1. Your cousin from Minnesota has asked your advice on a financial matter. She has inherited the family farm, 350 acres of land 10 miles south of St. Paul, Minnesota’s second largest city. She has asked your advice on these alternatives:
   (a) Farm the land, which she estimates would generate a net income of 250k/year but require her to give up her 100k/year job.
   (b) Rent as a farm. This would deliver 250k/year, but would require an initial investment of 400k, which could be financed for 40k/year.
   (c) Sell to a real estate magnate. The magnate would pay you 230k/year to develop the land into a shopping mall, but would have to make further investments in infrastructure improvements (roads, drainage) that would cost 90k/year to finance.

Your cousin is seriously considering farming, but would like to know what she is giving up financially to make such a decision.

2. Raggar is a natural food additive that gives bread and other products longer shelf lives. It is currently produced by two firms using different technologies. Alpha Products grows the rag plant, from which Raggar is extracted, in Pennsylvania. Production is very efficient for modest quantities, but large quantities force them to use progressively worse land, with lower yields and higher costs. Alpha’s cost function is

   \[ C(q_1) = 8 + 2q_1 + (q_1)^2. \]

Beta Corp grows rag plants in greenhouses in upstate New York. The greenhouses are expensive, but supply is effectively unlimited. Beta’s cost function is

   \[ C(q_2) = 10q_2. \]

Demand for the entire market is

   \[ Q = q_1 + q_2 = 30 - 2p. \]
(a) Graph the average and marginal cost curves for the two firms. [Reminder: If \( y = ax^n \), then \( \frac{dy}{dx} = nax^{n-1} \).]
(b) If Alpha Products and Beta Corp both act competitively (they act as if they cannot affect the price through their own actions), what is the market supply curve?
(c) What is the equilibrium price? At this price, how much does each firm produce?
(d) What are the profits of the two firms?

3. Analysts estimate that AT&T Wireless faces a demand curve with an elasticity of about \(-1.5\). The marginal cost is estimated to be \(3\)¢ a minute. Their current price is \(10\)¢ a minute for new retail customers.
   (a) If AT&T reduces its price to \(9\)¢/minute, by how much would you expect demand to increase? What would happen to revenue? Are these numbers exact or approximate?
   (b) If the analysts have estimated correctly, what price should AT&T charge?
   (c) AT&T uses a far more complex pricing system, which includes (among other things): cheap off-peak minutes ($5/month for 1000 off-peak minutes), minimum usage ($30/month for 300 minutes), and volume discounts ($60/month for 750 minutes). What logic do you see for each of these features?

Suggested Answers

1. Farming.
   (a) The net financial benefit from farming is \(150\)k/year. (The lost job is an opportunity cost of \(100\)k.)
   (b) Net financial benefit is \(210\)k = \(250 - 40\).
   (c) Net financial benefit is \(230\)k. (The magnate’s investment is irrelevant to you.)
   The best alternative is (c), which generates \(230\)k/year. Thus the opportunity cost of farming is \(230 - 150 = 80\), the difference between farming and the best available alternative. This covers only the financial analysis. If your cousin really likes farming (\(80\)k worth!), then she might very well choose (a).

2. Raggar.
   (a) Alpha has a U-shaped AC curve and an increasing (linear) MC curve. Beta has a horizontal AC=MC curve. For small quantities, Alpha has lower MC, but for large quantities Beta has lower MC.
   (b) The supply curves for the two firms are their MC curves. Using calculus,
   \[
   MC(q_1) = \frac{dC}{dq_1} = 2 + 2 q_1 \\
   MC(q_2) = \frac{dC}{dq_2} = 10.
   \]
   If we graph the market supply curve, we start with Alpha’s MC curve. When it hits MC=10, it levels off as Beta’s production takes over.
   (c) You might guess that the demand curve cuts the supply curve on its flat section. If so, \(p=MC=10\) and \(Q=30-2(10)=10\). At this price, Alpha would produce \(q_1=4\), leaving \(q_2=10-4=6\).
(d) Beta breaks even: $AC = MC = p = 10$. Alpha is a little more complicated: $AC = C(q)/q = \frac{[8+2(4)+4^2]}{4} = \frac{32}{4} = 8$. So it makes a profit of 2 per unit, 8 overall.

3. AT&T Wireless.

(a) We’re reducing price by 10%. As a result, volume should rise about 15% and revenue rise about 5%. These are both approximations, good for small changes in price but possibly inaccurate for large changes.

(b) The elasticity formula again: $p = MC/(1+1/\varepsilon) = \frac{3}{(1–1/1.5)} = 9$. This is exact: it’s the “first-order condition” for a maximization problem (choose output to maximize profit) as long as we know $\varepsilon$.

(c) This is the interesting part in real life. Some thoughts (you might have others):

- Off-peak: Peak minutes are more costly, in the sense that the system has a limited capacity – once it’s hit, the MC is effectively higher than 3¢. Cheap (or sometimes “free”) off-peak minutes are a way of shifting some use away from peak times where usage may approach capacity.

- Minimum: Part of this is the cost structure (there are fixed costs to handling an account). There may also be psychological issues: eg, people often prefer a fixed price to paying a series of small charges.

- Volume: Again, costs may decline with volume. In addition, high-volume users may be more sensitive to price than low-volume users. An example of price discrimination.

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