Stock Index Futures

Calculating the size of the futures hedge:

\[ N_f = (\beta_d - \beta_p) \times \frac{MV_{\text{portfolio}}}{\text{multiple on futures} \times \text{Index value}} \]

where:
- \( \beta_p \) denotes the current beta of the portfolio that we wish to hedge
- \( \beta_d \) denotes the desired beta of the portfolio
- \( MV_{\text{portfolio}} \) is the current value of the portfolio we wish to hedge
- \$ multiple on futures is the exchange specified multiple (\$250 for S&P futures)
- Index value is the current value of the underlying spot index (not the futures)

Note: If \( N_f > 0 \) it means we need to buy that many futures
If \( N_f < 0 \) it means we need to sell that many futures
Stock Index Futures

\[ N_t = \frac{(\beta_d - \beta_p) \times \frac{MV_{portfolio}}{\text{multiple on futures} \times \text{Index value}}} \]

Application:

A portfolio manager who believes he has superior stock picking ability holds a $400 mm U.S. equities portfolio having a beta of 1.20, as measured against the S&P 500. The S&P index current stands at 940.10.

The portfolio manager has become nervous about the market in general and would like to reduce his exposure to systematic (market) risk while retaining the stocks he holds in order to benefit from his special stock picking ability. He would like to reduce the portfolio’s beta to 0.50. What should he do?
Stock Index Futures

\[ N_f = (0.50 - 1.20) \times \frac{$400,000,000}{250 \times 940.10} \]

\[ = -1,191.36 \]

Notice that this hedge is an incomplete hedge. We deliberately did not remove all risk.
Stock Index Futures

Suppose that the portfolio manager wanted to reduce the systematic risk to 0?

\[ N_f = (0 - 1.20) \times \frac{\$400,000,000}{\$250 \times 940.10} \]

\[ = -2042.34 \]

Diagram:
- Risk profile of portfolio
- Payoff profile of futures hedge
- Combined payoffs
Hedging with bond futures:

Most hedging with T-bond futures is conducted using the dollar value of a basis point (DV01).

A DV01 is defined as the dollar amount by which the price of a $100 par value bond will change if its yield changes by 1 basis point (1 bp). DV01 can be derived from modified duration.

\[
DV01 = D_M \times \text{Price per $100 of par} \times 0.0001
\]

where: \( D_M \) is the modified duration

We must calculate the DV01 of the bond futures.
Treasury Bond Futures

DV01 of bond futures:

\[
DV01_{\text{futures}} = \frac{DV01_{\text{CTD}}}{\text{Conversion Factor}}
\]
Treasury Bond Futures

Hedging Cash Bonds (corporates, municipals, agencies, etc.):

Step 1: Determine the DV01 of the cash bond.

Step 2: Determine the CTD T-bond for the futures that will be used as the hedging instrument.

Step 3: Determine the DV01 of the CTD.

Step 4: Determine the DV01 of the futures.

Step 5: Determine the yield beta between the cash bond and CTD.

Step 6: Calculate the hedge ratio.

Step 7: Determine the risk equivalent position in bond futures

Step 8: Place the hedge.
Treasury Bond Futures

Example:

\[ HR = \frac{DV01_{\text{cash bond}}}{DV01_{\text{futures}}} \times \beta_y \]

Risk equivalent position = \( FV_{\text{cash}} \times HR \)
Treasury Bond Futures

Application:

A corporate bond dealer makes markets in a large number of different bonds. He earns profits for his Firm from his bid/ask spread, not from a view on the direction of interest rates. He practices complete hedging. His positions are financed in the repo market.

For purposes of illustration, we assume that the dealer has positions in only four bonds.

Note: A bond dealer might have a view on rates and might be authorized to speculate on this view, but we assume not for purposes of this application.
Treasury Bond Futures

Hedging Instrument: June Bond Futures: DV01 = 0.0875

Dealer JE
Risk Management Report

<table>
<thead>
<tr>
<th>POS</th>
<th>ISSUER</th>
<th>CPN</th>
<th>MATURITY</th>
<th>YLD</th>
<th>CALC PRICE</th>
<th>MOD DUR</th>
<th>DV01</th>
<th>YLD BETA</th>
<th>HR</th>
<th>EQUIV POS</th>
</tr>
</thead>
<tbody>
<tr>
<td>-3.0</td>
<td>GM</td>
<td>8.125</td>
<td>06/01/00</td>
<td>7.28%</td>
<td>103.03</td>
<td>3.5663</td>
<td>0.03674</td>
<td>0.58</td>
<td></td>
<td></td>
</tr>
<tr>
<td>+6.5</td>
<td>McDonalds</td>
<td>7.250</td>
<td>11/01/00</td>
<td>7.09%</td>
<td>100.61</td>
<td>3.9130</td>
<td>0.03937</td>
<td>0.54</td>
<td></td>
<td></td>
</tr>
<tr>
<td>-5.0</td>
<td>Delta</td>
<td>9.750</td>
<td>03/01/12</td>
<td>9.98%</td>
<td>98.18</td>
<td>7.9441</td>
<td>0.07799</td>
<td>0.66</td>
<td></td>
<td></td>
</tr>
<tr>
<td>+2.0</td>
<td>MCI</td>
<td>8.375</td>
<td>06/01/15</td>
<td>9.45%</td>
<td>90.52</td>
<td>8.9971</td>
<td>0.08144</td>
<td>0.51</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Complete the risk management report and determine the size of the position in June T-bond futures that JE needs to fully hedge his interest rate risk.
# Treasury Bond Futures

**Hedging Instrument:** June Bond Futures  
**DV01 = 0.0875**  
**2/28/96**

**Dealer JE**  
**Risk Management Report**

<table>
<thead>
<tr>
<th>POS MM</th>
<th>ISSUER</th>
<th>CPN</th>
<th>MATURITY</th>
<th>YLD</th>
<th>CALC PRICE</th>
<th>MOD DUR</th>
<th>DV01</th>
<th>YLD BETA</th>
<th>HR</th>
<th>EQUIV POS MM</th>
</tr>
</thead>
<tbody>
<tr>
<td>-3.0</td>
<td>GM</td>
<td>8.125</td>
<td>06/01/00</td>
<td>7.28%</td>
<td>103.03</td>
<td>3.5663</td>
<td>0.03674</td>
<td>0.58</td>
<td>0.2435</td>
<td>-0.7306</td>
</tr>
<tr>
<td>+6.5</td>
<td>McDonalds</td>
<td>7.250</td>
<td>11/01/00</td>
<td>7.09%</td>
<td>100.61</td>
<td>3.9130</td>
<td>0.03937</td>
<td>0.54</td>
<td>0.2430</td>
<td>+1.5793</td>
</tr>
<tr>
<td>-5.0</td>
<td>Delta</td>
<td>9.750</td>
<td>03/01/12</td>
<td>9.98%</td>
<td>98.18</td>
<td>7.9441</td>
<td>0.07799</td>
<td>0.66</td>
<td>0.5882</td>
<td>-2.9413</td>
</tr>
<tr>
<td>+2.0</td>
<td>MCI</td>
<td>8.375</td>
<td>06/01/15</td>
<td>9.45%</td>
<td>90.52</td>
<td>8.9971</td>
<td>0.08144</td>
<td>0.51</td>
<td>0.4747</td>
<td>+0.9494</td>
</tr>
</tbody>
</table>

**total equivalent position**  
**-1.1432 mm**

Equivalent number of futures  
\[ \frac{-1.1432 \text{ mm}}{0.1 \text{ mm}} = -11.432 \text{ futures} \]
Treasury Bond Futures

Interpretation:

Since the dealer’s cash position is risk equivalent to a short position in 11 bond futures, he would hedge by going long 11 bond futures. This matches the DV01 of his bond book to the DV01 of his hedge. By implication, it also matches the durations.
Stripping versus Stacking and Rolling

Suppose that a corporation just issued a $100 million four-year floating rate note that pays a quarterly coupon pegged to 3-month LIBOR. The floater’s coupon-accrual periods begin on the third Wednesday of March, June, September, and December. Each coupon reset is based on the spot 3-month LIBOR prevailing two business days prior to the beginning of the coupon-accrual period. Today is the reset date for the floater’s first coupon period, which begins on the third Wednesday of December 1998.

The corporation has decided that it wants to remove the uncertainty surrounding its future interest payments on this liability.

How can it do so using futures?
Eurodollar Futures

Notice that the floater’s coupon reset dates match the rate setting dates of the IMM’s Eurodollar futures contracts. This is convenient.

Since the LIBOR payment each quarter is based on $100 million of principal, we need to lock in LIBOR rates on that much principal. We don’t need to worry about the first coupon, because that coupon is already set. We only need to worry about the next 15 resets.
The corporation’s concern is that future spot LIBORs might rise, relative to the current forward LIBOR rates. This would make the floating coupon payments more costly. The hedge, therefore, should be structured to generate an offsetting profit should the firm find that the LIBORs rises.

Because of their structure, a short position in ED futures is profitable if LIBOR rises (i.e., $F_{ED} = 100 - LIBOR$).
Eurodollar Futures

There are two different ways to approach this problem. One is called *strip hedging* and the other is called *stacking and rolling*.

**Strip Hedging:**

While this is an oversimplification, essentially what we do is sell a series of sequentially maturing ED futures. A series of sequentially maturing futures is called a *strip*. This strip would consist of about 100 Mar 99 ED, 100 Jun 99 ED, 100 Sep 99 ED, and so forth out for 15 quarters.
Eurodollar Futures

Strip Hedging: Sell at the outset 100 of each of the Mar 99, Jun 99, Sep 99 and so on. These positions are all entered at the same time in Dec 98. As each settlement comes due, allow the maturing contracts to cash settle.

Strip hedging provides a very effective hedge, but can involve transacting in illiquid contracts. If contracts are not available far enough forward, complete strip hedge is not possible.
Stacking: Instead of entering 100 of each of 15 different contracts as we did in the strip, we enter 1500 of the first contract (Mar 99). When we get to Mar 99, we allow the contracts to cash settle (or offset them) and then “roll out” into 1400 Jun 99 contracts.
Eurodollar Futures

Stacking: When we get to Jun 99, we allow the contracts to cash settle (or offset them) and then "roll out" into 1300 Sep 99 contracts.

This strategy has the advantage of keeping us in the nearby contract, which is the most liquid, but it is a less effective hedge (more basis risk).
**Eurodollar Futures**

**Strip and Stack:** Sometimes a strip hedge and a stack hedge will be used together. For example, suppose that you wanted to hedge a 12-year floater. ED futures go out quarterly for ten years, but are only liquid out to about six years. We could strip hedge out to six years and stack hedge the rest.
Interest Rate Futures

Tailing the hedge:

In the strip hedge using ED in the preceding example, we assumed that to hedge $100 million of floaters you need 100 of each futures in the strip.

But, earlier we argued that the BPV of a futures is $25 because it is not discounted.

A change of a basis point in a forward 3-month LIBOR on $1 mm of real principal, however, will cause less than a $25 impact on a floater. Why?
Interest Rate Futures

The further out the LIBOR payment on the floater, the fewer futures contracts it will take.

Suppose that current ED prices for the first four contracts in the strip are as follows:

<table>
<thead>
<tr>
<th>Contract</th>
<th>Futures Price</th>
<th>Implied LIBOR</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dec 98</td>
<td>95.40</td>
<td>4.60%</td>
</tr>
<tr>
<td>Mar 99</td>
<td>95.20</td>
<td>4.80%</td>
</tr>
<tr>
<td>Jun 99</td>
<td>95.05</td>
<td>4.95%</td>
</tr>
<tr>
<td>Sep 99</td>
<td>94.95</td>
<td>5.05%</td>
</tr>
</tbody>
</table>
Interest Rate Futures

Let's determine how many futures we need to hedge the Jun 99 payment on the floater.

To get this, we need to first get the BPV in Dec 98 of the payment that will reset in Mar 99 and payout in Jun 99. There are 91 days between Dec 98 and Mar 99 and also 91 days between Mar 99 and Jun 99.

\[
BPV = \frac{\$100 \text{ mm} \times 91/360 \times 0.0001}{[1 + (0.046 \times 91/360)] \times [1 + (0.0480 \times 91/360)]}
\]

\[
= \$2441.64
\]
### Interest Rate Futures

Number of Mar 99 futures = \[
\frac{\text{BPV of payment on floater}}{\text{BPV of a single futures}}
\]

\[
= \frac{$2441.64}{$25}
\]

\[
= 97.66 \text{ contracts}
\]
Interest Rate Futures

Next we calculate the number of Jun 99 futures necessary to hedge the Sep 99 payment on the floater.

\[
\text{BPV of payment on floater} = \frac{\$100 \text{ mm} \times 0.0001 \times 91/360}{(1+0.046 \times 91/360)(1+0.0480 \times 91/360)(1+0.495 \times 91/360)}
\]

\[
= \$2411.47
\]

Therefore, we need:

\[
\text{Number of Jun 99 futures needed} = \frac{\$2411.47}{\$25} = 96.45
\]
Interest Rate Futures

This process is repeated for each contract that will be part of the strip.

Notice that the further out the floater's payment gets, the fewer contracts it takes to hedge the exposure.

This process of finding the number of futures contracts to properly hedge a cash market exposure is what we mean by "tailing the hedge."