INTERNATIONAL FINANCIAL MARKETS

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Also due to these market imperfections, the currency forward or futures price may only approximately equal the expected future spot exchange rate. A discrepancy between the contract price and expected future spot rate may arise because of these frictions. For instance, suppose the expected spot rate at contract maturity is $1.7000/E while the observed futures price is currently $1.7002/E. Arbitrageurs may not attempt to profit from this small discrepancy because of the transaction costs involved.

**CURRENCY FORWARD AND FUTURES PRICES**

The IRP theorem implies that currency forward and futures prices are equal if interest rates are constant and investors are risk-neutral. These assumptions were implicit in the preceding analysis. However, forward and futures prices for foreign exchange can differ because of nonconstant interest rates and risk aversion on the part of market participants. Also for these reasons, both forward and futures prices may differ from the expected future spot exchange rate at contract delivery. In this sec-

**Forward**

Suppose we suppose suppose if these were purchase gates us and a transparency, where therefore the forward price in $T ye.

13.7

where $\delta_T$ is unknown.

13.8

Combining the equation:

13.9

Thus we have that

$$(1 + y_0)^T$$

To determine the price of the free rate of $$(1 + y_0)^T$$

13.10

Hence, the forward rate assumption:

**Currency**

For currency today, we pay the current daily mark bonds and each trader exchanges as that day, we buy $
tion we analyze pricing when interest rates change intertemporally and investors are risk-averse. This analysis, which is based largely on the works of Cox, Ingersoll, and Ross (1981) and Richard and Sundaresan (1981), stresses the role of market expectations in determining prices.

Forward Prices under Risk Neutrality

Suppose we are considering buying a foreign currency in $T$ years. Also suppose that the current yield to maturity on U.S. riskless bonds is $\tau Y_0$, if these bonds exhibit a maturity of $T$ years. Further, suppose that we purchase $(1 + \tau Y_0)^T$ forward contracts today, where each contract obligates us to buy one unit of the foreign exchange at the forward price. Additionally, we buy $\tau F_0$ of riskless U.S. bonds with $T$ years to maturity, where $\tau F_0$ is the currency forward price. Our initial investment is therefore $\tau F_0$, since the forward price is set so that the initial value of the forward contract is zero.

In $T$ years the forward contracts expire and their value is

$$13.7 \quad \$(1 + \tau Y_0)^T(\hat{S}_T - \tau F_0),$$

where $\hat{S}_T$ is the exchange rate at contract delivery, which is currently unknown. The payoff from the bond investment is

$$13.8 \quad \$\tau F_0(1 + \tau Y_0)^T.$$ Combining eqs. 13.7 and 13.8 gives the total proceeds from the investment:

$$13.9 \quad \$(1 + \tau Y_0)^T(\hat{S}_T - \tau F_0) + \tau F_0(1 + \tau Y_0)^T = \$(1 + \tau Y_0)^T\hat{S}_T.$$ Thus we invest $\tau F_0$ and receive $(1 + \tau Y_0)^T\hat{S}_T$. It must therefore be true that the current forward price is equal to the present value of $(1 + \tau Y_0)^T\hat{S}_T$.

To determine this present value, we discount the expected proceeds of the investment: $E[(1 + \tau Y_0)^T\hat{S}_T]$. We can discount at the domestic risk-free rate of interest if investors are risk-neutral. Since $E[(1 + \tau Y_0)^T\hat{S}_T] = (1 + \tau Y_0)^TE(\hat{S}_T)$, the present value is

$$13.10 \quad \frac{(1 + \tau Y_0)^TE(\hat{S}_T)}{(1 + \tau Y_0)^T} = E(\hat{S}_T) = \tau F_0.$$ Hence, the currency forward price is equal to the expected spot exchange rate at contract delivery if investors are risk-neutral. Under this assumption, the forward price is an unbiased predictor of $\hat{S}_T$.

Currency Futures Prices under Risk Neutrality

For currency futures we must adjust our trading strategy to reflect the daily marking to the market procedure. We now purchase one-day bonds and futures contracts, reinvesting the cash flows at the end of each trading day into new one-day bonds and purchasing futures contracts as they unwind. Specifically, at the beginning of the initial trading day, we buy an amount of bonds equal to the current futures price $\tau F_0$. We then purchase $(1 + \tau Y_0)$ futures contracts, where $\tau Y_0$ is the yield to
maturity on a current one-day riskless bond. At the end of the day the proceeds from the bond investment will be

\[ \tau F^0_0(1 + \gamma Y_0). \]

The amount credited (or debited) to our futures margin account will be

\[ $(1 + \gamma Y_0)(\tau_i - F^0_i) - \tau F^0_0. \]

Combining eqs. 13.11 and 13.12 gives our total proceeds from the one-day investment:

\[ \tau F^0_0(1 + \gamma Y_0) + (1 + \gamma Y_0)(\tau_i - F^0_i) - \tau F^0_0 = $(1 + \gamma Y_0)(\tau_i - F^0_i). \]

In turn, these proceeds are reinvested in one-day bonds at the new (but currently unknown) rate of \( \gamma Y_1 \). We also buy more futures contracts; namely, we buy \((1 + \gamma Y_0)(1 + \gamma Y_1)\) contracts. Thus, our proceeds at the end of the second day are

\[ $(1 + \gamma Y_0)(\gamma Y_1)(1 + \gamma Y_1) + (1 + \gamma Y_0)(1 + \gamma Y_1)(\gamma Y_1 - F^0_2) - \gamma Y_1 F^0_2. \]

Continuing the process (and recognizing the pattern emerging from payoffs in eqs. 13.13 and 13.14), at any day \( t \) the proceeds from our investment strategy will be

\[ $(1 + \gamma Y_0)(1 + \gamma Y_1) \ldots (1 + \gamma Y_{t-1} F^0_t). \]

Finally, since at contract delivery the futures price must equal the prevailing exchange rate, \( S_T \), we have proceeds at delivery equal to

\[ $(1 + \gamma Y_0)(1 + \gamma Y_1) \ldots (1 + \gamma Y_T F^0_T). \]

Thus we invest \( \tau F^0_0 \) and receive \( \delta R S_T \), where \( R \) is the product of 1 plus the one-day interest rates through delivery:

\[ R = (1 + \gamma Y_0)(1 + \gamma Y_1) \ldots (1 + \gamma Y_T). \]

It must therefore be true that the current futures price is equal to the present value of \( R S_T \).

To determine this present value, we discount the expected proceeds of the investment: \( E(R S_T) \). We can discount at the domestic risk-free rate if investors are risk-neutral:

\[ \tau F^0_0 = \frac{E(R S_T)}{(1 + \gamma Y_0)}. \]

Since no liquidity premiums exist in the term structure under risk neutrality, \( (1 + \gamma Y_0)^\gamma = E(R) \). Thus, we have:

\[ E(\tau F^0_0) = \frac{E(R S_T)}{(1 + \gamma Y_0)^\gamma} = \frac{E(R) E(S_T) + \text{COV}(R, S_T)}{(1 + \gamma Y_0)^\gamma} = \frac{(1 + \gamma Y_0) E(S_T) + \text{COV}(R, S_T)}{(1 + \gamma Y_0)^\gamma} = E(S_T) + \frac{\text{COV}(R, S_T)}{(1 + \gamma Y_0)^\gamma}. \]

\[ \text{Equation 13.19 follows from the mathematical identity: } E(AB) = E(A)E(B) + \text{COV}(A,B). \]

\[ \text{Specifically: } \frac{E(\tau F^0_0)}{(1 + \gamma Y_0)^\gamma} = \frac{E(R) E(S_T) + \text{COV}(R, S_T)}{(1 + \gamma Y_0)^\gamma} = \frac{(1 + \gamma Y_0) E(S_T) + \text{COV}(R, S_T)}{(1 + \gamma Y_0)^\gamma} = E(S_T) + \frac{\text{COV}(R, S_T)}{(1 + \gamma Y_0)^\gamma}. \]

\[ \text{See Chapter 4 for further details.} \]
\[ \tau F^*_0 = E(\tilde{S}_T) + \frac{\text{COV}(\tilde{R}, \tilde{S}_T)}{(1 + \tau Y_0)^T}. \]

Equation 13.19 states that under risk neutrality, the currency futures price is a biased estimate of the expected spot exchange rate at delivery and, thus, is also not equal to the currency forward price. There exists an additional term that depends on the covariance between \( \tilde{R} \), the product of one-day interest rates, and \( \tilde{S}_T \), the rate of exchange.

This additional term is called the \textit{reinvestment rate premium}, and it drives a wedge between the current forward and futures prices. It arises because of the different cash flow patterns exhibited by currency forward and futures contracts. This additional term also derives from the nonconstant nature of interest rates. If rates, and thus yields, were constant, then the covariance term in eq. 13.19 would be zero, since \( \tilde{R} \) would no longer be random. Thus we can conclude that currency forward and futures prices differ under risk neutrality because of (1) the daily resettlement procedure that exists in the futures market, and (2) the nonconstant nature of interest rates.

\textbf{The Economics Underlying the Reinvestment Rate Premium} To understand why the reinvestment rate premium occurs, suppose you are long a DM futures contract. If the U.S. interest rate falls and the $/DM rate rises (negative covariance), then you can reinvest your day's margin credit only in a lower-interest-bearing U.S. asset. In a sense, your gain from the long DM futures position is somewhat offset by the lower reinvestment rate experienced. This offset would not occur in the DM forward market, because no cash flows occur until contract delivery. Thus, you will demand a futures price that is below the forward price (the expected future spot rate) to compensate for your reinvestment loss. Conversely, under positive covariance you are willing to accept a higher futures price since any daily gains on the futures contract can be reinvested at a higher rate of interest.

Whether the covariance term \( \text{COV}(\tilde{R}, \tilde{S}_T) \) is positive or negative for currencies is somewhat vague, and constitutes an empirical issue. For instance, one can argue that the covariance term is negative in the short run. As U.S. interest rates rise, Germans will purchase dollars in order to buy higher-yielding U.S. securities. Such purchases will tend to drive up the dollar's value, lowering the $/DM exchange rate. On the other hand, higher U.S. interest rates can cause a dollar depreciation in the longer run. Higher rates presumably reflect greater expected U.S. inflation, and more expensive U.S. goods and services will lower the foreign demand for dollars.\(^4\)

\textbf{Currency Forward Prices under Risk Aversion} Recall the original investment strategy for determining currency forward prices under risk neutrality. If market participants are now risk-averse, we cannot determine currency forward prices by discounting expected

\(^4\) See Chapters 2 through 4 for a discussion of the various determinants of exchange rates.
payoffs at the riskless rate of interest. Instead, we must account for the risk associated with the initial investment.

In equilibrium, the market value of the investment now must be such that the foregone utility associated with the investment is equal to the expected utility gained from the payoff:

$$U_0[\tau F_0] = \sum_{i=1}^{n} h_i [\hat{U}_{T',i}((1 + \tau Y_0)^{T'}S_{T',i})].$$

In eq. 13.20, the term $U_0[\tau F_0]$ represents the total utility lost by making the investment. $U_0$ denotes the utility associated with each dollar increase in our current consumption. Multiplying $U_0$ by our initial outlay, $\tau F_0$, gives us the total loss in our current utility associated with making the investment. The term $\hat{U}_{T',i}$ denotes the utility associated with increasing our consumption by a dollar in the $i$th state of nature at contract delivery $T$. This utility is currently unknown, since the investment's payoff is currently uncertain. Given $n$ possible states of nature, the right-hand side of eq. 13.20 yields the expected increase in utility from the investment.

Dividing both sides of eq. 13.20 by $U_0$ and taking expectations yields

$$\tau F_0 = (1 + \tau Y_0)^T E(M_{T',\hat{S}_{T'}}),$$

where $M_{T',i} = \hat{U}_{T',i}/U_0$. The term $M_{T'}$ is the marginal rate of substitution of consumption at delivery $T$ for current consumption. For instance, if $M_{T',i} = 0.70$, then we would be indifferent between $\$1.00$ of consumption at $T$ in state $i$ and $\$0.70$ of consumption now.

Applying the mathematical identity previously described in footnote 3 to eq. 13.21 gives

$$\tau F_0 = (1 + \tau Y_0)^T[E(M_{T'}E(\hat{S}_T) + COV(M_{T'},\hat{S}_T)].$$

$E(M_{T'})$ is the expected marginal rate of substitution over all $n$ states of nature. Intuitively, it is the expected value for the implicit rate of discount relating consumption now to future consumption at delivery $T$. Thus:

$$E(M_{T'}) = [(1 + \tau Y_0)^T]^{-1}.$$

Substituting eq. 13.23 into 13.22 gives the following result:

$$\tau F_0 = E(\hat{S}_T) + (1 + \tau Y_0)^T COV(M_{T'},\hat{S}_T).$$

Equation 13.24 states that under risk aversion, the currency forward price is equal to the expected future spot exchange rate at contract delivery plus a hedging premium, also known as a risk premium, that is a function of $COV(M_{T'},\hat{S}_T)$, the covariance between the marginal rate of substitution and the exchange rate at delivery. The forward price is no longer an unbiased estimate of $\hat{S}_T$. Under risk neutrality, investors have linear utility functions and, thus, constant marginal rates of substitution. Consequently, the covariance term $COV(M_{T'},\hat{S}_T)$ would be zero, and eq. 13.24 reduces to eq. 13.10. Again, under risk neutrality the current prevailing utility consur first de of sub consumption is by the COV(N) payoff: 

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The current prevailing utility consumption is by the COV(N) payoff. You are that is very. W. rate is COV(N) payoff.
The Economics Underlying the Hedging Premium. A representative utility function for a risk-averse investor is shown in Figure 13.2. As the currency forward price is equal to the spot exchange rate expected to prevail at contract delivery, the forward exchange rate is a function of consumption. The covariance term COV(U', S) is positive or negative for currencies is somewhat vague, and represents an empirical issue. Also, the magnitude of the covariance term largely depends on an investor's overall portfolio from which wealth is drawn for consumption. If you are long the currency forward contract at different levels of consumption, you are likely to yield positive payoffs when the forward rate is high, as in a recessionary state. Thus, the payoff on the forward contract is negatively correlated with the expected consumption. The forward contract represents insurance for consumption. If consumption is low (a recessionary state), the marginal utility of consumption is high, and the marginal utility of substitution is low (a recessionary state). The marginal utility of substitution is high, and the marginal utility of consumption is low (a recessionary state). The marginal utility of substitution is low (a recessionary state). The marginal utility of substitution is high, and the marginal utility of consumption is low (a recessionary state). The marginal utility of substitution is low (a recessionary state).
Currency Futures Prices under Risk Aversion

Recall the original investment strategy for determining currency futures prices under risk neutrality. If market participants are risk-averse, then the futures price is likely to change (in much the same way that the currency forward price changed under risk aversion).

If we apply the expected utility argument used previously, the expected value of the investment strategy now is

\[ rF^*_0 = E(\tilde{M}_T \tilde{R} \tilde{S}_T). \]

By applying our mathematical identity that the expected value of a product is equal to the product of the expected values plus their covariances, we have

\[ rF^*_0 = E(\tilde{M}_T)[E(\tilde{R})E(\tilde{S}_T) + COV(\tilde{R}, \tilde{S}_T)] + COV(\tilde{M}_T, \tilde{R} \tilde{S}_T). \]

Using eq. 13.23 gives

\[ rF^*_0 = E(\tilde{S}_T) \left[ \frac{E(\tilde{R})}{(1 + \gamma Y_0)^T} \right] + COV(\tilde{R}, \tilde{S}_T) \left[ \frac{1}{(1 + \gamma Y_0)^T} \right] + COV(\tilde{M}_T, \tilde{R} \tilde{S}_T). \]

Equation 13.27 states that the current futures price, \( rF^*_0 \), differs from the expected spot exchange rate at contract delivery because of three premia. The first premium, \( E(\tilde{R})/(1 + \gamma Y_0)^T \), represents a term premium, which is constant across all currencies. It arises from the existence of liquidity premia in the term structure when investors are risk-averse. There is a reduction in the average maturity (duration) of the cash flows associated with the process of marking to the market. The second premium, \( COV(\tilde{R}, \tilde{S}_T)/(1 + \gamma Y_0)^T \), represents a reinvestment rate premium associated with daily ressettlement and nonconstant rates of interest. It is the same premium that occurs in eq. 13.19. The third premium, \( COV(\tilde{M}_T, \tilde{R} \tilde{S}_T) \), is a hedging or risk premium that derives from the relation between the futures payoff and an investor's marginal rate of substitution. It is analogous to the premium appearing in eq. 13.24 for currency forward prices under risk aversion.

Again, the signs and magnitudes of these premia represent an empirical issue. Also, the last two premia in eq. 13.27 may be nonconstant over time.

The Impact of the Premia

Equation 13.27 yields the currency futures price under the conditions that investors are risk-averse and interest rates are nonconstant. These are very realistic conditions. As a result of these conditions, currency forward and futures prices differ, and the futures price differs from the expected future spot rate because of three premia. The magnitudes and signs of these premia are vague, and two of these premia are likely to be nonstationary.

The impact of these premia on international investment and international financial management is threefold. First, the futures price cannot be used blindly as an indicator of the market's expected future spot exchange rate. It is a biased estimate. Second, the expected return on a futures contract may be negative. If the contract has a tendency to reduce risk, then a hedging premium will be commanded in the marketplace. And third, multinational corporations that employ currency futures should not necessarily anticipate a share price appreciation from...
Regression Estimates of the Unbiased Forward Rate Hypothesis

Equation 13.10 can be expressed in regression form as

\[
\ln(S_t) = \alpha + \beta \ln(F_{t-1}) + \epsilon_t
\]

where \( S_t \) is the spot rate for currency \( j \) at time \( t \), \( F_{t-1} \) is the forward rate for currency \( j \) at time \( t - 1 \), \( \alpha \) and \( \beta \) are time-invariant parameters, and \( \epsilon_t \) is an error term. Under the assumption that the forward rate is an unbiased predictor of the subsequent spot exchange rate, \( \alpha = 0 \) and \( \beta = 1 \), and \( \epsilon_t \) is white noise. Failure to reject this joint hypothesis implies that \( F_{t-1} \) contains all the relevant information for the prediction of \( S_t \).

The preceding regression is estimated using monthly exchange rate data over the floating rate period January 1974 to August 1983. All data are end-of-period values from Data Resources, Inc. for the Canadian dollar, French franc, deutsche mark, and British pound. Both spot and forward rates are expressed as U.S. dollar prices per unit of foreign currency, and 30-day forward rates are used to avoid the potential serial correlation involved with overlapping data.

The results of the regression analysis follow. The method of estimation is ordinary least squares, although similar results are obtained when using Zellner's seemingly unrelated regression technique. The high \( R^2 \) values suggest that all equations perform very well. With the exception of the constant term (\( \alpha \)) for the mark, neither the individual hypotheses that \( \alpha = 0 \) and that \( \beta = 1 \) nor the joint hypothesis that \( \alpha = 0 \) and \( \beta = 1 \) can be rejected at the 5% significance level. The Durbin-Watson (D.W.) statistics indicate the absence of first-order correlation.

These results generally support the unbiased hypothesis of the forward foreign exchange market, at least for the in-sample fit. The results do not necessarily imply that a hedging premium does not exist. However, they do suggest that any such premium is likely to be very small.

<table>
<thead>
<tr>
<th>Currencies</th>
<th>( \alpha )</th>
<th>( \beta )</th>
<th>( R^2 )</th>
<th>( F(2,113)^b )</th>
<th>D.W.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Canadian dollar</td>
<td>-0.002</td>
<td>0.989</td>
<td>0.97</td>
<td>0.455</td>
<td>2.133</td>
</tr>
<tr>
<td>(0.002)</td>
<td>(0.015)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>French franc</td>
<td>0.006</td>
<td>1.005</td>
<td>0.97</td>
<td>0.168</td>
<td>2.054</td>
</tr>
<tr>
<td>(0.029)</td>
<td>(0.017)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Deutsche mark</td>
<td>-0.037</td>
<td>0.956</td>
<td>0.94</td>
<td>2.289</td>
<td>1.976</td>
</tr>
<tr>
<td>(0.019)</td>
<td>(0.022)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>British pound</td>
<td>0.005</td>
<td>0.990</td>
<td>0.95</td>
<td>0.245</td>
<td>1.761</td>
</tr>
<tr>
<td>(0.014)</td>
<td>(0.021)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

a. The numbers in parenthesis are standard errors.
b. \( F(2,113) \) is for testing the joint hypothesis that \( \alpha = 0 \) and \( \beta = 1 \). The critical value for \( F(2,113) \) is 3.09 at the 5% significance level.

hedges. If the currency futures contract indeed reduces the riskiness of the MNC's stock, hedging will result in a commensurate cost that reduces the stock's expected return such that market value is unchanged. This condition occurs if markets are integrated and risk is priced uniformly across all market participants.

### EMPIRICAL EVIDENCE

The preceding analysis implies that (1) currency forward and futures prices may differ from the expected future spot rate, and (2) currency forward and futures prices may differ from one another. These results in turn imply that currency forward and futures contracts may offer different degrees of hedging effectiveness. We now summarize a body of empirical evidence that addresses these issues.

**Forward and Futures as Predictors of the Future Spot Rates**

Equation 13.10 implies that the forward rate is an unbiased predictor of the actual future spot exchange rate under risk neutrality. The conventional test of this unbiased forward rate hypothesis uses a regression...