1) If \( \{x_t\} \) is \( AR(p) \), show that

\[
c_0 = \sigma^2 + \sum_{k=1}^{p} a_k c_{-k}.
\]

This formula is the Yule-Walker equation for \( r = 0 \).

2) If \( \{x_t\} \) is the \( MA(1) \) process \( x_t = \varepsilon_t - 0.9\varepsilon_{t-1} \) where \( \{\varepsilon_t\} \) is zero-mean white noise, show that \( \{x_t\} \) is invertible, and find its \( AR(\infty) \) representation.

3) The **Yule-Walker estimates** of an \( AR(p) \) model are obtained by solving the sample version of the first \( p+1 \) Yule-Walker equations (with \( \hat{c}_r \) in place of \( c_r \))

\[
\hat{c}_0 = \sigma^2 + \sum_{k=1}^{p} a_k \hat{c}_{-k},
\]

\[
\hat{c}_r = \sum_{k=1}^{p} a_k \hat{c}_{r-k} \quad (r = 1, \ldots, p),
\]

and solving for the \( p+1 \) unknowns \( \sigma^2, a_1, \ldots, a_p \).

Obtain an explicit expression for the Yule-Walker estimates of an \( AR(1) \) model in terms of \( \hat{c}_0 \) and \( \hat{c}_1 \).

Give an interpretation of the estimate of \( a_1 \) in terms of autocorrelation.

4) Construct the Dow returns, using \( \text{DowRet} = \text{diff}(\log(\text{Dow})) \). Assume that the Dow returns were generated by a time series with zero (theoretical) mean. Estimate an \( AR(1) \) model for this data using the Yule-Walker estimates. Is the estimated model closer to a white noise process, or to a random walk? In view of the fact that we are looking at daily returns on a stock index, does your answer make sense? Use your fitted model to predict the return one day ahead and two days ahead.

5) Suppose \( \{x_t\} \) is an \( AR(1) \) process, \( x_t = a x_{t-1} + \varepsilon_t \), where the \( \varepsilon_t \) are iid with zero mean, and \( |a| < 1 \).

Prove that \( \{x_t\} \) is a linear process.
6) Consider the model $x_t = e_t + 2e_{t-1} + e_{t-2}$, where the $e_t$ are iid with zero mean and $e_t$ is independent of $x_{t-1}, x_{t-2}, \ldots$.

a) Prove that $\{x_t\}$ is zero-mean white noise.

b) What is the best linear predictor of $x_{t+1}$ based on $x_t, x_{t-1}, \ldots$? Prove your answer.

c) What is the optimal predictor (not necessarily linear)? Prove your answer.