Solutions for The Problems – Chapters 16-23

Chapter –16

16.2. In this case, $S = 60$, $X = 60$, $r = 0.1$, $\sigma = 0.45$, $T = 0.25$, and $\Delta t = 0.0833$. Also

\[
\begin{align*}
    u &= e^{r \Delta t} = e^{0.45 \times 0.0833} = 1.1387 \\
    d &= \frac{1}{u} = 0.8782 \\
    a &= e^{r \Delta t} = e^{0.1 \times 0.0833} = 1.0084 \\
    p &= \frac{a - u}{u - d} = 0.4998 \\
    1 - p &= 0.5002
\end{align*}
\]

The output from DerivaGem for this example is shown in the Figure 16.1. The calculated price of the option is $\$5.16$.

Figure 16.1  Tree for Problem 16.2
16.4. In this case $F = 198$, $X = 200$, $r = 0.08$, $\sigma = 0.3$, $T = 0.75$, and $\Delta t = 0.25$. Also

$$u = e^{\alpha \sqrt{2 \Delta t}} = 1.1618$$

$$d = \frac{1}{u} = 0.8607$$

$$a = 1$$

$$p = \frac{a - d}{u - d} = 0.4626$$

$$1 - p = 0.5373$$

The output from DerivaGem for this example is shown in the Figure 16.2. The calculated price of the option is 20.34 cents.

![Figure 16.2 Tree for Problem 16.4](image-url)
16.10. In this case, $S = 50$, $X = 49$, $r = 0.05$, $\sigma = 0.30$, $T = 0.75$, and $\Delta t = 0.25$. Also

\[ u = e^{\sqrt{\Delta t}} = e^{0.30\sqrt{0.25}} = 1.0126 \]
\[ d = \frac{1}{u} = 0.9874 \]
\[ \alpha = e^{r\Delta t} = e^{0.1\times0.0833} = 1.0084 \]
\[ p = \frac{\alpha - u}{u - d} = 0.5043 \]
\[ 1 - p = 0.4957 \]

The output from DerivaGem for this example is shown in the Figure 16.3. The calculated price of the option is $4.29. Using 100 steps the price obtained is $3.91.

![Figure 16.3 Tree for Problem 16.10](image-url)
Chapter –17–

17.16 The calculations are shown in the following table. For example, when the strike price is 34, the price of a call option with a volatility of 10% is 5.926, and the price of a call option when the volatility is 30% is 6.312. When there is a 60% chance of the first volatility and 40% of the second, the price is $0.6 \times 5.926 + 0.4 \times 6.312 = 6.080$. The implied volatility given by this price is 23.21. The table shows that the uncertainty about volatility leads to a classic volatility smile similar to that in Figure 17.1 of the text. This is an illustration of the point made in the text that when volatility is stochastic with the stock price and volatility uncorrelated we get a pattern of implied volatilities similar to that observed for currency options.

<table>
<thead>
<tr>
<th>Strike Price</th>
<th>Call Option Price 10% Volatility</th>
<th>Call Option Price 30% Volatility</th>
<th>Weighted Price</th>
<th>Implied Volatility (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>34</td>
<td>5.926</td>
<td>6.312</td>
<td>6.080</td>
<td>23.21</td>
</tr>
<tr>
<td>36</td>
<td>3.962</td>
<td>4.749</td>
<td>4.277</td>
<td>21.03</td>
</tr>
<tr>
<td>38</td>
<td>2.128</td>
<td>3.423</td>
<td>2.646</td>
<td>18.88</td>
</tr>
<tr>
<td>40</td>
<td>0.788</td>
<td>2.362</td>
<td>1.418</td>
<td>18.00</td>
</tr>
<tr>
<td>42</td>
<td>0.177</td>
<td>1.560</td>
<td>0.730</td>
<td>18.80</td>
</tr>
<tr>
<td>44</td>
<td>0.023</td>
<td>0.988</td>
<td>0.409</td>
<td>20.61</td>
</tr>
<tr>
<td>46</td>
<td>0.002</td>
<td>0.601</td>
<td>0.242</td>
<td>22.43</td>
</tr>
</tbody>
</table>

Chapter –18–

Table 18.1  Positions in Options in Problem 18.27

<table>
<thead>
<tr>
<th>Option Type</th>
<th>Strike Price</th>
<th>Maturity (years)</th>
<th>Position</th>
</tr>
</thead>
<tbody>
<tr>
<td>Call</td>
<td>1.00</td>
<td>2.00</td>
<td>+1.0000</td>
</tr>
<tr>
<td>Put</td>
<td>0.80</td>
<td>2.00</td>
<td>-0.1255</td>
</tr>
<tr>
<td>Put</td>
<td>0.80</td>
<td>1.50</td>
<td>-0.1758</td>
</tr>
<tr>
<td>Put</td>
<td>0.80</td>
<td>1.00</td>
<td>-0.0956</td>
</tr>
<tr>
<td>Put</td>
<td>0.80</td>
<td>0.50</td>
<td>-0.0547</td>
</tr>
</tbody>
</table>
20.7. In this case $L = 1000$, $\delta_k = 0.25$, $F_k = 0.12$, $R_X = 0.13$, $r = 0.115$, $\sigma_k = 0.12$, $t_k = 1.25$, $P(0, t_{k+1}) = 0.8416$.

\[ L\delta_k = 250 \]

\[ d_1 = \frac{\ln(0.12/0.13) + 0.12^2 \times 1.25/2}{0.12 \sqrt{1.25}} = -0.5295 \]

\[ d_2 = -0.5295 - 0.12 \sqrt{1.25} = -0.6637 \]

The value of the option is

\[ 250 \times 0.8416 \times [0.12N(-0.5295) - 0.13N(-0.6637)] \]

\[ = 0.59 \]

or $0.59$.

20.18. In equation (20.14), $L = 10,000,000$, $R_X = 0.05$, $F_0 = 0.05$, $d_1 = 0.2 \sqrt{4}/2 = 0.2$, $d_2 = -.2$, and

\[ A = \frac{1}{1.05^5} + \frac{1}{1.05^6} + \frac{1}{1.05^7} = 2.2404 \]

The value of the swap (in millions of dollars) is

\[ 10 \times 2.2404[0.05N(0.2) - 0.05N(-0.2)] = 0.178 \]

This is the same as the answer given by DerivaGem. (For the purposes of using the DerivaGem software note that the interest rate is 4.879% with continuous compounding for all maturities.)
20.23. The payoff from the swaption is a series of five cash flows equal to \( \max[0.076 - R, 0] \) in million of dollars where \( R \) is the five-year swap rate in four years. The value of an annuity that provides \$1 per year at the end of years 5, 6, 7, 8, and 9 is

\[
\sum_{i=5}^{9} \frac{1}{1.08^i} = 2.9348
\]

The value of the swaption in millions of dollars is therefore

\[
2.9348[0.076N(-d_2) - 0.08N(-d_1)]
\]

where

\[
d_1 = \frac{\ln(0.08/0.076) + 0.25^2 \times 4/2}{0.25\sqrt{4}} = 0.3526
\]

and

\[
d_2 = \frac{\ln(0.08/0.076) - 0.25^2 \times 4/2}{0.25\sqrt{4}} = -0.1474
\]

The value of the swaption is

\[
2.9348[0.076N(0.1474) - 0.08N(-0.3526)] = 0.039554
\]

or $39,554. This is the same answer as that given by DerivaGem. Note that for the purposes of using DerivaGem the zero rate is 7.696% continuously compounded for all maturities.
23.16. (a) Define $u_i$ as the expected proportion of the no-default value lost from defaults between time $i-1$ years and time $i$ years and $u_i$ as the exposure at time $i$ years. The expected loss from defaults is

$$\sum_{i=1}^{5} u_i v_i$$

In this case

$$u_1 = 1 - e^{-0.0020 \times 1} = 0.0020$$ $$u_2 = e^{-0.0020 \times 1} - e^{-0.0040 \times 2} = 0.0060$$ $$u_3 = e^{-0.0040 \times 2} - e^{-0.0055 \times 3} = 0.0084$$ $$u_4 = e^{-0.0055 \times 3} - e^{-0.0065 \times 4} = 0.0093$$ $$u_5 = e^{-0.0065 \times 4} - e^{-0.0075 \times 5} = 0.0111$$

The parameter $v_i$ is the value of an $i$ year European option to enter into a swap with life $5-i$ when fixed is received and floating is paid. From DerivaGem (in millions of dollars), $v_1 = 0.9114$, $v_2 = 0.9852$, $v_3 = 0.9060$, $v_4 = 0.5455$, and $v_5 = 0$. The expected loss from defaults is 0.020.

(b) In this case $v_1 = 2.47$, $v_2 = 2.50$, $v_3 = 1.74$, $v_4 = 0.91$, and $v_5 = 0$. The $u_i$’s are the same. The expected loss from defaults is 0.0424. This is greater than the expected loss from defaults when fixed is received because the term structure is upward sloping. Floating payments are expected to increase. A swap where fixed is received is likely to move out of the money (lowering the credit risk) and a swap where floating is received is likely to move in the money (increasing the credit risk).

(c) The value of an annuity paying $1 per year for five years is

$$e^{-0.04 \times 1} + e^{-0.048 \times 2} + e^{-0.053 \times 3} + e^{-0.055 \times 4} + e^{-0.056 \times 5} = 4.28$$

The spread required to compensate for credit risk (in million of dollars) is therefore $(0.020 + 0.0424)/4.28 = 0.0149$ per year. This is about 1.5 basis points.

(d) The payment made on the day of the default depends on the one-year LIBOR rate one year earlier. A swap option that includes this payment is therefore path dependent. It cannot be evaluated in a straightforward way using Black’s model or a tree. Monte Carlo simulation can be used.