Assignment 3: Answer
Prima Donna Pricing: Raising Revenue at the New York City Opera
Revised: November 21, 2001

Here’s a solution and an intuitive procedure for finding it. Grades are based not on whether you reproduce this plan, but on how close you come to the revenue it generates. Money talks!

(a) Before we start, it’s worth thinking about who the various customers are, what they’re willing to pay, and how many of them there are. A quick summary in somewhat different form from the question:

<table>
<thead>
<tr>
<th>Willingness to pay for an “n”-performance ticket</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Patrons</td>
</tr>
<tr>
<td>-------------------</td>
</tr>
<tr>
<td>Die-hard opera fans</td>
</tr>
<tr>
<td>Regular opera fans</td>
</tr>
<tr>
<td>Occasional opera goers</td>
</tr>
<tr>
<td>First-time opera goers</td>
</tr>
<tr>
<td>Tourists who are opera fans</td>
</tr>
<tr>
<td>Other tourists</td>
</tr>
<tr>
<td>Students who are opera fans</td>
</tr>
<tr>
<td>Other students</td>
</tr>
</tbody>
</table>

Based on the table, we can make some informed guesses:
- First-timers and tourists never buy multi-performance plans.
- Die-hard fans always buy a plan.
- Regular and occasional goers will buy a plan if the difference between total valuation and price is greater than zero and greater than any other alternative.
- Students would be willing to buy a plan, but to reach them you’d have to lower the price well below what it would take to attract the first three groups.

Regarding the latter, two possibilities spring to mind:
- Policy 1: sell 8-performance plan to regular goers and no plan to occasional goers
- Policy 2: sell 8-performance plan to regular goers and 4-performance plan to occasional goers
- Policy 3: sell 4-performance plan to regular goers and no performance plan to occasional goers
We’ll look at each plan, starting with:

Policy 1.
Suppose we want to target the 8-performance plan to regulars. The most we can charge is $80 + 3 \times 80 + 4 \times 60$, or $560$. Clearly, a 4-performance ticket can’t cost more than $280$. But at this price, occasionals won’t buy it. They’ll buy single tickets if the price is right. The most we can charge them is $80$. Students: if we want to attract as many students as possible, then the price should be set at $40$ per individual student ticket. Finally, the maximum we can charge a same-day ticket is $(100\%-20\%) \times 40 = $32, for non-fan-tourists are willing to pay up to $40$ for a “regular” ticket.

Under this scheme, total yearly revenues are given by

\[
\begin{align*}
$80 \times (300 \text{ tourists} + 1,400 \text{ occasional goers}) & \quad $136K \\
$560 \times (800 \text{ regulars} + 200 \text{ die-hard x 2}) & \quad $672K \\
$40 \times (100 \text{ student fans x 16} + 800 \text{ other students}) & \quad $96K \\
$32 \times (2,400 \text{ other tourists}) & \quad $76.8K \\
\text{TOTAL} & \quad $980.8K
\end{align*}
\]

The explanation for the second line is that die-hard fans will choose to buy two 8-performance plans, as this saves on the total cost of buying tickets for all performances.

Notice that the total number of tickets sold per performance is $200$ die-hard fans $+$ $800\times 8/16$ regular goers $+$ $1,400\times 1/16$ occasional goers $+$ $300\times 1/16$ fan tourists $+$ $2,400\times 1/16$ other tourists $+$ $100$ student fans $+$ $800\times 1/16$ other students, a total of $1006.25$ (we ignore the difficulty of admitting $25\%$ of a person to an opera). So we’re ok on capacity.

Variations. Alternatively, we might set the individual price at $100$. This would imply greater revenue tourist fans, but a loss of demand from occasional buyers. Total revenues would decrease. Likewise, we could set the student price at $50$, but this would also reduce revenues.

Policy 2.
We now want to induce occasional buyers to purchase a 4-performance plan.
Willingness to pay for such a plan is $80+50\times 3=230$. We now have to worry about the incentive compatibility constraint for regular goers. If we continue charging $540$ for an 8-performance plan, they will simply buy two 4-performance plans. Thus the maximum we can charge for an 8-performance plan is $2\times 230 = $460$. Since occasional buyers no longer go for individual tickets, we can now jack up the price of individual tickets to $100$. Student and same-day prices are kept as before.

Under this scheme, total yearly revenues are given by
$100 \times (300 \text{ tourists}) = \$30K \\
$460 \times (800 \text{ regulars} + 200 \text{ die-hard} \times 2) = \$552K \\
$230 \times (1,400 \text{ occasional}) = \$322K \\
$40 \times (100 \text{ student fans} \times 16 + 800 \text{ other students}) = \$96K \\
$32 \times (2,400 \text{ other tourists}) = \$76.8K \\
\text{TOTAL} = \$1076.8K

The total number of tickets sold per performance is 200 die-hard fans + 800*8/16 regular goers + 1,400*4/16 occasional goers + 300*1/16 fan tourists + 2,400*1/16 other tourists + 100 student fans + 800*1/16 other students, a total of 1,268.75. Clearly, this is not feasible as it exceeds total seating capacity.

The optimal solution is then to ration same-day tickets, selling only the seats left after the others are sold, since same-days make the lower contribution to revenue. Total revenue is then

$100 \times (300 \text{ tourists}) = \$30K \\
$460 \times (800 \text{ regulars} + 200 \text{ die-hard} \times 2) = \$552K \\
$230 \times (1,400 \text{ occasional}) = \$322K \\
$40 \times (100 \text{ student fans} \times 16 + 31.26 \times 16 \text{ other students}) = \$84K \\
\text{TOTAL} = \$988K

This is the best you can do.

Variations. We could instead charge $50 for a student ticket and not sell same-day tickets. This produces total revenue of $948k (and a bunch of empty seats). A slightly more complicated version is to set a student price of $42 and a same-day price of $32. The $42 price is only purchased by student fans, the same-day is rationed to the first-coming students and tourists. This produces $987.2k. Why not charge more of student fans? Because if we do, they will prefer same-day tickets.

Clearly, Policy 2 is better than Policy 1. The latter leaves too many seats unused. Policy 3 would leave even more seats unused. It can be shown that it is also dominated by Policy 2.

(b) As of August 7, 2001, ticket prices quoted by the NYCO were as follows (all prices for 3rd ring, weekends):

- individual: $70
- 4 performances: $236
- 8 performances: $472
- students: 50% off individual price.

Moreover, there were “last-minute” student tickets at $10 (based on availability).
Five performance and six performance plans were also offered. Finally, a different option allowed patrons to pay an upfront fee in exchange for a guaranteed lower price for individual tickets.

These values seem roughly consistent with the optimal schedule derived in (a). Notice that the cost of an 8-performance plan is twice the cost of a 4-performance plan, i.e., the “incentive compatibility constraint” is binding as in the optimal solution derived above.

The main differences between (a) and (b) are that the individual and last-minute prices are lower at the NYCO than determined in (a). Moreover, the NYCO offers more plans than considered in (a)

(c) The fact that we obtained different values for the optimal price levels may simply be the result of having started from different numbers regarding demands. In particular, the fact that the optimal solution consists of two plans only depend crucially on the limited number of demand types we considered. The more the different types of patrons, the greater the number of optimal plans is.

Notes

Written by Luis Cabral and David Backus for the purpose of class discussion rather than to illustrate either effective or ineffective handling of an administrative situation. While this case was inspired by real-world events, some of the figures presented above are fictional. © 2001 David Backus and Luis Cabral.