Constructive Sales and Contingent Payment Options

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Abstract

This paper presents a product that obviates the constructive sales rules promulgated by the 1997 Tax Reform Act. The product is a contingent payment put option. It permits high net worth retail clients to postpone gains taxes, and is cost free when expiring out of the money. The product also accommodates a large loan-advance ratio.
1. Introduction

Envision a high net worth retail client whose wealth is heavily concentrated in a single common stock. Examples include investors who have held a strongly appreciating stock for many years, such as Cisco Systems or Home Depot. Or founders of successful initial public offerings, such as E-Bay, and venture capitalists or "angels" of start ups that in turn went public and exhibited substantial appreciation, such as E-phiphany. These individuals lack diversification and may lack liquidity, but selling their positions - either in whole or in part - may trigger large capital gains taxes and reduced corporate control as voting rights are lost. In addition, these investors may be precluded from immediately selling their stock holdings due to lock up agreements.

Prior to the 1997 Tax Reform Act, the two most common trading strategies for resolving such a dilemma were zero-cost collars and equity swaps. In the former an investment or commercial bank would sell a European-style put to the client and simultaneously buy from the client a counterpart call. The strike prices on the two options were established so as to insure that the collar had a zero cost at inception. In some cases, the strikes would be set the same, in which case the collar holder may be described as holding a synthetic short forward position on the stock, thus agreeing to sell the stock at option maturity for a pre-specified price. In other cases, the strikes would be set such that the call strike is above the current spot price of the stock and the put strike is below the current spot price of the stock. In these cases, the collar holder has limited his exposure to changes in the stock's price to the range between the put strike and the call strike. In either case, the stock would serve as collateral on the client's short call position, and the dealer could provide a loan to the client for a substantial fraction of the value of the client's stock holding. We will limit further discussion of collars to those cases constituting a synthetic forward.

The collar would be unwound prior to maturity and a new collar would then be established in order to perpetuate the arrangement. Thus the collar trades would have the economic effect of getting the client out of the concentrated and illiquid position while postponing the gains tax, maintaining the voting rights, and, in some circumstances, circumventing a lock up agreement. The dealer would delta hedge the collar by selling short the underlying stock, or by other means involving index trades. In the former case, short selling could occasion considerable downward price pressure or "slippage" in the stock price - particularly if the stock had little float. This slippage cost would commonly be passed to the client by setting the strike prices on the options around the average selling price of the stock sold short by the dealer.
In the equity swap strategy, the client would agree to pay the swap dealer the total return on the stock in return for receiving either a fixed swap rate.\(^1\) The notional principal on the swap would reflect the client's stock position value, the swap payments would be netted, and the swap rate would reflect the slippage occasioned by the dealer's delta hedging. The stock would again serve as collateral on the client's position, and the swap would therefore occasion a large loan-advance ratio. The swap would be unwound and a new position established in order to perpetuate the strategy.

However, part of the 1997 Tax Reform Act established that these (and similar) trading strategies constituted "constructive sales", meaning that capital gains taxes would be triggered just as if the stock position was sold outright by the client.

In reaction to this legislation, dealers now commonly offer clients a zero-cost collar where the strike prices on the call and put options are set at least 15 percent apart. For example, the call strike equals 110% and put strike equals 95% of the average selling price occasioned by the dealer's shorting. This "15% rule" apparently presents enough future variability in the client's position value that the collar trade does not trigger the constructive sales rules.

The purpose of this paper is to present an alternative product for these clients. The product is a contingent payment put option. Such an option has zero cost at inception and zero cost if it expires out of the money. Thus the product gives the client full upside capture should the stock increase in value. The product can also occasion less price slippage because dealers need to short less of the stock (relative to a collar trade) in order to delta hedge. In our opinion, the contingent payment put obviates the constructive sales rules and therefore offers clients the ability to postpone gains taxes while maintaining voting rights.\(^2\) In addition, the product occasions a large loan-advance ratio, albeit likely not as large as that occasioned by a collar strategy.

The next section describes the contingent payment put and discusses its pricing. The subsequent section addresses more subtle points related to slippage, loan-advance ratios, and dealer hedging.

### 2. Contingent Payment Puts

In a contingent payment put trade, the client pays nothing at contract inception and nothing if the underlying stock appreciates. However, the client pays a pre-specified amount if the stock depreciates such that the put expires in the money.

Here is an example. Suppose that the stock price at inception is $100 per share and that the put is for six months. If the stock price in six months is $100 or higher, then no payments are made by the client or by the dealer. If the terminal stock price is below $100, then the dealer pays the client the difference between the terminal price and $100,

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\(^1\) Alternatively, the fixed swap rate can be replaced by a floating swap rate, usually linked to LIBOR.

\(^2\) Traders should seek a legal opinion regarding this matter.
and the client pays the dealer the pre-specified amount. Let this amount be denoted by $V$. In our example, suppose that $V$ is $15 per share. Thus if the terminal price is, say $80, the dealer makes a net payment of $5 per share to the client. If the terminal price is $50, the dealer makes a net payment of $35. If the terminal price is $99, the client makes a net payment of $14. And so on. Thus the client has full upside capture and is guaranteed (abstracting from the credit risk presented by the dealer) a minimum value of $85 per share for the stock. Because of this, the dealer should be willing to advance a loan to the client in an amount approaching $85 per share held. The client's value diagram appears as follows:

Figure 1: Client's Value Diagram

An interesting question, of course, is, How should dealers and clients quantify the contingent payment amount, $V$?

To begin the task of addressing this question, suppose that, consistent with our illustration, the stock price ($S$) is now $100. Consider a six-month ($T = 0.50$) European-style put struck at $100 ($X$). Let the applicable six-month zero rate with continuous compounding ($r$) be 4%, and let the stock's six-month at-the-money volatility ($\sigma$) be 28.28427%. Also, let the stock be non-dividend paying.

Under the standard assumptions of Black and Scholes (1973), it can be shown that the price of this at-the-money put should be $6.94 (ignoring bid-ask spreads). Also, it can be shown that the probability that this put will expire in the money is exactly 0.50.
Now, there was once a product introduced by First Boston called a "zero-cost option". Here the client would pay nothing up front, but instead would pay the option premium plus interest at the end of the option's life. In our example the premium plus accrued interest would be $6.94e^{.04(.50)} = $7.08. So the zero-cost put would cost nothing at inception and have a payoff of $\max[0,\$100 - S_T] - $7.08, where $S_T$ is the terminal price of the stock. Equivalently, the payoff is $\max[-$7.08, $92.92 - S_T]$.

Continuing the analysis, note that the contingent payment put represents a variation on this theme - because the client pays nothing up front and nothing if the option expires out of the money. To determine the contingent payment $V$ in our example, simply divide the $7.08 by the probability that the put will expire in the money - which recall is 50%. That is, $V$ should be $7.08/0.50 = $14.16. (In our illustration, $V$ was set to $15 to accommodate a modest price spread for the dealer.) The intuition here is simple: To make the dealer whole, the dealer must set $V$ so that the expected receipt - in six months - is $7.08: $7.08 = 0.50 x S_T + 0.50 x V$, and solving for $V$ gives $14.16$.

Let us take another example. Suppose that a stock is priced at $60, the time period of interest is three months ($T = .25$), the interest rate is 5%, $X = 60, the stock's volatility is 50%, and the stock pays no dividends. Here the at-the-money put should be priced at $5.56 and the probability of the put expiring in the money is 52.985%. The dealer should receive $5.63 at the end of a zero-cost put. For the contingent payment put, the value of $V$ should be $10.63: 5.63 = .47015 x S_T + .52985 x V$, and solving for $V$ gives $10.63$. Here the dealer might set $V$ at, say $12.50 (a substantial spread). The client would then receive all of the upside capture of the stock and at most would lose $12.50 per share while never triggering the constructive sales rules. The dealer could extend to the client a loan of nearly $47.50 per share held.

Here is a general equation for determining $V$:

\[ V = S \left( 1 - \frac{N(-d1)e^{rT}/N(-d2)}{N(-d2)} \right), \]

where

\[ d1 = \left( r + \frac{(\sigma^2/2)}{T} \right) / \sigma \sqrt{T}, \]

\[ d2 = d1 - \sigma \sqrt{T}, \]

and

$N(\bullet) = \text{the standard normal distribution with upper limit of integration } \bullet$.

Consider our two previous examples. For the first, we have $S = 100, T = .50, r = .04, and \sigma = .2828147$. Using these input gives $d1 = .20$ and $d2 = 0$, and $N(-d2) = .5793$ and $N(0) = .5000$. Thus we have $N(-d2) = .4207$ and $N(0) = .5000$. Finally we have

$V = S \left( 1 - \frac{N(-d1)e^{rT}/N(-d2)}{N(-d2)} \right)$.

3 The derivation of this equation is quite complex and is available from the authors upon request. It is based on many of the same assumptions found in Black and Scholes (1973). The equation assumes 100% upside capture, that is, $S = X$, and an underlying stock that does not pay dividends. The equation can be readily modified to accommodate dividend-paying stocks, stock indexes, currencies, less than 100% upside capture, et cetera.
\[ V = 100 \{ 1 - \left( \frac{(0.4207)e^{0.04 \cdot 0.50}}{0.5000} \right) \} = $14.16. \]

For the second example we have input \( S = 60, T = 0.25, r = 0.05, \) and \( \sigma = 0.50. \) This gives \( d_1 = 0.1750 \) and \( d_2 = -0.0750, \) and \( N(-0.1750) = 0.43055 \) and \( N(0.0750) = 0.52985. \) Finally we have

\[ V = 60 \{ 1 - \left( \frac{(0.43055)e^{0.05 \cdot 0.25}}{0.52985} \right) \} = $10.63. \]

3. Subtle Issues

This section briefly addresses more subtle product issues related to dealer hedging, loan-advance ratios, and slippage.

For now, abstract from the issue of slippage and focus on the matter of loan-advance ratios. Also, recall our first example where the current stock price is $100 per share. In a traditional zero-cost collar trade (where for simplicity we assume that the strike prices of the call and put are both $100), the client is effectively short a forward contract with a forward price of $100 per share, and so the dealer is long the forward contract. Here the dealer should be willing to extend the client a loan for approximately the present value of the forward price (reflecting interest costs on the loan). On the other hand, with our contingent payment put where \( V = 15, \) the client is guaranteed no less than $85 per share, not $100. Thus the dealer will only extend a loan for approximately the present value of $85. In general, loan-advance ratios will be lower for contingent payment puts than forward-like collars (under the caveat of no slippage).

Now turn your attention to the matter of slippage. Under the collar trade, the dealer must short a substantial amount of stock in order to delta hedge his position. The dealer is long a call that has a positive delta. In addition, the dealer is short a put that has a negative delta, resulting in another positive delta position. Thus the dealer must short stock against both option positions in order to delta hedge, and this can occasion substantial downward price pressure or slippage. Once again, this slippage is traditionally passed through to the client by making the strike prices on the call and put equal to the average price received by the dealer on the short stock trades. In turn, this slippage can erode the loan-advance ratio under the collar strategy. For example, if the slippage is $10 per share, then the strikes in our first illustration will be set at about $90 per share, and the dealer will only extend a loan amount equal to about the present value of $90 per share. On the other hand, the contingent payment put should occasion less slippage. In our first example where \( S = 100 \) and \( V = 15, \) the dealer is effectively short an out of the money put, that is, a put with strike price $85. Thus the dealer will need to short sell far less stock than with the collar trade in order to delta hedge his position. In turn, this will occasion less slippage and the subsequent slippage-induced erosion of loan-advance ratios. At the end of the day, the contingent payment put could actually occasion a larger loan advance ratio if the slippage is substantially lower than that presented by a collar trading strategy.
Finally, focus on other hedging concerns faced by the dealer, specifically gamma and vega. With a forward-like collar trade, the dealer's gamma and vega positions are about neutral. With the contingent put, however, the dealer will be short gamma and vega. This can present some obstacles to the dealer (especially in light of recent years in which dealers who have been delta neutral and long gamma have enjoyed huge profits). To overcome this matter, a dealer should look to include the contingent payment put trades in a large, diversified book that is naturally long the broad equity market in both delta and volatility. Also, a dealer should keep in mind that the short gamma position is compensated, at least somewhat, by being long theta. Moreover, the product might be accorded hedge accounting treatment, or offered by a desk located outside the United States.

4. Conclusion

The constructive sales rules promulgated by the 1997 Tax Reform Act have made traditional tax postponing products - such as standard zero-cost collars and retail equity swaps - obsolete. This paper proposed an alternative product - the contingent payment put. This product offers the client full upside capture with a guaranteed downside given by the contingent payment. A model was presented for quantifying said payment. This model can be easily modified to accommodate variations of the basic contingent payment put as well as underlying stocks that pay dividends. The product occasions a large loan-advance ratio, albeit likely not as large as that of a zero-cost collar. A dealer needs to short less stock to delta hedge the contingent payment put than a collar, and so the product should occasion less slippage cost for the client. Still, a dealer will have greater gamma and vega exposure with a contingent payment put than with a collar product. Whether these risks are significant depends on the rest of the dealer's trading book, the accounting treatment accorded the product by the middle office, and the geographic location of the product desk.

References