SOLUTIONS TO THE PROBLEMS

2.13. The clearinghouse member is required to provide 20 x $2,000 = $40,000 as initial margin for the new contracts. There is a gain of (50,200 - 50,000) x 100 = $20,000 on the existing contracts. There is also a loss of (51,000 - 50,200) x 20 = $16,000 on the new contracts. The member must therefore add
40,000 -20,000 + 16,000 = $36,000
to the margin account.

2.14. The optimal hedge ratio is
0.8 x0.65/0.81 = 0.642
This means that the size of the futures position should be 64.2% of the size of the company's exposure in a 3-month hedge.

2.25. The minimum variance hedge ratio is
0.7 x 1.2/1.4 = 0.6
The beef producer requires a long position in 200000 x 0.6 = 120,000 lbs of cattle.
The beef producer should therefore take a long position in 3 contracts.

3.11. (a) The forward price, Fo, is given by equation (3.5) as:
Fo = 40e^{0.1} = 44.21
or $44.21. The initial value of the forward contract is zero.
(b) The delivery price K in the contract is $44.21. The value of the contract, f, after six months is given by equation (3.9) as:
f = 45- 44.21e^{-0.1*0.5}
= 2.95
i.e., it is $2.95. The forward price is given by:
45e^{0.1*0.5} = 47.31
or $47.31.

3.12. Using equation (3.7) the six month futures price is
150e^{(0.07-0.032)0.5} = 152.88
or $152.88.

3.16. The theoretical futures price is
0.65e^{0.1667*(0.08-0.03)} = 0.6554
The actual futures price is too high. This suggests that an arbitrageur should borrow U.S. dollars, buy Swiss francs, and short Swiss franc futures.

3.24. One S&P contract is on 250 times the index. The formula for the number of contracts that should be shorted gives
1.2x10,000,000/900x250 = 44.4
Rounding to the nearest whole number, 44 contracts should be shorted. To reduce the beta to 0.3, 44.4 x 0.7 = 31 contracts should be shorted.
4.7. Value of a contract is $108,468.75 \times 1,000 = 108,468.75$. The number of contracts that should be shorted is 

\[
6,000,000/108,468.75 \times 8.2/7.6 = 59.7
\]

Rounding to the nearest whole number, 60 contracts should be shorted.

4.15. There are 177 days between February 4 and July 30 and 182 days between February 4 and August 4. The cash price of the bond is therefore:

\[
110 + 177/182 \times 6.5 = 116.32
\]

The rate of interest with continuous compounding is \(2 \ln 1.06 = 0.1165\) or 11.65% per annum. A coupon of 6.5 will be received in 5 days (= 0.01366 years) time. The present value of the coupon is

\[
6.5e^{-0.01366 \times 0.1165} = 6.490
\]

The futures contract lasts for 62 days (= 0.1694 years). The cash futures price if the contract were written on the 13% bond would be

\[
(116.32 - 6.490)e^{0.1694 \times 0.1165} = 112.02
\]

At delivery there are 57 days of accrued interest. The quoted futures price if the contract were written on the 12% bond would therefore be

\[
112.02 - 6.5 \times 57/182 = 110.01
\]

Taking the conversion factor into account the quoted futures price should be:

\[
110.01/1.5 = 73.34
\]

4.18. The forward interest rate for the time period between months 6 and 9 is 9% per annum. This is because 9% per annum for three months when combined with 7.5% per annum for six months gives an average interest rate of 8% per annum for the nine-month period. For there to be no arbitrage the Treasury bill futures contract should lock an interest rate of 9% per annum for the period between 6 months and 9 months. Its price should therefore be

\[
1,000,000e^{-0.09 \times 0.25} = 977,751 \text{ or } $977,751.
\]

This analysis assumes no difference between forward and futures prices.

4.21. The company treasurer can hedge the company's exposure by shorting Eurodollar futures contracts. The Eurodollar futures position leads to a profit if rates rise and a loss if they fall. The duration of the commercial paper is twice that of the Eurodollar deposit underlying the Eurodollar futures contract. From equation (4.6) the contract price of a Eurodollar futures contract is $980,000. The number of contracts that should be shorted is, therefore,

\[
4,820,000/980,000 \times 2 = 9.84
\]

Rounding to the nearest whole number 10 contracts should be shorted.

4.22. The treasurer should short Treasury bond futures contract. If bond prices go down, this futures position will provide offsetting gains. The number of contracts that should be shorted is

\[
10,000,000 \times 7.1/91,375 \times 8.8 = 88.30
\]

Rounding to the nearest whole number 88 contracts should be shorted.
5.2. Company X has a comparative advantage in yen markets but wants to borrow dollars. Company Y has a comparative advantage in dollar markets but wants to borrow yen. This provides the basis for the swap. There is a 1.5% per annum differential between the yen rates and a 0.4% per annum differential between the dollar rates. The total gain to all parties from the swap is therefore 1.5% - 0.4% = 1.1% per annum. Since the bank requires 0.5% per annum, this leaves 0.3% per annum for each of X and Y. The swap should lead to X borrowing dollars at 9.6% - 0.3% = 9.3% per annum and to Y borrowing yen at 6.5% - 0.3% = 6.2% per annum. The appropriate arrangement is therefore as shown in Figure 5.2. All foreign exchange risk is borne by the bank.

![Figure 5.2](image)

5.3. The swap rates imply that the LIBOR yield curve is flat at 10% with semiannual compounding. In 4 months $6 million will be received and $4.8 million will be paid. In 10 months $6 million will be received and the LIBOR rate prevailing in 4 months' time will be paid. The value of the fixed rate bond underlying swap is

\[ 6e^{-0.3333 \times 0.1} + 106e^{-0.8333 \times 0.1} = $103.33 \text{ million} \]

The value of the floating rate bond underlying swap is

\[ (100 + 4.8)e^{-0.3333 \times 0.1} = $101.36 \text{ million} \]

The value of the swap to party paying floating is $103.33 - $101.36 = $1.97 million. The value of the swap to party paying fixed is -$1.97 million. These results can also be derived by decomposing the swap into forward contracts. Consider the party paying floating. The first forward contract involves paying $4.8 million and receiving $6 million in 4 months. It has a value of 1.2e^{-0.3333 \times 0.1} = $1.16 million. To value the second forward contract we note that the forward interest rate is 10% per annum with continuous compounding or 10.254% per annum with semiannual compounding. The value of the forward contract is

\[ 100 \times (0.12 \times 0.5 - 0.10254 \times 0.5)e^{-0.0833 \times 0.1} = $0.80 \text{ million} \]

The total value of the forward contract is therefore $1.16 + $0.80 = $1.96 million.

5.5. The swap involves exchanging the sterling interest of 20 x 0.14 = 2.8 million for the dollar interest of 30 x 0.1 = 3 million. The principal amounts are also exchanged at the end of the life of the swap. The value of the sterling bond underlying the swap is:

\[ 2.8/(1.11)^{1/4} + 22.8/(1.11)^{5/4} = 22.74 \text{ million} \]

The value of the dollar bond underlying the swap is

\[ 3/(1.08)^{1/4} + 33/(1.08)^{5/4} = $32.92 \text{ million} \]

Value of swap to party paying sterling is therefore

\[ 32.92 - 22.74 \times 1.65 = -$4.60 \text{ million} \]
The value of the swap to the party paying dollars is +$4.60 million.
The results can also be obtained by viewing the swap as a portfolio of forward contracts. The continuously compounded interest rates in sterling and dollars are 10.43% per annum and 7.70% per annum. The 3-month and 15-month forward exchange rates are $1.65e^{0.025 \times 0.0273} = 1.6388$ and $1.65e^{1.25 \times 0.0273} = 1.5946$. The value of the two forward contracts corresponding to the exchange of interest for the party paying sterling is therefore:

$$
(3 - 2.8 \times 1.6388)e^{-0.077 \times 0.25} = -$1.56 \text{ million}
$$

$$
(3 - 2.8 \times 1.5946)e^{-0.077 \times 1.25} = -$1.33 \text{ million}
$$

The value of the forward contract corresponding to the exchange of principals is:

$$
(30 - 20 \times 1.5946)e^{-0.077 \times 1.25} = -$1.72 \text{ million}
$$

The total value of the swap is -$1.56 - $1.33 - $1.72 = -$4.61 million.

**9.4.** Consider a portfolio consisting of
-1 : put option
+Ä : shares

If the stock price rises to $55, this is worth 55Ä. If the stock price falls to $45, it is worth 45Ä - 5. These are the same when

$$
45Ä - 5 = 55Ä
$$

or Ä = -0.50. The value of the portfolio in one month is -27.5 for both stock prices. Its value today must be the present value of -27.5 or -27.5e^{-0.05} = -26.16. This means that

$$
-pp + 50Ä = -26.16
$$

where pp is the put price. Since Ä = -0.50, the put price is $1.16. As an alternative approach we can calculate the probability, p, of an up movement in a risk-neutral world. This must satisfy:

$$
55p + 45(1 - p) = 50e^{-0.05}
$$

so that

$$
l0p = 50e^{0.05} - 45
$$

or p = 0.7564. The value of the option is then its expected payoff discounted at the risk-free rate or

$$
(0 \times 0.7564 + 5 \times 0.2436)e^{-0.05} = 1.16
$$

This agrees with the previous calculation.

**9.5.** In this case $u = 1.10$, $d = 0.90$, and $r = 0.08$ so that

$$
p = (e^{0.08 \times 0.5} - 0.90)/1.10 - 0.90 = 0.7041
$$

The tree for stock price movements is shown in Figure 9.1. We can work back from the end of the tree to the beginning as indicated in the diagram to give the value of the option as $9.61. The option value can also be calculated directly from Equation (9.8):

$$
e^{2x0.08 \times 0.5}(0.7041^2 \times 21 + 2 \times 0.7041 \times 0.2959 \times 0 + 0.2959^2 \times 0) = 9.61
$$
9.6. Figure 9.2 shows how we can value the put option using the same tree as in Figure 9.1. The value of the option is $1.92. The option value can also be calculated directly from Equation (9.8):

$$e^{-2x0.5x0.08}[0.7041^2 \times 0 + 2 \times 0.7041 \times 0.2959 \times 1 + 0.2959^2 \times 19] = 1.92$$

The stock price plus the put price is 100 + 1.92 = 101.92. The present value of the strike price plus the call price is $100e^{-0.08} + 9.61 = 101.92$. These are the same, verifying that put-call parity holds.