1. (30 points) You are the price manager of a retail bank. Your market research team tells you that there are two types of customers. Low-use consumers, of which there are 10,000, are willing to pay up to 12¢ per check up to 4 checks a month; beyond that there are not willing to pay any positive price. High-use consumers, 4,000 in total, are willing to pay up to 17¢ per check for the first 4 checks a month; up to 14¢ per check for the next 6 checks a month; and zero thereafter. These customers cannot be distinguished \textit{a priori}. The marginal cost of processing a check is 2¢.

You offer two plans. Plan A allows a maximum of 4 checks a month and charges $p_A$ per check. Plan B has no limits on the number of checks and a price $p_B$ per check. What values of $p_A$ and $p_B$ produce the highest profit?

2. (40 points) 3D Karaoke (3DK) is the new rage in Manhattan. In case you don’t know, 3D Karaoke is a system that, in addition to traditional karaoke, comes with special 3D glasses that give you the image of an enthusiastic audience that screams and shouts as you sing along. (Actually, there are several options of audience you can choose from.)

There is currently one 3DK bar in the Village. Two companies have announced plans for creating their own bars: K-R-Us and the company you work for, BAStar. Analysts believe these will be the only entrants in the years to come. It is also believed that each entrant will open only one bar each.

There are two options regarding location, the Village and Midtown. If both bars are located in the Village, then each will receive a net profit of $10m. If both bars located in midtown then BAStar gets $19m and K-R-Us $21m. If BAStar locates in the Village and K-R-Us in Midtown, then BAStar gets $22m and K-R-Us $9m. Finally, if BAStar locates in Midtown and K-R-Us in the Village then BAStar gets $9m and K-R-Us $18m.
(a) Suppose BAStar and K-R-Us must decide simultaneously where to locate. Describe the game played between the two firms. Comment on the logic for the profits generated by various location combinations.

(b) Are there any dominant or dominated strategies in this game? What is the Nash equilibrium?

(c) What assumptions regarding player rationality did you need to make in answering the previous question?

(d) Suppose now that your firm, BAStar, has the possibility of an early credible commitment to a location. In other words, you can choose a location before K-R-Us does so, and when the latter makes its location choice it will do so knowing what location BAStar has chosen. Represent and solve this game.

(e) Based on the answers to the previous questions, calculate the value of commitment. How might you achieve such credible commitment?

3. (30 points) OYes! is a leading software developer. It owns one of the best known fifth generation operating systems (OS). A competitor, OS5, is suing OYes! for using anticompetitive practices in selling its OS. The key to this suit is a series of memos that are currently in the possession of OYes!.

The defendant has now offered the plaintiff to settle out of court and asked the latter to make an offer.

(a) If you are a consultant to OS5, how would you advise them to proceed?
(b) Can you think of other situations that lead to similar problems to this one?

Suggested Answers

1. This is a price discrimination problem, similar (but thankfully less complicated than) the opera problem. You want to choose prices that generate maximal profit, subject to the condition that the two types of customers will choose the plan that generates the most surplus for them: willingness to pay minus what they actually pay.

The idea is to offer a plan that will be chosen by low-use consumers (Plan A) and one that will be used by high-use consumers (Plan B). The maximum we can charge low-use consumers is \( p_A = 12 \) (this and what follows are in cents). How much can you then charge high-use consumers? The value \( p_B \) must be low enough that they would not prefer to go for Plan A. By choosing Plan B, high users would get a surplus of \( 4*(17-12) = 20 \). By choosing Plan B, they get \( 4*(17-p_B) + 6*(14-p_B) \). For high-end consumers to choose Plan B, we must have

\[
4*(17-p_B) + 6*(14-p_B) \geq 4*(17-12) = 20.
\]

Thus the highest possible value of \( p_B \) is \( p_B = 13.2 \). The profit from this plan is

\[
(12-2)*4*10,000 + (13.2-2)*10*4,000 = $4,000 + $4,480 = $8,480
\]
A detail: we need to make sure that we don’t make more by charging a higher price for Plan B and ignoring the low-use customers altogether. We get the most out of the high-use customers by charging $p_B = 14$. The profit generated is then

$$(14-2)*10*4,000 = \$6,000.$$ 

This is lower than the other, so it’s not the best plan.

2. (a) The payoff matrix for the game is

<table>
<thead>
<tr>
<th></th>
<th>Village</th>
<th>Midtown</th>
</tr>
</thead>
<tbody>
<tr>
<td>Village</td>
<td>10</td>
<td>9</td>
</tr>
<tr>
<td>Midtown</td>
<td>18</td>
<td>21</td>
</tr>
</tbody>
</table>

The logic? Since there already exists one bar in the Village, two more would lead to very aggressive competition. It would be better for both to locate in Midtown. However, if only one locates in Midtown then there won’t be enough critical mass to attract customers.

(b) Village is a dominant strategy for BAStar and (Village, Village) is the Nash equilibrium. It’s an unfortunate game for both players. Note that K-R-Us would benefit if it paid BAStar (say) $2m to choose Midtown.

(c) In determining the Nash equilibrium, each player maximizes its payoff and assumes that the other player knows the game (its structure and payoffs). Moreover, K-R-Us must assume that BAStar will choose the strategy that gives it the highest payoff (i.e., is “rational”). But you never know.

(d) A somewhat different game is for BAStar to move first. We could represent this with a tree:

In this case, the equilibrium is (Midtown, Midtown).
(e) Commitment (moving first) increases BAStar’s profit from 10 to 19, so its value is 9. Just as interesting, it increases K-R-U’s profit, too, from 10 to 21. But how do you accomplish it? Good question! Sign a contract with a developer? The key points: the action must be observable and irreversible.

3. This is a classic asymmetric information problem – adverse selection, to be specific.
   (a) OYes! has superior information on how good the case is. The difficulty from OS5’s point of view is that an offer is most likely to be accepted when it is weaker than what OS5 could expect to get in court. It’s Groucho Marx all over again: if OYes! accepts an offer, OS5 can be confident that it’s a bad one from their perspective. The question for them is whether they can get more information; it might be worth spending some money to get it, since it changes their negotiating position. Court costs may make settlement mutually attractive, but you still need to take adverse selection into account. Prepare to go to court!
   (b) There are lots of examples: life and health insurance, buying tickets from a scalper at the Garden (this from Dana: how do you know they’re real tickets?), used cars, bank loans, etc.

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