Demand
Revised: October 1, 2001

An existing or new business needs to have some idea how many people might buy its product at different prices. This construct is called a demand curve. Or stated the other way around, the demand curve tells us how much people are willing to pay for a given number of units of a product. In this lecture, we describe the demand curve’s origins in the tastes and budgets of individuals and go on to describe properties of demand: how sensitive it is to the product’s price, to prices of other products, to income, and so on. The concepts are basic tools of business analysis.

Tastes and Budgets

The traditional theoretical setup for thinking about demand starts with the tastes or preferences of an individual consumer. To see how this works, suppose there are two products. We might express different combinations of purchases in a diagram with quantities of product 1 on the horizontal axis and quantities of product 2 on the vertical axis.

Indifference curves. We can represent our tastes by how various combinations are regarded. We do this graphically with “indifference curves”: lines that represent combinations of the two products that we regard as equally good. For example, we might think of 2 units of product 1 and 1 of product 2 as equal to the reverse (1 unit of product 1 and 2 units of product 2). A line connecting all such points about which we are indifferent (ie, like equally well) is called an indifference curve. Generally indifference curves are downward-sloping, since we need more of a product to compensate us for less of the other. (We call this the “more is better” principle.) We also assume they are curved away from the origin, since as we consider combinations with more and more of one product, we need increasing amounts of the other to keep us equally satisfied.

If we put lots of indifference curves on our graph, we can get a complete description of our tastes. Since “more is better,” indifference curves that are up and to the right (farther from the origin) represent higher levels of satisfaction.

Budgets. The other ingredient in our analysis is what the consumer can afford: the “budget line.” With two products, the budget might be expressed in the equation:

\[ p_1 q_1 + p_2 q_2 = \text{Income}. \]

If we solve for \( q_2 \) we see this is a downward-sloping line:
\[ q_2 = \frac{\text{Income}}{p_2} - \left( \frac{p_1}{p_2} \right) q_1 \]

This is a straight line whose position and slope depend on income and prices. If you increase income, for example, the budget line shifts out. If you increase \( p_1 \), the line twists clockwise (around the vertical intercept, \( \frac{\text{Income}}{p_2} \)). And if you increase \( p_2 \), the line twists counterclockwise (around the fixed horizontal intercept, \( \frac{\text{Income}}{p_1} \)).

**Demand.** Putting together tastes (represented by indifference curves) and possibilities (represented by the budget line) we can find out what our hypothetical consumer should do. With indifference curves and budget line on the same graph, the consumer’s best choice is the highest indifference curve the budget line touches. The point where they touch gives us the quantities demanded of products 1 and 2. Implicitly, these demands depend on tastes (these are built into the indifference curves). They also depend on income (since a change in income shifts the budget line and therefore leads to a change in demand) and prices (for the same reason).

If we step back from the details a little, we see that the analysis reminds us that the quantity of a product demanded by a consumer depends on:

- The tastes of the individual, expressed by indifference curves.
- The price of the product. Generally the lower the price the higher the demand. Depending on the curvature of the indifference curves, a change in price might have a small or large impact on the number of units demanded.
- The price of other products. Decisions are not made in isolation. If we spend less on one product, that necessarily leaves more to spend on others.
- Income. At higher levels of income, we can buy more of everything (and generally do).

More on each of these shortly.

**Price Elasticity**

We presume (based on lots of evidence) that the demand for a product – any product! – increases if we lower the price. This doesn’t have to be the case, but it invariably is. The question is how sensitive demand is to price. We measure the sensitivity by the “price elasticity of demand,”

\[ \varepsilon = \frac{\Delta q}{\Delta p} \cdot \frac{p}{q} \approx \frac{(dq/dp) \cdot (p/q)}{1}. \]

In words, the price elasticity is the ratio between the percentage change in quantity and the percentage change in price for a small change in price. Note that elasticity is not the same thing as slope, although slope is an input to it; see above, where the slope is \( dq/dp \). One of the advantages of elasticity over slope is that it does not depend on units of measurement and thus is more easily compared across products.

Note that elasticities are generally negative, since demand declines with price. The question is how negative. We say that products in which \( \varepsilon > 1 \) are “elastic,” meaning
the quantity demanded is sensitive to price. The higher is $|\varepsilon|$, the more sensitive to price. Conversely, if $|\varepsilon|$ is small, we say that demand is “inelastic,” meaning that demand is relatively insensitive to price. Note that the elasticity is defined at a point: it generally differs from one point to another along a demand curve.

This property of demand depends on tastes (about which there is no argument!), but we can nevertheless get some insight through examples. Demand for personal computers is elastic, with the result that the demand for PCs has expanded dramatically as the price has fallen. Demands for gas and energy are relatively inelastic, at least in the short run, since there is little people can do easily to alter the amount they use.

Let’s return to the gasoline example to highlight an important point about elasticities over time. If gasoline increased from $1.50 per gallon to $3 per gallon overnight, most people who commuted by car would still fill their tanks in the morning and drive to work. In the short run, they would not be willing or able to find a substitute means to avoiding the higher gasoline prices. Thus, as we said above, gasoline would be inelastic in the short term.

However, if a news report revealed that the government was planning to levy a 100% tax on gasoline for the next 3 years, people’s behavior would undoubtedly change. In the next day or week they would probably still drive their cars, but their demand for gas could change for many reasons. They might buy more fuel-efficient cars, carpool, or take the bus or train to work. Perhaps some would work from home. As a result, the quantity of gasoline demanded at the new price would decrease. This illustrates a general feature of elasticities: demand tends to be more elastic in the long run than in the short run.

Another useful fine point is the difference between the elasticity of demand for a product category (personal digital assistants, say) and specific products in that category (Handspring’s Visor Deluxe). Generally the demand for a specific product is more elastic than the demand for the category as a whole. Why? Because when the price of a specific product rises, people are willing to buy fewer units. Some of this reduction leads to purchases of other products in the same category (a Palm IV) and part to a reduction in the category as a whole. Only the latter shows up in the elasticity of the category as a whole, so it’s typically less elastic.

**Elasticity and Revenue**

From a firm’s perspective, the elasticity of demand is a critical piece of information, since it determines the change in revenue that results from a given change in price. Recall that revenue is simply price times quantity demanded. It is easy to see that by increasing a product’s price by 15%, each unit sold will yield more money. But if overall demand for the product drops as a result of the price increase, the positive effects of higher per unit prices will be offset by a decline in the number of units demanded. Which effect is larger?
Formally, the change in revenue induced by a (small) change in price is
\[
\frac{d(pq)}{(pq)} = \frac{dp}{p} + \frac{dq}{q} = \frac{dp}{p} + \left(\frac{dq}{dp}\right)\left(\frac{p}{q}\right)\left(\frac{dp}{p}\right) = (1 + \varepsilon) \frac{dp}{p}.
\]
That is, the percent change in revenue following a price change is \((1 + \text{elasticity}) \times \% \text{ change in price}\). Since \(\varepsilon < 0\), the direct effect of the price change (the “1”) and the indirect effect of demand (the “\(\varepsilon\)”) are opposite. If demand is elastic (\(\varepsilon < -1\)), the demand effect is larger and an increase in price reduces revenue. This is a basic point, but one that some have missed: that to increase revenue in markets with elastic demand, you need to lower price, not raise it.

**Cross-Price Elasticity**

We have seen that demand for a product depends not only on its own price, but on prices of other goods. When there are lots of other products it’s easy to lose sight of this, but it’s always there. Some examples are obvious. The demand for ski boots depends on the demand for skis: if skis get more expensive, we might expect people to buy fewer boots, too. And as we saw above, the demand for commuter rail tickets may be influenced by the price of gasoline.

We summarize the sensitivity of demand to the price of another product with the “cross price elasticity”:

\[
\text{Cross-price elasticity} = \frac{dq_1}{q_1} / \left(\frac{dp_2}{p_2}\right).
\]

That is, the ratio of the percent change in demand for product 1 to the percent change in the price for product 2.

The essential distinction here is between substitutes and complements. If the cross-price elasticity is positive, we say that the products are “substitutes.” Hence Coke and Pepsi are substitutes: if Coke gets more expensive, we’d expect some people (but not all) to switch to Pepsi. Similarly, gas and commuter rail tickets are substitutes, since an increase in the price of gas would lead some people to switch from car to train travel.

Conversely, if the cross-price elasticity is negative, we say the products are “complements.” The language of business is filled with competitive metaphors for which substitutes seem appropriate (Coke v. Pepsi). But there are lots of examples of complements, in which a price reduction for one product increases demand for others. Skis and boots are one example. Others include: Windows and Intel microprocessors, beer and pretzels, gas and cars.

**Income Elasticity**

Changes in income also affect demand. Higher income generally means greater demand for all products, but some products benefit more than others. We define the “income elasticity” of a product by
Income elasticity = \((dq/q) / (dy/y)\).

That is, the percent change in demand induced by a one percent change in income.

Economists have names for products with different income elasticities. “Inferior goods” have negative income elasticities. Although inferior goods aren’t all that common, it’s fun to try to think of examples. Spam comes to mind, on the assumption that anyone with enough money would buy something else. You can see the source of the term. “Normal goods” have positive income elasticities. Within normal goods, those with elasticities between zero and one are referred to as necessities, and those with elasticities greater than one as luxuries. Can you explain why?

Measuring Demand

In an ideal world, firms would know the demands for their products. In practice, it’s not so easy. One reason is that it’s hard to get reliable market data: how much was bought by whom and at what price? Another is that it’s inherently difficult (for reasons we won’t go into) to infer the sensitivity to (say) price from other variables, when all of these variables might be changing at the same time and not all of them are known to the analyst. We have colleagues who provide this service for a reasonable fee. Another approach – one that is increasingly common – is to do experiments in markets. Thus catalog companies sometimes send out catalogs to different customers in which some of the prices are different. These experiments run the risk of alienating customers (what if you find out you got the high price?), but you can see the value to the firm of knowing the demand for its products.

Consumer Surplus

One last thing. The demand curve tells us what consumers are willing to pay for a product. In a market, though, some people are able to buy the product for less than they’d be willing to pay. Consumer surplus is the difference between the total benefit to consumers and expenditures on the product.

Recall from our discussion of supply and demand that, in equilibrium, supply intersects demand at a certain point to determine the market-clearing price. Now think of the section of the demand curve that lies above this price. This section represents individuals who would have paid more for the good, but did do not have to. Thus, on a market demand curve, the “triangle” that lies above the price and below the demand curve represents the consumer’s surplus associated with the purchase of the good. The area below the price is the actual expenditure on the good. This total area – consumer surplus plus actual expenditure – is known as total willingness to pay (or “value in use”).

More on this later. One thing for you to think about in the meantime: How might a firm go about claiming some or all of the consumer surplus for itself?
Numerical Examples

Example 1. Demand for a product is estimated to be

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<th>Price</th>
<th>Quantity</th>
<th>Revenue</th>
<th>Elasticity (approx)</th>
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<tr>
<td>10</td>
<td>6.31</td>
<td>63.10</td>
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<td>5.63</td>
<td>61.90</td>
<td>-1.09</td>
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<td>4.21</td>
<td>58.99</td>
<td></td>
</tr>
<tr>
<td>15</td>
<td>3.88</td>
<td>58.18</td>
<td></td>
</tr>
</tbody>
</table>

What is the elasticity of demand? We can approximate it by the “change formula,”

$$\varepsilon \approx \left( \frac{\Delta q}{\Delta p} \right) \left( \frac{p}{q} \right) = \frac{(5.63-6.31)/(11-10)}{10/6.31} = -1.08. $$

This is approximate, since we’re using discrete changes. In fact, the numbers were generated by the demand curve

$$\log q = \log 100 - 1.2 \log p,$$

which has an elasticity of −1.2.

Example 2. Suppose the demand for “product 1” is given by the demand curve,

$$q_1 = a - b_1 p_1 + b_2 p_2, $$

which you’ll note also depends on the price of product 2. The price elasticity is

$$\varepsilon = (dq_1/dp_1) \left( \frac{p_1}{q_1} \right) = - b_1 \left( \frac{p_1}{q_1} \right). $$

Note the demand is elastic for high values of \((p_1/q_1)\) and inelastic for low values. “The” cross-elasticity is

$$(dq_1/dp_2) \left( \frac{p_2}{q_1} \right) = b_2 \left( \frac{p_2}{q_1} \right). $$

Suppose \(a=500, b_1=10, b_2=5, \) and \(p_1=p_2=50. \) Then \(q_1=250, \) the elasticity is −2, and the cross-elasticity is 1. The positive sign of the cross-elasticity means the products are substitutes.

Consumer surplus is the area under the demand curve but above the price. Since demand is linear, this is a triangle. With the same numbers, the consumer surplus is

$$250 \times (75-50)/2 = 31,250.$$
Further Reading

Similar material can be found in many economics texts, including:


Written by Chris Chamberlain under the supervision of Luis Cabral and David Backus. © 2001 Luis Cabral and David Backus.