I. INTRODUCTION

The title of this essay is taken, of course, from the Gurley/Shaw (1960) monograph to remind the reader at the outset that the objective of constructing a unified theory of money and finance is an old one, one that has challenged theorists at least since J.R. Hicks's (1935) "Suggestion." That the attainment of this objective is still regarded as part of an agenda for future research suggests that there must be something difficult about the problem that earlier writers either did not see or did not adequately face. This paper is an attempt to identify this difficulty and to offer one way of dealing with it.

If it is easier today than it was in 1960 to identify exactly in which respects the theory of finance fails as monetary theory, this is largely due to rapid recent progress in the theory of finance. Theoretical research in finance is now conducted almost entirely within the contingent-claim general equilibrium framework introduced by Arrow (1964) and Debreu (1959). This is not an historical statement, for each of the three pillars of modern financial theory—portfolio theory, the Modigliani-Miller Theorem, and the theory of efficient markets—was discovered within different (and mutually distinct) theoretical frameworks, but all three have since been reformulated in contingent-claim terms, and it was this reformulation that revealed their essential unity and set the stage for many further theoretical advances.

This paper begins in Section II with a review of a simple version of the Fisherian model of real capital theory in contingent-claim terms and a review of the relationship of this model to various aspects of financial economics. A central feature of this model is that all trading occurs in a

*I am grateful to Nancy Stokey for criticism of an earlier draft and for comments by Arthur Kupferman, Allan Meltzer, Bennett McCallum, and Manuel Sanchez.
centralized market, with all agents present. In such a setting, the position of each agent is fully described by a single number: his wealth, or the market value of all the claims he owns. The command any one claim has over goods is fully described by its market value, which is to say all claims are equally "liquid."

If the point of a theory of money, or of "liquidity preference," is to capture the fact that, in some situations in reality, money has a relative command over other goods in excess of its relative value in centralized securities trading, then a successful theoretical model must place agents in such situations, at least some of the time. How, as a matter of modeling strategy, might this best be done?

I do not believe we have enough experience with alternative formulations to answer this question now, but the monetary model introduced in Section III employs a device used in Lucas (1982), Townsend (1982), and Lucas and Stokey (1983), in which agents alternate between two different kinds of market situations. Each period, they all attend a securities market in which money and other securities are exchanged. Subsequent to securities trading, agents trade in (implicitly) decentralized goods markets in which the purchase of at least some goods is assumed subject to the cash-in-advance constraint of the form suggested by Clower (1967).

The assumption of this model that agents regularly, if not continuously, trade in a centralized securities market admits a theory of securities pricing that is close to the standard barter theory reviewed in Section II. Yet interesting and fully operational modifications are required for a monetary system, so that pricing formulas differ in important ways from the barter versions that have been subjected to much recent empirical testing. These are reviewed in Section IV.

Section V turns to the question, central to the objectives of monetary theory though traditionally peripheral to the theory of finance, of methods for constructing monetary equilibria under alternative fiscal and monetary regimes. Here a simplified version of the model of Section III is studied to the point where one can begin to see what a full analysis would involve, and various simple examples are fully "solved." Section VI contains concluding comments.

II. THE THEORY OF FINANCE

The theory of finance, as the term is now generally understood, con-
sists of various specializations and applications of the Arrow-Debreu contingent-claim formulation of a competitive equilibrium for an economy operating through time, subject to stochastic shocks. As background for summarizing several of the main results in the theory of finance, and also for considering how this theory might be extended to include monetary elements, it will be useful to state a highly simplified version.

We consider an economy subject to exogenously given stochastic shocks, \( \{s_t\} \), where the realization of the vector \( s_t \) is public knowledge prior to any consumption or production activity in \( t \) and where the joint density \( f^t \) of \( (s_1, \ldots, s_t) \) is known to all agents. Use \( s^t = (s_1, \ldots, s_t) \) to denote the full history of shocks up to and including time \( t \). A commodity or good in this setting is idealized as a function \( c_t(s^t) \), the value of which denotes the quantity of the good to be exchanged (or consumed or produced) at date \( t \) contingent on the occurrence of the history \( s^t \). I will confine attention here to two consumption goods: a nonstorable, produced good, \( c_t \), and leisure, \( x_t \). The sequence \( \{c_t, x_t\} \) of pairs of functions \( c_t(s^t), x_t(s^t) \), each defined over all possible histories \( s^t \), provides a catalogue of an individual consumer's consumption for all dates, under all possible circumstances.1

Consumers will be taken to maximize expected utility:

\[
\max_{c^t, x^t} \sum_{t=0}^{\infty} \beta^t U(c_t(s^t), x_t(s^t)) f^t(s^t) ds^t.
\]

The shorthand

\[
\sum_{t=0}^{\infty} \beta^t U(c_t, x_t) f^t ds^t \tag{2.1}
\]

is taken to mean the same thing and will be used repeatedly below. Firms are assumed to have a technology:

\[
c_t + g_t + k_{t+1} = F(k_t, l-x_t, s_t) \tag{2.2}
\]

1Here and below I am simply setting out a notation useful for discussing technically-elementary aspects of various models. If a mathematically-rigorous exposition were to be provided, it would be necessary to specify the commodity space and functions defined in more detail, and phrases like "defined over all possible histories" would need elaboration or replacement.
describing the combinations of private consumption goods \( c_t \), government consumption goods \( g_t \), and end-of-period capital stocks \( k_{t+1} \) that can be produced when beginning-of-period capital stocks are \( k_t \), labor is \( 1-x_t \), and the shock history is \( s^t \). At this formal level one could consider many different consumers and firms, but it will economize on subscripts to consider one ("representing" many) of each.

Questions of government finance of the expenditure stream \( \{g_t\} \) will be kept simple, here and throughout the paper, by assuming that government has the ability to levy distortion-free, lump-sum taxes on consumers.\(^2\) Let \( e_t(s^t) \) denote contingent tax obligations of consumers at \( t \). To admit the possibility of deficit finance, let \( B_0 \) denote initial, goods-denominated debt obligations owed consumers by government.

To describe the trading possibilities open to these agents, and hence to formulate a definition of equilibrium, it is useful to keep in mind two quite different, but highly complementary scenarios, one of which is standard in general equilibrium theory and the other of which is closer to the traditions of financial and monetary theory. In the first, Arrow-Debreu, scenario, all agents are taken to convene at time 0, knowing \( s_0 \) and the distributions \( f^1, f^2, \ldots \) of future shocks, to trade in a complete range of sequences \( \{c_t, x_t\} \) of contingent claims on goods. In this trading the price \( n_t(s^t) \) of the contingent consumption claim \( c_t(s^t) \) and the price \(-n_{xt}(s^t)\) of a contingent claim on leisure \( x_t(s^t) \) are both dated functions of the shock history \( s^t \), so that, for example, the value of the claim \( c_t(s^t) \) is the product \( n_t(s^t)c_t(s^t) \) and the present value of an entire sequence \( \{c_t\} \) is\(^3\)

\[
\sum_{t=0}^\infty \int n_t(s^t)c_t(s^t)ds^t
\]

Here prices are quoted in an abstract unit-of-account, so a normalization like \( n_0 = 1 \) is permitted.

In this setting, firms choose \( \{c_t + g_t, 1-x_t, k_{t+1}\} \), given \( k_0 \) and \( \{n_t, n_{xt}\} \), to maximize

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\(^2\)See Lucas and Stokey (1983) for a normative analysis, in a context similar to this one, of government finance when all taxes distort.

\(^3\)Here and below, the normalization \( \int ds^t = 1 \) at all \( t \) is assumed.
subject to (2.2) for all \( t, s^t \). Call the value of this maximized objective function \( \pi \). Consumers are endowed with one unit of labor-leisure per period, they are liable for taxes, they own the firms, and they hold the outstanding government debt, so their budget constraint is:

\[
\sum_{t=0}^{\infty} \int [\pi_t(s^t)(c_t(s^t) + \theta_t(s^t)) - \pi_{xt}(s^t)(1-x_t(s^t))] ds^t,
\]

subject to (2.2) for all \( t, s^t \). Consumers choose \( (c_t, x_t) \), given \( (\pi_t, \pi_{xt}), (\theta_t), \pi, \) and \( B_0 \), to maximize (2.1) subject to (2.4).

This scenario, in which all equilibrium quantities and prices are set at time 0, conflicts (though very superficially) with the observation that in reality trading goes on all the time, concurrent with consumption and production of goods rather than prior to these activities. It also presupposes, though I believe this observation is equally superficial, a "large" number of traded securities. The next, alternative, scenario deals with the first observation or objection fully, sheds some light on the second, and in general permits a reinterpretation of a contingent-claim equilibrium of the sort sketched above that is much closer to traditional capital and monetary theory.

First, imagine that all agents meet at the beginning of every period \( t \), trading in contingent claims of exactly the same character as those traded in the Arrow-Debreu, time 0 market. Then, utilizing a notational complication that will shortly be dropped, let \( \pi_{0t}(s_{00}^t) \) denote the original prices (\( \pi_t(s^t) \) above) and, in general, use \( \pi_{t, s^t} \) to denote the price established at the time-market, if \( s^t \) has occurred up to that date, for goods dated \( s^t \) contingent on the history \( t \) from \( t+1 \) through \( t \). Then these markets at \( t > 0 \) are simply redundant, for arbitrage will enforce

\[
\pi_{0, t}(s_{00}^t) = \pi_{0, t}(s_{00}^t) \pi_{t, t}(s_{t, t}^t)
\]

over all dates and histories, and future trading will simply reconfirm trades agreed to at \( t = 0 \).
In this "sequence economy" reinterpretation of an Arrow-Debreu economy, one is free, without affecting the analysis of equilibria, to think of prices like $\pi_{t,s}(s^{t},t^{s^2}), \tau > t$ not as being set at time 0 but rather as being correctly or rationally expected (as of $t = 0$) to be set in the time-$t$ market should the history $s^{t}$ be realized. That is, one thinks of certain prices as being formally established at each date, in light of rational expectations as to how certain other prices will be set later. This reinterpretation evidently permits one to economize drastically on the variety of securities assumed to be traded at any one date. Special assumptions on preferences, technology, and shocks often permit still further economies, as will be seen below. In what follows, the formalism of the timeless Arrow-Debreu scenario will be used to generate equilibrium conditions, but it will be useful to keep this alternative-sequence interpretation in mind and, where possible, to think of these equilibrium conditions as describing the evolution of a competitive system with rational expectations.

The first-order conditions for the consumer's problem: maximize (2.1) subject to (2.4) include:

$$0 = \beta^{t} U_{C}(c_{t},x_{t})f^{t} - \lambda \pi_{t}^{t} , \text{ all } t, s^{t}, \quad (2.6)$$

$$0 = \beta^{t} U_{X}(c_{t},x_{t})f^{t} - \lambda \pi_{xt}^{t} , \text{ all } t, s^{t} \quad (2.7)$$

where the number $\lambda$ is the multiplier associated with (2.4).

The first-order conditions for the firm's problem: maximize (2.3) subject to (2.2) include:

$$0 = \pi_{t}^{t} - u_{t}^{t} , \text{ all } t, s^{t}, \quad (2.8)$$

$$0 = \pi_{xt}^{t} - u_{t}^{t}F_{x}(k_{t},l-x_{t},s^{t}) , \text{ all } t, s^{t} \quad (2.9)$$

and

$$0 = \int u_{t}^{t} F_{k}(k_{t},l-x_{t},s^{t})ds_{t} - u_{t-1}^{t} , \quad (2.10)$$

all $t \geq 1, s^{t-1}$.
where the functions $\mu_t - \mu_t(s^t)$ are the multipliers associated with the constraints (2.2). In addition, under suitable restrictions, certain boundary or transversality conditions "at infinity" are also necessary.

Equations (2.2) and (2.6)-(2.10) together with boundary conditions implicitly define the set of stochastic processes for quantities and prices that are equilibria for this economy. Eliminating multipliers from (2.6) and (2.7), equilibrium prices are given by:

$$\frac{\pi_t}{\pi_t} = \frac{U_x(c_t, x_t)}{U_c(c_t, x_t)}$$

and

$$\frac{\pi_t}{\pi_0} = \beta^t \frac{U_c(c_t, x_t)}{U_c(c_0, x_0)} f^t.$$  (2.12)

Eliminating prices as well from (2.6) - (2.10), equilibrium quantities must satisfy:

$$\frac{U_x(c_t, x_t)}{U_c(c_t, x_t)} = F_x(k_t, 1-x_t, s^t),$$  (2.13)

$$U_c(c_{t-1}, x_{t-1}) =$$

$$\beta \int U_c(c_t, x_t) F_k(k_t, 1-x_t, s^t) f^t_{t-1} ds_t, $$  (2.14)

where $f^t_{t-1} = f^t/f^{t-1}$ is the density of $s_t$ conditional on $s^{t-1}$. The marginal interpretations of (2.13) - (2.14) and their connections (2.11) - (2.12) to prices are familiar.

Equations (2.13) and (2.14) together with the technology (2.2) and suitable boundary conditions can sometimes be used to construct the equilibrium resource allocation $(c_t, x_t, k_{t+1})_{t=0}^\infty$, given the shocks $(s_t)_{t=0}^\infty$: they are the "stochastic Euler equations" of the system. See Brock (1982) for a useful illustration together with an exposition of their intimate connection to the Capital Asset Pricing Model of the theory of finance. Much of the existing theory of finance is a collection of obser-
vations based on these conditions or on more basic properties of the model from which they are obtained. In reviewing the main elements of this theory, in the remainder of this section, it will be convenient to divide these observations into three categories.

One important category of results consists of the various "equivalence theorems" that rest on the linearity of equilibrium price systems and the nature of consumers' budget sets. Thus, the value $\Pi$ of (2.3) is the equilibrium value of a claim to the entire net receipts stream, $R = (R_t(s^*))$, say, of the firm. Should the firm market this stream in the form of $n$ claims to the receipts streams $R_1, \ldots, R_n$, with $\varepsilon_i R_i = R_i$, each with value (at equilibrium prices) $\Pi_1, \ldots, \Pi_n$, it is evident that $\sum \varepsilon_i \Pi_i = \Pi$. The proof is simply an observation on the consumer's budget constraint (2.4): if consumers can choose whether the right-hand side of (2.4) is $\Pi$ or $\sum \varepsilon_i \Pi_i$, and if they are indifferent between these choices, these values must be the same. This is Hirshleifer's (1966) proof of the justly-celebrated Modigliani-Miller (1958) Theorem. The fact that the component receipts streams $R_i$ can be arbitrary sequences of contingent claims gives the arbitrage reasoning underlying the Modigliani-Miller Theorem a power, in applications, that may not be apparent, given the simplicity of its proof.

The "Ricardian equivalence theorem" of government finance is another application of the same reasoning.4 The budget constraint facing the government can be derived from the consumer's constraint (2.4) and the definition (2.3) of the firm's value $\Pi$. It is:

$$\tau_0 B_0 = \sum_{t=0}^{\infty} \int_{s} \tau_t (g_t - \theta_t) ds^t$$  \hspace{1cm} (2.15)

Clearly, for given $g_t$, any two debt-tax patterns $B_0, (\theta_t)$ and $B_0', (\theta_t')$ that satisfy (2.15) will imply the same budget sets for consumers, hence be consistent with the same equilibrium prices and quantities, and hence be "equivalent" economically.

The government budget constraint more frequently appears in the literature in a "flow" form that can be derived from the "stock" form (2.15) as follows. Let the time 0 deficit $\tau_0 (g_0 - \theta_0)$ plus retirement of outstanding debt $\tau_0 B_0$ be financed by the issue of new, contingent one-period debt $B_1(s_1)$. Then, for all realizations of $s_1$

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4See Barro (1979) for references as well as for a proof that does not rely on the infinitely-lived household device used here.
must hold, analogously to (2.15). Integrating (2.16) with respect to \( s_1 \) and subtracting from (2.15) gives:

\[
\pi_0(q_0 - \theta_0) + \pi_0q_0 - \int \pi_1 B_1 ds_1 = 0 ,
\]

which states that a current account deficit must be offset by net issues of new debt. The "flow constraint" (2.17) appears more frequently in the literature than does the "stock constraint" (2.15), but it should be clear that the latter is more fundamental and contains more information than does (2.17); a boundary condition is "lost" when (2.15) is differenced.

Notice that these equivalence results do not depend in any detailed way on the nature of technology or consumer preferences. They are simply consequences of the hypothesis that the system is in competitive equilibrium. A second category of results involves the manipulation of the first-order conditions (2.6)-(2.10), similarly taking the existence of equilibrium prices and quantities for granted but involving the technology and/or preferences in an essential way.

Let \((z_t)_{t=0}^\infty\) be an arbitrary sequence of contingent-goods claims, and consider the problem of pricing the remaining terms \((z_t)_{\tau=t}^\infty\) in terms of goods at time \( t \). In terms of the sequence economy prices \( \pi_{t \tau} \) defined above, this price, \( Q_t(z) \) say, is

\[
Q_t(z) = \sum_{\tau=t}^\infty \int \pi_{t \tau} z d(s^\tau) .
\]

The time-0 prices \( \pi_t \) are given by (2.12). Similarly, using the normalization on sequence economy prices \( \pi_{tt} = 1 \), (2.6) implies:

\[
\pi_{tt} = \beta^{t-t} \frac{U_c(c_t, x_t) f_t}{U_c(c_t, x_t) f_t} , \text{ all } t, \tau \geq t, s^\tau .
\]

Since \( f_t / f^t \) is the density of \( s^\tau \) conditional on \( s^t \), integrating with respect to this density involves applying the operator \( E_t(\cdot) \). Therefore, inserting the prices (2.19) into (2.18) gives the pricing formula:

\[
Q_t(z) = z_t + \sum_{\tau=t+1}^\infty \beta^{\tau-t} E_t \left[ \frac{U_c(c_t, x_t)}{U_c(c_t, x_t)} z_\tau \right] .
\]
The formula (2.20) implies, in turn,

\[ U_c(c_{t+1},x_{t+1})Q_{t+1} = \beta E_t[U_c(c_{t+1},x_{t+1})Q_{t+1}(z)] . \]  

(2.21)

If marginal utility \( U_c \) is roughly constant, either because utility is linear or because consumption does not vary much, if the discount factor \( \beta \) is near one, as would be the case if the time unit is short, and if no "dividend" \( z_t \) is paid in \( t \), (2.21) reduces to

\[ Q_t = E_t(Q_{t+1}) . \]  

(2.22)

This is the famous Martingale property of securities prices that formed the basis for the early tests of "market efficiency." Since the operator \( E_t(\cdot) \) is conditional on all components of the shock history \( s_t \), a vast variety of specific predictions is implicit in (2.22), so these efficiency tests introduced a degree of empirical stringency without precedent in economic research. Perhaps of even more lasting importance, they introduced a class of statistical tests in which stochastic elements were intrinsic to the economic reasoning underlying the hypothesis, as opposed to being added as an afterthought to a relationship motivated by deterministic theoretical arguments.

More recently it has been recognized that these two virtues do not depend crucially on the accuracy of the narrow conditions under which (2.22) follows from (2.21). Dividends can be measured, whether zero or not, the discount factor can be assigned any value or treated as a parameter to be estimated, and the utility function can be parameterized and its arguments measured from observed time series. Hence, (2.21) as it stands is a statistical hypothesis or can easily be made into one, with implications not appreciably weaker than the original efficiency hypothesis (2.22). A number of studies have pursued this idea in a variety of econo-

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5See Fama (1970) for a valuable, early survey of a literature that begins with Fama (1965).
metrically sophisticated ways.\textsuperscript{6}

It should perhaps be emphasized that the hypothesis (2.22) is merely an example of a variety of conceptually similar hypotheses. The return stream \( \{z_t\} \) is arbitrary and can be matched to observed streams in many ways. There are many possible specifications of the set on which \( \mathbb{E}_t(\cdot) \) is conditioned. Moreover, with many consumers, (2.22) must hold separately for all. Finally, similar tests could as well be based on the firm's first-order conditions (2.10), for each firm separately, and for each type of capital good.

Neither the equivalence theorems based on the linearity of price systems nor the efficiency tests based on marginal conditions require solving the model for the behavior of economically-determined variables, given the behavior of shocks of various kinds or even the verification that such solutions exist. They are simply implications that follow from the hypothesis that the model (which in practice is typically not even fully specified) has an equilibrium, and they follow from vacuous systems as easily as from internally-consistent ones. A third category of results, perhaps of less interest from the point of view of the theory of finance but obviously of essential importance to monetary theories designed to evaluate the consequences of alternative policies, consists of methods for verifying the existence of, constructing, and characterizing solutions.

Bewley (1972) provides an existence theorem for a class of models much broader than the one discussed in this section, though its usefulness for calculating solutions has not been tested in practice. For an exchange economy with homogeneous agents the question of existence of equilibria is trivially resolved, and the formula (2.20) can be regarded as a solution for prices, given the behavior of quantities (and hence of marginal utilities). With production and capital accumulation, one can exploit the equivalence of optimal and equilibrium allocations (again, with homogeneous consumers) together with the possibility of calculating the former by dynamic programming methods to view (2.20) as an operational solution for the price of an arbitrary security. In Lucas and Stokey (1982), a method is provided for constructing optimal allocations with heterogeneous consumers,

\textsuperscript{6}See, for example, Hall (1978), Grossman and Shiller (1981), and Hansen and Singleton (1983). With the exception of Hall's original paper, the tests reported in these papers, as in Mehra and Prescott (1983), strongly reject the implications of their particular versions of the one-consumer, barter model reviewed in this section.
but with the environment restricted to be deterministic. A recent paper by Mehra and Prescott (1983) uses simulations of an exchange-economy version, adapted for stationarity in rates of growth of output, as the basis for a test on United States time series of output and securities prices.

From the point of view of classical hypothesis testing, nothing is gained in restricting attention to models that have solutions or solutions that can be characterized or simulated. If a first-order condition such as (2.22) is tested and rejected, one can view as rejected all models carrying this equality as an implication, without having to spell out each model or verify its internal consistency. Since there is no doubt that with rich enough data sets any such condition will be rejected, a research program based on purely negative application of first-order conditions has, in a sense, inexhaustible possibilities. Yet I think it is clear that pursuit of this line is at best a useful adjunct in the effort to obtain simulatable, necessarily "false" models that have the potential for shedding light on the questions that lead us to be interested in monetary theory in the first place.

III. MONEY IN A THEORY OF FINANCE

Insofar as the model of the preceding section succeeds in capturing the main features of the modern theory of finance, it is surely well-suited to illustrate what I identified in the Introduction as the main difficulty in integrating that theory with monetary theory. Whether trading in this model is viewed as occurring once, at time 0, or repeatedly, all trades occur in centralized markets with all agents simultaneously trading, and no security can enjoy a "liquidity" advantage over any other.

In this section and those following, I will interpret the cash-in-advance constraint suggested by Clower (1967) as capturing the idea that at least some trades are carried out away from centralized markets, so that money can be used to effect purchases that other securities, equally valued in centralized trading, cannot effect. This interpretation, elaborated on below, seems to me consistent with the many earlier applications of this
constraint. The present treatment will follow Lucas and Stokey (1983) in applying the cash-in-advance constraint to a subset of consumption goods only, permitting the possibility that consumers can substitute against the holding of money without substituting against consumption in general.

As in Section II, it will be convenient to shift back and forth between the timeless Arrow-Debreu scenario and its sequence-economy interpretation. To motivate the introduction of money, it is easier to think in terms of a sequence of markets, meeting each period. Think of trade in securities—the full range considered in Section II together with fiat currency and contingent claims on future currency—at the beginning of each trading day, say 9:00-9:15 a.m. After securities trading is concluded, production and exchange of current goods is carried out in the remainder of the day, in what I think of as a decentralized fashion.

By "decentralized" I mean firms that are spatially scattered, with workers selling labor to a particular firm going to a specific location, losing contact with other buyers of labor, and shoppers purchasing goods from a particular firm similarly obliged to go to its specific location. Insofar as goods and labor have been contracted for in advance, evidence of such contracts is simply presented by buyers and/or sellers at each location, with the indicated exchange then taking place. Equivalently, under rational expectations and the information structure assumed here, one may think of sellers in these transactions issuing invoices, or trade credit, to be cleared at tomorrow's securities market. In either case, the relevant price and quantity determination is made in the competitive securities markets.

\[7\text{See Kohn (1980), where the idea of a finance constraint is traced to Robertson (1940) and Tsiang (1956). See also Grandmont and Younes (1973); Foley and Hellwig (1975); Lucas (1980), (1982); and Townsend (1982).}\]

The convention adopted in this paper that all traders alternate synchronously between centralized and decentralized markets is only one of many ways of utilizing the cash-in-advance constraint to study situations with incomplete markets. For example, Grossman and Weiss (1982) and Rotemberg (1982) examine models in which some agents are always engaged in securities trading but never all agents at the same time. Comparison of their results with those cited above makes it clear that the characteristics of the equilibrium depend critically on the nature of the assumed trading rules and timing conventions.

The intergenerational models introduced by Samuelson (1958) provide another context for analyzing monetary issues within the general equilibrium framework used in the theory of finance. See Wallace (1980) for a useful description of recent developments.

I do not see any way of judging which of these approaches will prove most useful for which questions that does not involve working out the implications of theories of both types. By pursuing the particular Clower-type approach used here, I do not mean to suggest that I view this question as closed at the present time.
market, with only the actual execution of trades taking place elsewhere.

In the complete-markets model of Section II, all exchange can be thought of as executed in this way, so that while one may think of much economic activity as occurring in a decentralized way, nothing is lost, and much analytical simplicity is gained, by thinking of all economic decisions as arrived at in a single, centralized market. In this section, a subset of consumption goods—"cash goods"—will be thought of as exchanged in circumstances where the buyer is unknown to the seller, so that the latter is unwilling either to accept as payment claims issued in earlier securities trading or to issue trade credit to be discharged later. Such goods, if purchased at all, must be paid for with currency acquired in advance: at the securities market of that morning, or earlier.

With trading in securities and in goods assumed to take place at different times within a given trading period $t$, the information structure is complicated somewhat, relative to the last section. As before, let the history of shocks $s^{t-1} = (s_1, ..., s_{t-1})$ be public knowledge prior to all period $t$ trading. Let period $t$'s shock, $s_t$, be the pair $(s_{1t}, s_{2t})$, where $s_{1t}$ is realized and publicly known prior to any securities trading in $t$ and where $s_{2t}$ is known prior to any trading in goods and labor, but unknown until securities trading for $t$ is closed. Hence, agents must commit themselves to a portfolio decision on the basis of partial current information. Use $f_t$ as before to denote the density of $(s_t', s_{1t})$, and $f_{2t} = f_t f_{1t}$ the conditional density of $s_{2t}$, given $(s_{t-1}^t, s_{1t})$.

Let preferences distinguish between cash goods, $c_{1t}$, and credit goods, $c_{2t}$, as follows:

$$\sum_{t=0}^{\infty} \theta^t U(c_{1t}, c_{2t}, x_t) f_{20} f_t ds_{20} ds_t$$

(3.1)\]

The technology is assumed unchanged from Section II:

$$c_{1t} + c_{2t} + g_t + k_{t+1} = F(k_t, 1-x_t, s_t).$$

(3.2)\]

It remains to formulate the budget constraints of the household and the objective function of the firm in a way consistent with the trading scenario sketched above.

It is most convenient, as in Section II, to begin by picturing firms and households at 9:00 a.m. at $t=0$, reviewing the possibilities for $t=0, 1, 2, ...$ under all contingencies $s_{20}, s_1, s_2, ..., s_{10}$ being known and $k_0$...
being given. Let us begin with the firm.

In terms of the un-normalized, unit-of-account prices \( \{\pi_t, \pi_{xt}\} \), the firms wishes to choose history-contingent plans for total output \( \{c_{1t}(s^t) + c_{2t}(s^t) + g_t(s^t)\} \), labor input \( \{1-x_t(s^t)\} \), and capital \( \{k_{t+1}(s^t)\} \) to maximize expression (2.3). For the monetary economy, let \( p_t(s^t) \) be the dollar spot price of goods in \( t \) and \( W_t(s^t) \) be the dollar spot wage. Then, net dollar inflow in \( t \) is

\[
P_t(c_{1t} + c_{2t} + g_t) - W_t(1-x_t).
\]

These dollar receipts are available for distribution as dividends of securities trading in period \( t+1 \). This is true whether goods are sold (or labor bought) in \( t \) for currency, which is carried into \( t+1 \) as overnight balances, or for credit, with payment due at \( t+1 \). At this point, \( (s^t, s_{1t}, s_{1t+1}) \) is known. Let \( q_t(s_{t-1}, s_{1t}) \) be the price at 0 of a claim to one dollar at \( t \), contingent on \( (s_{t-1}, s_{1t}) \). Since this net-receipts expression is a function of \( s^t \) but not of \( s_{1t}, s_{1t+1} \), each dollar is valued at time 0 at \( \int q_t+1 ds_{t+1,t+1} \).

Then, in time-0 dollars, the firm's objective is to maximize:

\[
\sum_{t=0}^{\infty} \int q_{t+1} ds_{1,t+1} [p_t(c_{1t} + c_{2t} + g_t)] - W_t(1-x_t) ds_{20} ds^t = v_n
\]

subject to (3.2).

For the firm to be indifferent between contracting in advance at \( \{\pi_t, \pi_{xt}\} \) or at the dollar prices \( \{q_t, p_t, W_t\} \), the proportionality conditions

\[
\int q_{t+1} ds_{1,t+1} \cdot p_t = v_{\pi_t}
\]

\[
\int q_{t+1} ds_{1,t+1} \cdot W_t = v_{\pi_{xt}}
\]

must hold. Clearly, one may interchangeably think of firms as trading real goods claims in advance or trading claims to dollars. Similarly, one may view \( \int q_{t+1} ds_{1,t+1} \cdot p_t \) as a future price at 0 for goods in \( t \), or view \( p_t(s^t) \) as price expectation rationally held at 0 about prices in \( t \), and \( \int q_{t+1} ds_{1,t+1} \) as the price at 0 of one dollar at \( t+1 \), contingent on
The household, in contrast, is not indifferent between purchasing for cash and on trade credit as it is, by convention, obliged to acquire cash in advance:

\[ p_t(s^t)c_{1t}(s^t) \leq M_t(s^{t-1},s_{1t}) \]  

(3.6)

where \( M_t(s^{t-1},s_{1t}) \) is currency holdings at the close of securities trading in \( t \) and \( p_t c_{1t} \) is cash goods purchases during \( t \). This constraint (3.6) must hold for all realizations of the shock \( s_{2t} \), which is to say that money must be acquired to cover cash goods spending in advance of the realization of information relevant to this spending decision. This formulation is one way (suggested to me by Edwin Burmeister and Robert Flood) of introducing a precautionary demand for cash balances.\(^8\)

To develop the household's budget constraint, consider the household's sources of dollars as of 9:00 a.m. in period \( t+1 \) and its uses of dollars. Sources of dollars at \( t+1 \) include (I assume) wages earned during \( t \), \( W_t(1-x_t) \) and dollars held for cash good purchases in \( t \) and carried over unspent, \( M_t - p_t c_{1t} \). The time-0 price of these two items, contingent on \( (s_{20},s^t) \), is \( s_{q^{t+1}d_{s_{1,t+1}}} \). Uses of dollars at \( t+1 \) include payment for goods bought on trade credit in \( t \), \( p_t c_{2t} \), for taxes accrued in \( t \), \( p_t \delta_t \), and (possibly contingent on \( s_{1,t+1} \)) acquisitions \( M_{t+1} \) of cash for spending in \( t+1 \) or later. The time-0 price of the first two items is \( s_{q^{t+1}d_{s_{1,t+1}}} \) of the third, \( q_{c_{t+1}} \).

These sources and uses apply to all times \( t=0,1,2,\ldots \). In addition, the household owns the firm, with value at 0 of \( w_H \), initial cash (prior to securities trading at 0), \( \bar{M} \) (say), and initial holdings of dollar denominated government debt, \( B_0 \). These considerations motivate, after summing up and collecting terms, the budget constraint:

\[ \text{See Svensson (1983) for a useful development of some aspects of this formulation.} \]

Though an individual agent would, in this setup, be willing to pay to observe \( s_{2t} \) before committing himself to money holdings, it is the case (as Marianne Baxter pointed out to me) that if all agents know \( s_{2t} \) at the same time as \( s_{yt} \) (as opposed to later, as assumed in the text), the equilibrium resource allocation will not be affected. This is clear from the equation (5.16) derived below, in which the decomposition of \( s_t \) into \( (s_{yt},s_{2t}) \) is immaterial.
\[
\sum_{t=0}^{\infty} \int [q_{t+1} + l + p_t(x_{1t} + c_{2t} + \theta_t) - W_t(1-x_t) - M_t]
\]
\[+ q_t M_t] d\tau_2 d\tau_1 \leq \nu_1 + M + B_0. \tag{3.7}
\]

The Lagrangean for the consumer's problem involves a multiplier \(\lambda\) associated with (3.7) and multipliers \(\nu_t(s^t)\) associated with the constraints (3.6). It is:
\[
L = \sum_{t=0}^{\infty} \beta_t U(c_t, x_t) f_{20} f_{10} d\tau_2 d\tau_1
\]
\[-\lambda \sum_{t=0}^{\infty} \int q_{t+1} d\tau_1 d\tau_2 [p_t(c_{1t} + c_{2t} + \theta_t) - W_t(1-x_t) - M_t] + q_t M_t d\tau_2 d\tau_1.
\]
\[+ \lambda [\nu_1 + M + B_0] + \sum_{t=0}^{\infty} \nu_t M_t - p_t c_{1t} ds_2 d\tau_1 ds_1. \tag{3.8}
\]

The first-order conditions for the household's problem include, then,
\[
0 = \beta T U_1(c_t, x_t) f_{20} f_{10} - \lambda p_t q_{t+1} d\tau_1 d\tau_2 - \nu_t p_t,
\]
all \(t, s^t, s_{20}\), \tag{3.8}
\[
0 = \beta U_2(c_t, x_t) f_{20} f_{10} - \lambda p_t q_{t+1} d\tau_1 d\tau_2,
\]
all \(t, s^t, s_{20}\), \tag{3.9}
\[
0 = \beta U_3(c_t, x_t) f_{20} f_{10} - \lambda W_t q_{t+1} d\tau_1 d\tau_2,
\]
all \(t, s^t, s_{20}\), \tag{3.10}
\[
0 = \lambda q_{t+1} d\tau_2 d\tau_1 - \lambda q_t + \int \nu_t d\tau_1 d\tau_2,
\]
all \(t, s^{t-1}, s_{1t}, s_{20}\), \tag{3.11}
\[
M_t = p_t c_{1t}, \text{ with equality if } \nu_t > 0,
\]
all \(t, s^t, s_{20}\). \tag{3.12}
Here (3.11) sets to zero the derivative of \( L \) with respect to \( M_t \), its form reflecting the fact that \( M_t \) is a function of \((s_{20},s_{t-1},s_t)\), not \((s_{20},s_t)\).

The firm's first-order conditions include

\[
0 = p_t F_x(k_t,1-x_t,s_t) - W_t, \quad \text{all } t,s_t,s_{20} \tag{3.13}
\]

\[
0 = \int q_{t+1} \cdot ds_{1,t+1} p_t F(k_t,1-x_t,s_t) ds_t - \int q_{t} ds_{1,t} p_{t-1}, \\
\text{all } t \geq 1, s_t,s_{20} \tag{3.14}
\]

together with suitable transversality conditions. An additional equilibrium condition is given by the technology (3.2).

As in Section II, one may eliminate multipliers from (3.8)-(3.14) to obtain various, familiar relationships among marginal rates of substitution, of transformation, and relative prices. Thus, from (3.9), (3.10), and (3.13),

\[
\frac{U_x(c_t,x_t)}{U_2(c_t,x_t)} = F_x(k_t,1-x_t,s_t) = \frac{W_t}{p_t} \tag{3.15}
\]

analogous to (2.11) and (2.13), and from (3.9) and (3.14):

\[
U_2(c_{t-1},x_{t-1}) = 8 \int U_2(c_t,x_t) F(k_t,1-x_t,s_t) f_{t-1} ds_t, \tag{3.16}
\]

analogous to (2.14). These margins between credit goods and leisure and between credit goods at different dates are not disturbed by the addition of money.

The margin between cash and credit goods is, of course, affected by monetary considerations. From (3.8) and (3.9),

\[
\frac{U_2(c_t,x_t)}{U_1(c_t,x_t)} = \frac{\lambda/q_{t+1} ds_{1,t+1}}{\lambda/q_{t+1} ds_{1,t+1} + \nu_t} \tag{3.17}
\]

Consider first the case in which all uncertainty in \( t \) is resolved prior to
securities trading, so that \( s_t = s_{lt} \) and \( \int x_t ds^2_t = x_t \) for any variable \( x_t \). Then (3.11) gives:

\[
u_t = \lambda q_t - \lambda q_{t+1} ds_{t+1}
\]

and (3.17) becomes

\[
\frac{U_2(c_{t}, x_t)}{U_1(c_{t}, x_t)} = \frac{\int q_{t+1} ds_{t+1}}{q_t} = (1+r_t)^{-1},
\]

(3.18)

where \( r_t \) is the nominal rate of interest. Here the marginal-transactions benefit to holding cash is determined simultaneously with the one-period nominal bond price, so the latter measures exactly the relative price of cash and credit goods.

More generally, in the presence of a precautionary motive \( s_{2t} \), the nominal interest rate at \( t \) is given in terms of the time-0 bond prices \( q_t \) by

\[
(1+r_t)^{-1} = \frac{\int q_{t+1} ds_{2t} ds_{1,t+1}}{q_t}.
\]

(3.19)

Then dividing (3.8) and (3.9) through by \( p_t \), integrating both with respect to \( s_{2t} \), and substituting for \( \int u_t ds_{2t} \) from (3.11);

\[
(1+r_t)^{-1} = \frac{\int p_{t+1}^{-1} U_2(c_{t}, x_t) f_{2t} ds_{2t}}{\int p_t^{-1} U_1(c_{t}, x_t) f_{2t} ds_{2t}}.
\]

(3.20)

Roughly speaking, nominal interest rates must be equated to an expected marginal-transactions benefit of holding cash in situations where the money-holding decision must be made before these benefits can be known exactly. Further uses of these marginal conditions will be considered in Section IV and V.

In addition to these first-order conditions, the budget constraints of households and the government must hold in equilibrium. Using (3.3) and (3.7), the government budget constraint is
\[ \tilde{M} + B_0 = \sum_{t=0}^{\infty} \int \left( \frac{M_t(q_t - \eta_{t+1})}{q_t} \right) ds_{t+1} ds_t \]  

(3.21)

or: initial government liabilities \( \tilde{M} + B_0 \) must equal the present value of fiscal surpluses, \( \eta_t - q_t \), plus the present value of seigniorage profits, \( \tilde{M}_t(q_t - \eta_{t+1}) \).

A flow version of (3.21), analogous to (2.17), can be derived from the observation that condition (3.21) must hold for all \( t, s^t \), with \( M_{t-1} \) playing the role of \( \tilde{M} \) for \( t > 0 \). That is, for all \( t, s^t \):

\[ \int q_t(M_{t-1} + B_t) = \sum_{t=0}^{\infty} \int \left( \frac{M_t(q_t - \eta_{t+1})}{q_t} \right) ds_{t+1} ds_t \]  

(3.22)

Now updating (3.22) from \( t \) to \( t+1 \), integrating with respect to \( s_{2t} \) and \( s_{1,t+1} \), and subtracting from (3.22) gives:

\[ \int p_t \int q_{t+1} ds_{1,t+1} (g_t - \eta_t) ds_{2t} \]  

(3.23)

That is, a current fiscal deficit, exclusive of debt service, must be financed by net issues of new debt or by issues of money.

For the case, discussed above, in which all uncertainty is resolved prior to the close of securities trading (that is, \( s_{1t} = s_t \)), the nominal interest rate is defined in (3.18) and (3.23) becomes:

\[ (1 + r_t)^{-1} p_t(g_t - \eta_t) = \int \frac{B_{t+1} q_{t+1}}{q_t} ds_{t+1} - B_t + M_t - M_{t-1} \]  

(3.24)
If, in addition, one-period government debt is uncontingent,

\[(1+r_t)^{-1}p_t(g_t-\theta_t) = (1+r_t)^{-1}B_{t+1} - B_t + M_t - M_{t-1} \]

In general, with the government assumed to buy on trade credit, expenditures \(g_t\) that depend on \(s_{2t}\) involve the implicit issue of \(s_{2t}\)-contingent "bonds."

### IV. IMPLICATIONS FOR THE THEORY OF FINANCE

The incorporation of monetary elements into the real theory of finance as carried out in the last section has no effect on the "equivalence theorems" of private finance: i.e., the Modigliani-Miller Theory and its applications. The linearity of equilibrium price systems on which they rest is not altered by the addition of monetary complications.

The Ricardian equivalence theorem of public finance requires modification in a monetary setting, as follows. Government policy consists of contingent sequences of government expenditures, taxes, and money supply: \([g_t, \theta_t, M_t]\). If there is an equilibrium for a given policy in this sense, so that in particular (3.21) holds, and if \([g_t, \theta_t', M_t]\) is another policy also satisfying (3.21), then the same equilibrium is associated with this second policy. As with the barter version of the theorem, the proof follows from the observation that at given prices the policy change in question does not alter budget sets.

Notice that this equivalence argument does not go through if the policy-change involves \([M_t]\) as well as \([\theta_t]\), \([g_t]\) being held fixed. In general, different \(M_t\) paths will be associated with different equilibrium quantities and/or prices, and the seigniorage term

\[
\sum_{t=0}^{\infty} M_t [q_t - q_{t+1} ds_1, t+1] ds_2 ds_t
\]

on the right-side of (3.21) will not represent the proceeds from a lump-sum tax. One way to interpret this monetary amendment to the Ricardian equivalence theorem is as an "irrelevance theorem" about open-market operations. The path \([M_t]\) matters, in this economy, as does the path \([g_t]\) of real government consumption, but the route by which money is injected into or withdrawn from the system, changes in \([\theta_t]\), or in securities trading is of no independent importance.
Though these modifications for a monetary economy are technically minor, they completely reverse a popular reading of the equivalence theorem for barter economies to the effect that government deficits, being simply announcements of future taxes, do not matter. In a rational-expectations equilibrium, what is "announced" by a change in the current deficit or in the second term in the sum on the right of (3.21) is that something must change in subsequent terms so as to maintain (3.21). In the barter system, this "something" is either \( g_t \) or \( e_t \). In the monetary system, it could as well be future monetary policy that changes. One could catalogue various possibilities, but the main lessons are, first, the futility of trying to assess policy changes in terms other than changes in policy processes and, second, the impossibility of analyzing changes in monetary and fiscal processes independently of each other.

The securities-pricing formulas of Section II also require significant modification or reinterpretation for a monetary economy. Let \( z_t \) as before be an arbitrary sequence of claims to credit goods (and hence, in value, to goods-in-general) and let \( Q_t(z) \) be the price in terms of time-\( t \) goods of the remaining terms \( \{z_{\tau \mid \tau = t} \}^\infty_{t=1} \). Then

\[
Q_t(z) = \frac{p_t}{q_{t+1}} z_{t+1} \sum_{\tau=t}^{\infty} \frac{p_{\tau}}{q_{\tau+1}} ds_{\tau+1} \cdot z_{\tau} ds(t^T)
\]

and, using (3.10),

\[
Q_t(z) = z_t + \sum_{\tau=t+1}^{\infty} g_{\tau-t} \int_{t}^{T} \frac{U_2(c_{t+1,\tau})}{U_2(c_{t+1,\tau})} z_{\tau} ds(t^T)
\]

which exactly replicates (2.20) with \( U_C \) replaced by \( U_2 \). (The notation \( E_t(\cdot) \) is no longer useful because uncertainty is resolved at two points within period \( t \).) The analogues to (2.21) and (2.22) then follow from (4.1) as from (2.20).

The marginal utility \( U_2 \) of credit goods can be interpreted as the marginal utility of goods-in-general if the function \( V \) is defined by:

\[
V(c_{1t}, c_{2t}, x_t) = U(c_{1t}, c_{2t}, x_t)
\]

so (4.1) and (2.20) are very close. The arguments of these marginal utilities differ in the monetary and real cases, however. For example, in the absence of a precautionary motive \( s_{1t} = s_t \), \( c_{1t} = \frac{M_t}{P_t} \) in equilibrium,
and
\[ U_2(c_{t+1}, x_t) = V_2(c_{1t}, c_{1t} + c_{2t}, x_t) = V_2\left(\frac{M_t}{p_t}, c_{1t} + c_{2t}, x_t\right) \]
so that both real balances and leisure affect the marginal utility of total consumption. Clearly one would not wish to impose the hypothesis that \( V \) is a separable function of \( c_{1t} \) and \( c_{1t} + c_{2t} \), so this is not an inessential amendment.

Perhaps more fundamentally, securities in a monetary economy will not in general be claims to streams of real goods. For example, the owner of an equity share in the firm has title to a stream of dollar receipts, payable at the beginning of the period after these receipts are earned. In general, let \( \{Z_t\} \) be an arbitrary dollar stream, and assume that \( Z_t \) is a function of \( s^t \), including events \( s_{2t} \) that are unknown at the time of securities-trading in \( t \). We wish to find the dollar price \( R_t(Z) \), as a function of the information \( (s^{t-1}, s_{1t}) \) available at time \( t \) securities-trading, of the stochastic stream \( Z_t(s^t), Z_{t+1}(s^{t+1}), \ldots \).

Since the dollars \( Z_t \) become available only as of securities trading at time \( t+1 \),
\[ R_t(Z) = \sum_{s_1,s_2 \in \mathcal{S}_{t+1}} \mathbb{E}_{s_1,s_2} \frac{Z_t(s_1, s_2)}{q_{t+1}} ds_1 ds_2 ds^t \]
From (3.9),
\[ q_{t+1} ds_1, s_2 \mid s_1, s_2 = \lambda^{-1} p_t^{-1} e_t U_2(c_t, x_t) f_20^T r^t \]
while from (3.8) and (3.10),
\[ q_t = \lambda^{-1} p_t^{-1} U_1(c_t, x_t) f_20^T ds_2 t \]
Substituting these values into (4.2) gives:
\[ R_t(Z) = \sum_{s_1,s_2 \in \mathcal{S}_{t+1}} \mathbb{E}_{s_1,s_2} \frac{Z_t(s_1, s_2)}{q_{t+1}} ds_1 ds_2 ds^t \]
From the interest-rate definition (3.20), (4.3) is equivalent to
\[ R_t(Z) = \sum_{\tau=t}^{\infty} B^{-\tau-t} (1+r_c^{-1})^{-1} \left( \frac{Z_{\tau}}{p_t} \right) \frac{E[U_2(c_{\tau}, x_{\tau}) | s_{t-1}, s_{1t}]}{E[p_t^{-1} U_2(c_t, x_t) | s_{t-1}, s_{1t}]} \]  

(4.4)

In general, it is clear that if \( z_t \) in (4.1) is identified with \( \frac{Z_{\tau}}{p_t} \), it will not be the case that \( Q_t(z) \) as given by (4.1) will match \( R_t(Z) \) as given by (4.4). The natural deflations do not reduce nominal securities-pricing in a monetary world to the pricing of real securities in a barter world.

Obviously, the predictions of any theory will be altered if one introduces into it elements from which the original theory abstracted, so these observations do not amount to serious criticism of the application of a "barter" model to a monetary world. On the contrary, successful empirical applications of financial theory that abstracts from monetary complications testify to the good judgment of financial theorists in leaving such complications aside. The virtue of introducing monetary complications as done here is not to show that they affect the predictions of the theory (how could it be otherwise?) but to show that they do so in a fully operational, testable way.

V. IMPLICATIONS FOR THE THEORY OF MONEY

Viewed as examples or prototypes of monetary theories, our interest in models such as that sketched in Section III is not so much in direct testing of first-order conditions as in whether their solutions can be constructed and characterized, given assumed behavior for the various shocks to the system. This section uses simple examples to address this issue.

Constructive proofs that equilibria exist for models as discussed in Section III are even less well-developed than for barter systems. For pure exchange economies equilibrium prices are easily obtained, as in Lucas (1982). With production but without capital, less trivial results can be obtained, as illustrated below. Townsend (1982) has built on earlier work by Bewley (1972) and Heller (1974) to sketch a quite general proof of existence for a model close to that in Section III. The device used by Brock (1982) of exploiting the link between optimally-planned and equilibria allocations is not available in monetary systems. In general, because of
the "wedge" which the inflation tax introduces between the marginal rate of 
substitution in cash and credit goods and their unit marginal rate of 
transformation.

In this section, the model of Section III will be specialized by (1) 
excluding capital goods, (2) restricting the technology to the form

\[ c_{1t} + c_{2t} + g_t = (1-x_t) \xi_t , \]  
(5.1)

where \( \xi_t \) is a component of \( s_t \), and (3) assuming that the shocks

\[ s_t = (\xi_t, g_t \text{ other variables \ldots}) \]

follow a Markov process with transition density \( p(s',s) \) given by

\[ \int_{s'} p(u,s)du = \Pr(s_{t+1} \leq s' | s_t = s) . \]  
(5.2)

That is, the joint density \( f^t(s_1,\ldots,s_t) \) takes the form:

\[ f^t(s^t) = f^{t-1}(s^{t-1})p(s_t,s_{t-1}) \]  
(5.3)

Finally, the money-supply process is assumed given by a fixed function \( m \) of 
the current state:

\[ M_t = m(s_t) M_{t-1} . \]  
(5.4)

Under these additional assumptions, I will first seek functions \( c,x \)
and \( \psi \) such that the allocations \( c_t = (c_{1t}, c_{2t}) = c(s_t) \), \( x_t = x(s_t) \) and real 
balances \( \psi(s_t) = M_t/p_t \) satisfy (3.14)-(3.16) for all \( t, s_t \). Associated 
with an equilibrium in this stationary sense will be a nominal interest-
rate function \( r_t = r(s_t) \) and similar recursive expressions for all other 
equilibrium prices.

As a useful intermediate step, define the functions \( v(s) \) and \( w(s) \) by

\[ v(s) = \xi(s)U_2(c(s),x(s)) \]  
(5.5)

and

\[ v(s) + w(s) = \xi(s)U_1(c(s),x(s)) \]  
(5.6)

so that from (3.8) and (3.9):
\[ V(s_t) = \frac{\lambda q_{t+1} ds_{t+1}}{s_{t+1}^2} \]

and

\[ V(s_t) + w(s_t) = \frac{\lambda q_{t+1} ds_{t+1} + v_{t+1} M_{t+1}}{s_{t+1}^2} \]

We are headed for a functional equation in this function \( V(s) \).

From (3.9) and (3.10), an additional equilibrium condition is:

\[ U'_x(c(s), x(s)) \frac{U'_x(c(s), x(s))}{U'_x(c(s), x(s))} = \zeta \] (5.9)

From (5.4), (3.12) and the definition of \( w(s) \) and \( \psi(s) \),

\[ \psi(s) \geq c_1(s) \text{ with equality if } w(s) > 0 . \] (5.10)

For given \( s = s_t \), then, (5.1), (5.5), (5.6), (5.9), and (5.10) provide five "equations" in the six "unknowns" \( c_1(s), c_2(s), x(s), \psi(s), w(s), \) and \( V(s) \). Alternatively, for given \( v(s) \) and \( (\zeta, \eta) \) we can think of solving this system for \( c_1, c_2, x, \psi, \) and \( w \) as functions of \( (v, \zeta, \eta) \) or of \( (v(s), s) \). For \( s \)-values where the constraint (5.10) is binding, (5.1), (5.5), and (5.9) are solved for \( (c_1, c_2, x) \) as functions of \( (v(s), s), \psi(s) \) equals \( c_1(s) \), and \( w(s) \) is given by (5.6). For values where (5.10) is slack, \( w(s) = 0 \), (5.1), (5.9), and the condition \( U_1(c, x) = U_2(c, x) \) are solved for \( (c_1, c_2, x) \). Then \( \psi \) is obtained from (5.5) or (5.6). Let us assume that this static system can be solved uniquely, and denote the solution for \( w \) in particular by

\[ w(s) = h(v(s), \zeta, \eta) = h(v(s), s) \] (5.11)

This is the first step toward constructing an equilibrium.

Next, note that from (5.7),

\[ v(s_{t-1}) = \frac{\lambda q_t ds_t M_{t-1}}{s_{t-1}^2} \] (5.12)

From (3.11), integrated with respect to \( s_{t+1} \),
where the second equality follows from (5.7) and (5.8). Inserting from (5.13) into (5.12):

\[
v(s_{t-1}) = \beta \int_{M_t}^{M_{t-1}} \frac{v(s_t) + w(s_t)}{M_t} f_t^t \, ds_t.
\]  

(5.14)

Since this derivation holds for all states \(s_{t-1}\), we have, using the definitions of \(m(s)\) and \(p(s',s)\),

\[
v(s) = \beta \int \frac{v(s')+w(s')}{m(s')} p(s',s) \, ds'.
\]  

(5.15)

Using (5.11), (5.15) becomes:

\[
v(s) = \beta \int \frac{v(s')+w(s',s)}{m(s')} p(s',s) \, ds'.
\]  

(5.16)

Equation (5.15) arises from the marginal balancing undertaken by an agent in the goods market in state \(s\), deciding whether or not to spend an additional dollar on credit goods. The utility of this marginal expenditure is

\[
\frac{1}{p_t} U_2(c_t, x_t) = \frac{1}{M_t} v(s_t).
\]

The utility foregone is, from (5.15),

\[
\frac{\beta}{M_t} E_{t} \left[ \frac{v(s_{t+1}) + w(s_{t+1})}{M_{t+1}/M_t} \right] = \beta E_{t+1} \left[ \frac{1}{p_{t+1}} U_1(c_{t+1}, x_{t+1}) \right] = \beta E_{t} \left[ \frac{1}{p_{t+1}} U_1(c_{t+1}, x_{t+1}) \right] | s_t \).
\]

That is, a dollar spent on credit goods today involves a dollar unavailable for spending on cash goods tomorrow. Since \(m\) is a given function, equation (5.16) is a functional equation in the single unknown function \(v(s)\).
Solving it is the second step toward constructing an equilibrium.

Third and finally, an equilibrium must satisfy the government budget constraint (3.21). Given initial values $s_0$ of $s_t$, the transitions $p(s',s)$, the function $m(s)$, and the functions $c, x, \psi, w$ and $\nu$ just solved, the right side of (3.21) is determined. Substituting from (5.7) and (5.8) into (3.21) gives

$$\lambda(M + B_0) = \sum_{t=0}^{\infty} \nu(s_t) - \nu(s_{t+1}) \int_{t=0}^{\infty} f_t ds_t.$$  \hfill (5.17)

From (5.7), the multiplier $\lambda$ is

$$\lambda = \frac{\nu(s_0) f_0 ds_0}{M_0 q_0 ds_0} = \frac{1 + r_0}{M_0} \int_{t=0}^{\infty} f_t ds_t.$$  \hfill (5.18)

Policies $[g_t, \theta_t], M, B_0,$ and $m(s_t)$ are associated with equilibrium allocations only if the implied values of $w, \psi, \nu$ and $\lambda$, calculated as above, satisfy (5.17).

The construction just sketched can be illustrated by examples. To begin with the simplest, consider the deterministic case where $s$ (and hence $m(s)$) is constant. Then (5.15) becomes

$$mv = \beta (v + \nu)$$

so that (5.5) and (5.6) give

$$\frac{U_1(c,x)}{U_2(c,x)} = \frac{m}{\beta}$$  \hfill (5.19)

(provided $m \geq \beta$). Then, with $(\varepsilon, g)$ constant as well as $m$, (5.17), (5.9), and (5.1) are solved for $(c, x)$ as functions of $(\frac{m}{\beta}, \varepsilon, g)$. Real balances $\psi$ are equal to $c_1$, and the equilibrium values of $\nu$ and $w$ are, respectively, $c_1 U_2(c, x)$ and $(\frac{m}{\beta} - 1)c_1 U_2(c, x)$. The constant nominal rate of interest $r$ is, from (3.18) and (5.19), given by

$$(1 + r)^{-1} = \frac{\beta}{m}.$$  \hfill (5.20)

This completes the first two steps in constructing an equilibrium for this deterministic case. For the third, insert the constant values of $w, \nu, g, \theta$, and $\psi$ into (5.17) and use (5.18) and (5.20) to get:
With a noninflationary monetary policy \( (M_0 = \bar{M} \text{ and } m = 1) \), the price level is constant at \( \psi^{-1} \bar{M} \) and (5.21) implies:

\[
\sigma \theta = \sigma g + r \psi \frac{B_0}{\bar{M}} .
\]

That is, tax receipts must equal government purchases plus service of the original real debt. (That \( \psi \) and \( g \) are discounted by \( \sigma \) reflects the convention that taxes and government spending are carried out on credit, and interest is due in cash.)

Treating \( M_0 \) as a free parameter amounts to permitting an initial, arbitrary open-market operation before pursuing the policy \( m \). Thus, if all debt is initially "monetized," \( M_0 = \bar{M} + B \), (5.21) gives

\[
g - \theta = \psi(m-1)
\]

so that real deficits of \( g - \theta \) can be perpetually financed by real seigniorage revenues \( \psi(m-1) \). As \( M_0 = \bar{M} \), \( g - \theta = \psi \left( \frac{m}{\bar{M}} - 1 \right) \), the maximum seigniorage revenue for given \( m \).

In general, it is clear from (5.21) that monetary and fiscal policies cannot be set in an unrestricted way. Equation (5.21) may be read as defining the equilibrium tax rate \( \psi \) consistent with given \( m, g, \bar{M}, M_0 \) and \( B \), or as fixing some other "free parameter" of the monetary-fiscal system.

As a second example, let the shocks \([s_t] \) be independent and identically distributed, with \( s_{2t} = (\xi_t, g_t) \), with \( s_{1t} \) determining monetary shocks, independent of \( s_{2t} \), and with \( \xi_t \) constant at \( \theta \). That is, in each period a monetary shock is realized, securities are traded, real shocks are realized (necessitating government deficits or surpluses on trade credit), and trading is concluded. From (5.16) in this case, \( \nu(s) = k \) for some constant \( k \). Then \( c, x, \psi \) and \( w \) are functions of \( k \) and the real shocks \( s_{2t} = (\xi_t, g_t) \). The value of \( k \) is implicit in

\[
k = \bar{M} E \left( \frac{k + h(k, s_{2})}{m(s_{1})^{2}} \right) .
\]

The nominal interest rate is, from (3.20), (5.5), and (5.6):
\[
[1 + r(s_1)]^{-1} = \frac{\int \frac{v(s)}{m(s_1)} p_2(s_2) ds_2}{\int \frac{v(s) + w(s)}{m(s_1)} p_2(s_2) ds_2} = \frac{k}{k + \int h(k, s_1) p_2(s_2) ds_2}.
\]

Applying (5.22) to the numerator and cancelling:

\[
(1 + r)^{-1} = AE(\frac{1}{m(s_1)})
\]

which may be compared to (5.20).

As a variation on this last example, retain the assumptions of independence but assume that all shocks are realized prior to securities trading. Then again, \(v(s)\) is constant at a value \(k\) satisfying (5.22). Nominal interest is now given by

\[
[1 + r(s)]^{-1} = \frac{k}{k + h(k, s, g)}
\]

which varies with the real shock \((s, g)\). The formula comparable to (5.20) is, in this case.

\[
[1 + r(s)]^{-1} = AE(\frac{1}{m_s}) \frac{k + E(h(k, s))}{k + h(k, s)}
\]

Inserting this information into (5.20) one finds that the magnitude

\[
[1 + r(s_t)]\frac{M + B_0}{M_0} - r(s_t) + \frac{g_t - \theta}{v(s_t)}
\]

must be constant with respect to \(s_t\). Since this condition will hold only coincidentally, the conclusion is that, in general, no equilibrium exists for this case. With taxes fixed at \(\theta\) and government spending \(g_t\) stochastically determined, the fact that interest rates vary with real shocks makes the required value equality (3.20) impossible to maintain under all circumstances. A compensating tax or open-market policy is required for internal consistency.

These three examples all involve serially independent shocks. From (5.16) it is clear that, in general, \(v(s_t)\) will vary with \(s_t\) due only to
the extent that $s_t$ conveys information about future shocks. The corresponding "general" expression for the one-period nominal interest rate is:

$$[1+r(s_1)]^{-1} = \frac{\int v(s)p_2(s_2|s_1)ds_2}{\int (v(s) + w(s))p_2(s_2|s_1)ds_2}.$$  \hspace{1cm} (5.25)

Equation (5.25) is consistent with any co-movements of interest rates and real and nominal variables: the kind of single-equation vacuity familiar from other rational-expectations models and, in the case of interest rates, from the financial pages of any newspaper. The content in (5.25) must come from the fact that both $r(s_t)$ and the transition function $p(s_{t+1}|s_t)$ are, at least in part, observable.

The examples just worked through certainly do not constitute a complete analysis of the functional equation (5.16) and of its implications for pricing and resource allocation in a monetary economy. That analysis must await another occasion. They do suggest, however, that an operational theory can be based on (5.16), that such a theory will reproduce familiar results from deterministic theory when specialized to that case, and that its scope can be extended to make predictions under stochastic conditions for which conventional theory is plainly inadequate.

VI. CONCLUDING REMARKS

This paper was motivated by reference to earlier research directed at "unifying" the theories of money and finance and by the idea that success in this enterprise will involve capturing in a single model the sense in which securities are traded and priced in centralized "efficient" markets as well as the sense in which other goods are traded outside of these centralized exchanges, in situations where at least one security ("money") is valued higher than it "ought" to be on efficient market grounds alone. I think that this idea, in various forms, is present in most writing on money. One way of developing it was proposed in Sections III-V.

Ultimately, however, financial and monetary theory have quite different objectives, and however desirable theoretical "unity" may be, one can identify strong forces that will continue to pull apart these two bodies of theory. The real capital theory reviewed in Section II can be modified in two distinct directions: toward increasing generality in its assumptions.
about technology, demography, and preferences or toward the specificity needed to permit the application of constructive solution methods. The empirical failures of the simplest "representative consumer" models indicate that increased generality is required to produce success in the sense of first-order conditions that can pass the modern descendants of the efficiency tests of finance. Such generality is not difficult to obtain, and I expect much additional fruitful work in this direction.

The objective of designing simulatable models, an objective central to monetary theory, necessarily pulls in the opposite direction. The introduction of monetary elements, with the associated "wedge" of inefficiency, renders solution methods that exploit the links between equilibrium and optimality inapplicable and requires new analytical approaches. Sections III and V of this paper outline one possible approach and pursue it to the point where one can begin to get some idea of its potential.

If I am right that the relationship between financial and monetary economics is not, even ideally, one of "unity," it is nevertheless surely the case that there is much to be gained by close interaction. The power in applications of the contingent-claim point of view, so clearly evident in finance, will be as usefully applied to monetary theory. (This is not so much a prediction as it is an observation on the best recent work in the area.) The source of this power, I think, is the ability of this framework to permit the reduction of the study of asset demands to the study of demands for the more fundamental attributes to which assets are claims. If the theory of finance had remained content to postulate preferences over such "goods" as "debt" and "equity," financial textbooks would still be "explaining" corporate capital structure as unique tangency points of indifference curves to budget lines, on the picture that Modigliani and Miller showed to be spurious in 1958.

Postulating preferences or demands for "real balances" together with other "goods" is no more (and no less) useful than postulating a demand for the debt of a particular corporation. To get beyond this point, it is necessary to think more specifically about what it is, exactly, that money gives one access to. The Clower convention, as applied in this paper, is one way to do this. I have tried to illustrate some aspects of the power and flexibility of this approach and, in doing so, I have also revealed some of its limitations. Ultimately, the merits of a particular approach to the theory of money (as to the theory of anything else) will be judged less by its axioms than by whether it seems capable of giving reliable answers to the substantive questions that lead us to be interested in mone-
tary theory in the first place. This is an inquiry that has clearly only just begun.
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