10.12. With the notation in the text

\[ \frac{\Delta S}{S} \sim \phi(\mu \Delta t, \sigma \sqrt{\Delta t}) \]

In this case \( S = 50, \mu = 0.16, \sigma = 0.30 \) and \( \Delta t = 1/365 = 0.00274 \). Hence

\[ \frac{\Delta S}{50} \sim \phi(0.16 \times 0.00274, 0.30 \times 0.00274) \]

\[ = \phi(0.00044, 0.0157) \]

and

\[ \Delta S \sim \phi(50 \times 0.00044, 50 \times 0.0157) \]

that is,

\[ \Delta S \sim \phi(0.022, 0.785) \]

(a) The expected stock price at the end of the next day is therefore 50.022
(b) The standard deviation of the stock price at the end of the next day is 0.785
(c) 95% confidence limits for the stock price at the end of the next day are

\[ 50.022 - 1.96 \times 0.785 \text{ and } 50.022 + 1.96 \times 0.785 \]

i.e.,

\[ 48.48 \text{ and } 51.56 \]
11.7. In this case $\mu = 0.15$ and $\sigma = 0.25$. From equation (11.7) the probability distribution for the rate of return over a 2-year period with continuous compounding is:

$$\phi \left( 0.15 - \frac{0.25^2}{2}, \frac{0.25}{\sqrt{2}} \right)$$

i.e.,

$$\phi(0.11875, 0.1768)$$

The expected value of the return is 11.875% per annum and the standard deviation is 17.68% per annum.

11.8. (a) The required probability is the probability of the stock price being above $40 in six months' time. Suppose that the stock price in six months is $S_T$

$$\ln S_T \sim \phi(\ln 38 + (0.16 - \frac{0.35^2}{2})0.5, 0.35\sqrt{0.5})$$

i.e.,

$$\ln S_T \sim \phi(3.687, 0.247)$$

Since $\ln 40 = 3.689$, the required probability is

$$1 - N \left( \frac{3.689 - 3.687}{0.247} \right) = 1 - N(0.008)$$

From normal distribution tables $N(0.008) = 0.5032$ so that the required probability is 0.4968. In general the required probability is $N(d_2)$. (See Problem 11.22).

(b) In this case the required probability is the probability of the stock price being less than $40 in six months' time. It is

$$1 - 0.4968 = 0.5032$$
11.13. In this case $S_0 = 52$, $X = 50$, $r = 0.12$, $\sigma = 0.30$ and $T = 0.25$.

\[
d_1 = \frac{\ln(52/50) + (0.12 + 0.3^2/2)0.25}{0.30\sqrt{0.25}} = 0.5365
\]
\[
d_2 = d_1 - 0.30\sqrt{0.25} = 0.3865
\]

The price of the European call is

\[
52N(0.5365) - 50e^{-0.12\times0.25}N(0.3865)
\]
\[
= 52 \times 0.7042 - 50e^{-0.03} \times 0.6504
\]
\[
= 5.06
\]

or $\$5.06$.

11.14. In this case $S_0 = 69$, $X = 70$, $r = 0.05$, $\sigma = 0.35$ and $T = 0.5$.

\[
d_1 = \frac{\ln(69/70) + (0.05 + 0.35^2/2) \times 0.5}{0.35\sqrt{0.5}} = 0.1666
\]
\[
d_2 = d_1 - 0.35\sqrt{0.5} = -0.0809
\]

The price of the European put is

\[
70e^{-0.05\times0.5}N(0.0809) - 69N(-0.1666)
\]
\[
= 70e^{-0.025} \times 0.5323 - 69 \times 0.4338
\]
\[
= 6.40
\]

or $\$6.40$.

11.16. In the case $c = 2.5$, $S_0 = 15$, $X = 13$, $T = 0.25$, $r = 0.05$. The implied volatility must be calculated using an iterative procedure.

A volatility of 0.2 (or 20% per annum) gives $c = 2.20$. A volatility of 0.3 gives $c = 2.32$. A volatility of 0.4 gives $c = 2.507$. A volatility of 0.39 gives $c = 2.487$. By interpolation the implied volatility is about 0.397 or 39.7% per annum.
11.19. Using DerivaGem we obtain the following table of implied volatilities

<table>
<thead>
<tr>
<th>Strike Price ($)</th>
<th>Maturity (months)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>3</td>
</tr>
<tr>
<td>45</td>
<td>37.78</td>
</tr>
<tr>
<td>50</td>
<td>34.15</td>
</tr>
<tr>
<td>55</td>
<td>31.98</td>
</tr>
</tbody>
</table>

The option prices are not exactly consistent with Black–Scholes. If they were, the implied volatilities would be all the same. We usually find in practice that low strike price options on a stock have significantly higher implied volatilities than high strike price options on the same stock. This phenomenon is discussed in Chapter 17.

Chapter - 12

12.9. In this case $S_0 = 0.52$, $X = 0.50$, $r = 0.04$, $r_f = 0.08$, $\sigma = 0.12$, $T = 0.6667$, and

$$d_1 = \frac{\ln(0.52/0.50) + (0.04 - 0.08 + 0.12^2/2)0.6667}{0.12\sqrt{0.6667}} = 0.1771$$

$$d_2 = d_1 - 0.12\sqrt{0.6667} = 0.0791$$

and the put price is

$$0.50N(-0.0791)e^{-0.04\times0.6667} - 0.52N(-0.1771)e^{-0.08\times0.6667}$$

$$= 0.50 \times 0.4685e^{-0.04\times0.6667} - 0.52 \times 0.4297e^{-0.08\times0.6667}$$

$$= 0.0162$$

12.13. In this case $F_0 = 19$, $X = 20$, $r = 0.12$, $\sigma = 0.20$, and $T = 0.4167$. The value of the European put futures option is

$$20N(-d_2)e^{-0.12\times0.4167} - 19N(-d_1)e^{-0.12\times0.4167}$$

where

$$d_1 = \frac{\ln(19/20) + (0.04/2)0.4167}{0.2\sqrt{0.4167}} = -0.3327$$

$$d_2 = d_1 - 0.2\sqrt{0.4167} = -0.4618$$

This is

$$e^{-0.12\times0.4167}[20N(0.4618) - 19N(0.3327)]$$

$$= e^{-0.12\times0.4167}(20 \times 0.6778 - 19 \times 0.6303)$$

$$= 1.50$$

or $1.50$. 

4
12.15. In this case \( S_0 = 696, X = 700, r = 0.07, \sigma = 0.3, T = 0.25 \) and \( q = 0.04 \). The option can be valued using equation (12.5).

\[
d_1 = \frac{\ln(696/700) + (0.07 - 0.04 + 0.09/2) \times 0.25}{0.3\sqrt{0.25}} = 0.0868
\]

\[
d_2 = d_1 - 0.3\sqrt{0.25} = -0.0632
\]

and

\[
N(-d_1) = 0.4654, \quad N(-d_2) = 0.5252
\]

The value of the put, \( p \), is given by:

\[
p = 700e^{-0.07\times0.25} \times 0.5252 - 696e^{-0.04\times0.25} \times 0.4654 = 40.6
\]

i.e., it is \$40.6.

Chapter – 13

13.10. The delta of a European futures option is usually defined as the rate of change of the option price with respect to the futures price (not the spot price). It is

\[
e^{-rT}N(d_1)
\]

In this case \( F_0 = 8, X = 8, r = 0.12, \sigma = 0.18, T = 0.6667 \)

\[
d_1 = \frac{\ln(8/8) + (0.18^2/2) \times 0.6667}{0.18\sqrt{0.6667}} = 0.0735
\]

\( N(d_1) = 0.5293 \) and the delta of the option is

\[
e^{-0.12\times0.6667} \times 0.5293 = 0.4886
\]

The delta of a short position in 1000 futures options is therefore –488.6.

13.11. In order to answer this problem it is important to distinguish between the rate of change of the price of a derivative security with respect to the futures price and the rate of change of the price of the derivative security with respect to the spot price. The former will be referred to as the futures delta; the latter will be referred to as the spot delta. The futures delta of a nine-month futures contract to buy one ounce of silver is by definition 1.0. Hence, from the answer to problem 13.11, a long position in nine-month futures on 488.6 ounces is necessary to hedge the option position.

The spot delta of a nine-month futures contract is \( e^{0.12\times0.75} = 1.094 \) assuming no storage costs. (This is because silver can be treated in the same way as a non-dividend-paying stock when there are no storage costs.) Hence the spot delta of the option position is \(-488.6 \times 1.094 = -534.6\). Thus a long position in 534.6 ounces of silver is necessary to hedge the option position.

The spot delta of a one-year silver futures contract to buy one ounce of silver is \( e^{0.12} = 1.1275 \). Hence a long position in \( e^{-0.12} \times 534.6 = 474.1 \) ounces of one-year silver futures is necessary to hedge the option position.
13.14. In this case \( S_0 = 0.80, X = 0.81, r = 0.08, r_f = 0.05, \sigma = 0.15, T = 0.5833 \)

\[
d_1 = \frac{\ln(0.80/0.81) + (0.08 - 0.05 + 0.15^2/2) \times 0.5833}{0.15\sqrt{0.5833}} = 0.1016
\]

\[
d_2 = d_1 - 0.15\sqrt{0.5833} = -0.0130
\]

\[N(d_1) = 0.5405; \quad N(d_2) = 0.4998\]

The delta of one call option is \( e^{-r_f T} N(d_1) = e^{-0.05 \times 0.5833} \times 0.5405 = 0.5250 \).

\[
N'(d_1) = \frac{1}{\sqrt{2\pi}} e^{-d_1^2/2} = \frac{1}{\sqrt{2\pi}} e^{-0.00616} = 0.3969
\]

so that the gamma of one call option is

\[
\frac{N'(d_1)e^{-r_f T}}{S_0\sqrt{T}} = \frac{0.3969 \times 0.9713}{0.80 \times 0.15 \times \sqrt{0.5833}} = 4.206
\]

The vega of one call option is

\[
S_0\sqrt{T}N'(d_1)e^{-r_f T} = 0.80\sqrt{0.5833} \times 0.3969 \times 0.9713 = 0.2355
\]

The theta of one call option is

\[
- \frac{S_0N'(d_1)e^{-r_f T}}{2\sqrt{T}} + r_f S_0 N(d_1)e^{-r_f T} - rXe^{-rT}N(d_2)
\]

\[
= - \frac{0.8 \times 0.3969 \times 0.15 \times 0.9713}{2\sqrt{0.5833}}
\]

\[
+ 0.05 \times 0.8 \times 0.5405 \times 0.9713 - 0.08 \times 0.81 \times 0.9544 \times 0.4948
\]

\[
= -0.0399
\]

The rho of one call option is

\[
XTe^{-rT}N(d_2)
\]

\[
= 0.81 \times 0.5833 \times 0.9544 \times 0.4948
\]

\[
= 0.2231
\]

Delta can be interpreted as meaning that, when the spot price increases by a small amount \( (\text{measured in cents}) \), the value of an option to buy one yen increases by 0.5255 times that amount. Gamma can be interpreted as meaning that, when the spot price increases by a small amount \( (\text{measured in cents}) \), the delta increases by 4.206 times that amount. Vega can be interpreted as meaning that, when the volatility \( (\text{measured in decimal form}) \) increases by a small amount, the option's value increases by 0.2355 times that amount. Theta can be interpreted as meaning that, when a small amount of time \( (\text{measured in years}) \) passes, the option's value decreases by 0.0399 times that amount. Finally, rho can be interpreted as meaning that, when the interest rate \( (\text{measured in decimal form}) \) increases by a small amount the option's value increases by 0.2231 times that amount.
13.22. The delta indicates that when the value of the euro exchange rate increases by $0.01, the value of the bank's position increases by $0.01 \times 30,000 = $300. The gamma indicates that when the euro exchange rate increases by $0.01 the delta of the portfolio decreases by $0.01 \times 80,000 = 800. For delta neutrality 30,000 euros should be shorted. When the exchange rate moves up to 0.93, we expect the delta of the portfolio to decrease by $(0.93 - 0.90) \times 80,000 = 2,400$ so that it becomes 27,600. To maintain delta neutrality, it is therefore necessary for the bank to unwind its short position 2,400 euros so that a net 27,600 have been shorted. As shown in the text (see Figure 13.8), when a portfolio is delta neutral and has a negative gamma, a loss is experienced when there is a large movement in the underlying asset price price. We can conclude that the bank is likely to have lost money.

Chapter – 14

14.1. The standard deviation of the daily change in the investment in each asset is $1,000. The variance of the portfolio's daily change is

\[ 1,000^2 + 1,000^2 + 2 \times 0.3 \times 1,000 \times 1,000 = 2,600,000 \]

The standard deviation of the portfolio's daily change is the square root of this or $1,612.45$. The standard deviation of the 5-day change is

\[ 1,612.45 \times \sqrt{5} = $3,605.55 \]

From the tables of $N(x)$ we see that $N(-1.645) = 0.05$. This means that 5% of a normal distribution lies more than 1.645 standard deviations below the mean. The 5-day 95 percent value at risk is therefore $1.645 \times 3,605.55 = $5,931.

14.2. The duration model gives

\[ \frac{\Delta B}{B} = -D\Delta y \]

where $\Delta B$ and $\Delta y$ are the change in the value of the bond portfolio and its yield in one day, and $D$ is the modified duration of the portfolio. Since $D = 3.7$ and the standard deviation of $\Delta y$ is 0.09%, it follows that the standard deviation of the return on the bond portfolio in one day is $0.09 \times 3.7 = 0.3332\%$. The value of the portfolio is $4$ million. The standard deviation of the change in its value in one day is therefore $4,000,000 \times 0.003332 = $13,320. Since $N(-1.282) = 0.9$, the 20-day 90 percent value at risk is

\[ 13,320 \times \sqrt{20} \times 1.282 = $76,367 \]

14.11. The contract is a long position in a sterling bond combined with a short position in a dollar bond. The value of the sterling bond is $1.53e^{-0.05 \times 0.8}$ or $1.492$ million. The value of the dollar bond is $1.5e^{-0.05 \times 0.8}$ or $1.463$ million. The variance of the change in the value of the contract in one day is

\[ 1.492^2 \times 0.0006^2 + 1.463^2 \times 0.0005^2 - 2 \times 0.8 \times 1.492 \times 0.0006 \times 1.463 \times 0.0005 = 0.0000000288 \]

The standard deviation is therefore $0.000537$ million. The 10-day 99% VaR is $0.000537 \times \sqrt{10} \times 2.33 = $0.00396$ million.