{BM} \sigma_B) / \sigma_M .
\)

Therefore,
\[
\sigma_B = (\beta_B \sigma_M) / \rho_{BM} = (1.2 \times 0.075) / 0.9 = 0.10 \text{ or } 10%.
\]

f. \[
\begin{array}{ccc}
\text{Asset} & E[r_i] & \sigma_i & \beta_i \\
A & 8\% & 6\% & 0.40 \\
C & 11.5\% & 11.25\% & 0.75 \\
M & 14\% & 7.5\% & 1.00 \\
B & 16\% & 10\% & 1.20 \\
\end{array}
\]

We just confirmed that there is indeed a linear-positive relationship between \( E[r_i] \) and \( \beta_i \), but there is no such relationship between \( E[r_i] \) and \( \sigma_i \).
S1. If we buy the bill (at the ask), the settlement price is \((1-0.058\times150/360)=0.9758\) (per $1 par). The BEY is \((1-0.9758)/0.9758 \times (365/150) = 6.03\%\) (APR with \(n=150\) day compounding). The EAR is \(FVIF(6.03\%/M, M) - 1\) (where \(M=365/150\)), so EAR=6.14%. If we sell the bill (at the bid), the settlement price is 0.9752. The BEY is 6.19%. The EAR is 6.30\% (so we need the bank to offer more to justify selling). The EAR of the bank account is \(FVIF(5.95\%/4, 4)-1 = 6.08\%\). Therefore, we should withdraw our money from the bank and buy a T-bill. (We give up an EAR of 6.08\% to get an EAR of 6.14\%).

S2. a. The situation is this:

When the investor is entirely at stock 1, \(E_r=10\%\) and \(\sigma=10\%\). A portfolio composed of stock 2 and \(r_f\) with the same risk can have a higher expected return. To construct this portfolio, let \(y\) be the amount in stock 2. Then \(10\%=\sigma_c=y\sigma_2=y(15): y=2/3\).

The expected return is \(E_{r_C}=(1/3)5 + (2/3)20 = 15\%\).

b. No, there are no guarantees. In any given year, stock 1 might do much better than stock 2, in which case the return on the new portfolio might be lower than the return on stock 1 alone.

S3. a. 

b. In the portfolio of risky assets, \(P, w_A=2/3\) and \(w_B=1.3\), so

\[
\sigma_p = \sqrt{(2/3)^2(25.6)^2 + (1/3)^2(10)^2 + (2/3)(25.6)(26)(0.577)} = 23.2\%
\]

\[
\sigma_c = (3/4) 23.2 = 17.4\%
\]

c. \(E_{r_A}=5+0.8(8)=11.4\%\)

d. The market model says \(r_A-r_f=1+0.8(r_m-r_f)+e_A\), or \(E_{r_A}-r_f=1+0.8(E_{r_m}-r_f)\), (since \(e_A\) is expected to be zero). Since \(r_f=5\%\) and \(E_{r_m}-r_f=8\%, E_{r_A}-5=1+0.8(8)=12.4\%\)

e. The difference might arise because the CAPM is completely wrong, because stock A is temporarily undervalued, or because the index model was estimated with error.
S4. a) The following (somewhat blurred) printout is what your POP spreadsheet should look like:

<table>
<thead>
<tr>
<th></th>
<th>Stock</th>
<th>IBM</th>
<th>Apple</th>
<th>Microsoft</th>
<th>Nike</th>
<th>ADM</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>weight</td>
<td>-0.52</td>
<td>-0.47</td>
<td>1.43</td>
<td>0.62</td>
<td>-0.06</td>
</tr>
</tbody>
</table>

b) The optimal weights in the tangent portfolio are:

<table>
<thead>
<tr>
<th>Stock</th>
<th>IBM</th>
<th>Apple</th>
<th>Microsoft</th>
<th>Nike</th>
<th>ADM</th>
</tr>
</thead>
<tbody>
<tr>
<td>weight</td>
<td>-0.38</td>
<td>-0.35</td>
<td>1.06</td>
<td>0.45</td>
<td>0.22</td>
</tr>
</tbody>
</table>

and the optimal weights in the tangent portfolio are:
d) In (b) you expect Nike and especially Microsoft to perform well, given their level of risk. Therefore, you suggest to invest in these two stocks, while shorting the remaining three stocks, which offer low reward and high risk. You are confident in your short positions because the stocks in this case overall move together, and you just use the short position to enhance the performance of the overall portfolio. You also note that ADM is useful in improving the risk-return tradeoff if weighted appropriately, and you suggest to include it as a small short position to improve the diversification level of the portfolio.

In (c) you are still bullish on Microsoft and Nike, and suggest to short IBM and Apple. However, now ADM is uncorrelated with all four assets. It can act as an insurance instrument in the sense that when other stocks will decline (as they will often do together because of their positive correlations), ADM may actually rise. Hence you suggest to invest in ADM 22% of the funds invested in risky assets.

[You then ask your clients to answer the Risk-Tolerance Questionnaire (see the WSJ article in the Additional Readings to Lecture Notes 5), and based on this can advice them on their overall asset allocation between the riskless asset and the above optimal (tangent) portfolio.]

S5. a) When the cost of capital is 15% the NPV of the day care center is $708, while the NPV of the health spa is only $425, thus you build the day care center. When the cost of capital is 5%, the NPV of the day care center is $1,808, and the NPV of the health spa is $2,040, thus you build the health spa. The attractiveness of the health spa, with its delayed cash flows relative to the day care center, increases when the cost of capital is low.

b) To get the IRR, in this example, you can use the financial calculator.
Day care: -5000 PV, 3 n, 2500 PMT, 0 FV, solving for I/YR gives 23.34%.
Health spa: The structure of the cash flow is irregular, so need to enter cash flows individually, -5000 CFj, 0 CFj, 1000 CFj, 7100 CFj, pressing [shift] IRR/YR yields 18.3%.

c) Tripling the size of the health spa project means that the initial outflow is $15,000, followed by zero in year one, $3,000 in year 2, and $21,300 in year 3. The IRR of this newly scaled project is still 18.3%, because the IRR is invariant with respect to scale (it measures dollars returned relative to dollars invested). Thus, the IRR decision rule is unaffected by size and the day care center is still always preferable to the health spa according to the IRR rule. On the other hand, the NPV of the “scaled up” health spa project at 15% is now $1,274, which is greater than the $708 NPV of the unaltered day care project at a cost of capital of 15%. Thus, at a cost of capital of 15% the NPV rule recommends the expanded health spa over the day care center because the expanded scale of the health spa adds more value to your firm (an increase of $1,274) compared with the day care center’s added value (only $708). In other words, although the day care center has a greater return per dollar invested it is not big enough and does not contribute as much to your firm, compared with the health spa, even at the 15% cost of capital. At 5% the expanded health spa will not be beneficial under all values of the cost of capital. See what happens for a cost of capital of 20% -- or anything above 18.3% -- when you triple the size of the health spa.

**Remark:** Another problem with the IRR is that when cash flow change sign more than once (unlike in this example), there will be as many IRR’s as there are changes of sign in the cash flows. Hence, the NPV rule is the correct one to use always (when comparing investments). The problem with the NPV is that you need to come up with the discount rate (e.g., by relying on the CAPM).