Modeling the Federal Reserve Board Production Index

Here, we consider the Federal Reserve Board Production Index, monthly from January 1948 to December 1978 \((n = 372)\). The time series plot (see attached Figures) shows a rather jagged appearance, with apparently exponential growth overall, although there was clearly some trouble in the mid 1970’s (presumably the energy crisis). It seems reasonable to take logs, so we will hereafter work with "LPROD", the natural log of the original data. The time series plot of log production is roughly linear, but the jagged appearance remains. Perhaps some of this jaggedness is due to seasonality. The acf decays very slowly, indicating that differencing is required. The pacf shows significantly negative values at lags 13, 25, 37, etc. Again, this may be related to seasonality at lag 12 (one year), but it is a bit puzzling why the values are negative, and why they are occurring at lags which are one month beyond multiples of one year.

The first differences show some evidence of nonconstant variance. It is not clear what to do about this problem, since we already took logs of the raw data. We will proceed in spite of the problem. The acf of the first differences shows significantly positive values at lags 12, 24, 36, etc. The decay is quite slow when viewed at multiples of 12, indicating that we have "seasonal nonstationarity" and should take a seasonal difference. In other words, we need \(D = 1\). And since we have already taken an ordinary difference, we need \(d = 1\). So at this point, we are considering the multiplicative seasonal ARIMA \((p,1,q)\times(P,1,Q)_{12}\) models.

The seasonal (that is, 12-month) difference of the ordinary first difference has an acf which is significantly negative at lag 12. The pacf shows significantly negative values at the first few multiples of 12. Overall though, the decays in the acf and pacf are much faster here than they were before, indicating that the seasonal difference of the first difference is reasonably close to stationarity. This is good, since it can be shown for the multiplicative seasonal ARIMA \((p,1,q)\times(P,1,Q)_{12}\) model to hold, the seasonal difference of the ordinary first difference must be stationary.

We now turn to MINITAB, and try out a few \((p,1,q)\times(P,1,Q)_{12}\) models for LPROD, using \(AIC_c\) for guidance. The effective sample size is \(N = 359\), since we lose one data point due to the ordinary difference, and 12 more due to the seasonal difference. We obtain the following results.
The \( \text{AIC}_C \) values are simply supposed to be compared to each other; these forecasts produces forecasts for the production series itself. A CIC most negative criterion is defined here as

\[
\text{AIC}_C = N \log \left( \frac{SS}{N} \right) + 2(p + q + P + Q + 1) - \frac{N}{N - p - q - P - Q - 2} .
\]

The preferred model is the one with the smallest value of \( \text{AIC}_C \), which is the \((0,1,0)\times(0,1,1)_{12} \), with \( \text{AIC}_C = -3001 \). Forecasts can now be obtained from MINITAB. Taking the exponential function of these forecasts produces forecasts for the production series itself.

(Note that it doesn’t matter that the \( \text{AIC}_C \) values were all negative in this example. The \( \text{AIC}_C \) values are simply supposed to be compared to each other, so an additive constant in \( \text{AIC}_C \) is irrelevant. This means that we should pick the model whose actual (not absolute) value of \( \text{AIC}_C \) is the smallest. In the case where some of the \( \text{AIC}_C \) values are negative, we therefore pick the model which has the most negative value of \( \text{AIC}_C \).)