Pricing
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Most firms have some control over the price they set. Although they may have competitors, they can charge a higher or lower price, and generate less or more demand as a result. The question is how high to set the price. A high price generates more revenue per unit, but fewer units are sold. A low price generates less revenue per unit, but more units are sold. The answer? It depends … on the elasticity of demand and marginal cost.

Monopoly Pricing: Intuition

To see this most clearly, consider a monopoly: an industry with a single producer. Generally a monopoly will set a higher price, and sell fewer units, than competitive firms in the same industry. Our task is to show why, and to give you some insight into how the price might be set.

Let’s start by reviewing price setting in a competitive industry. Recall that for a competitive firm, “marginal revenue” (the extra revenue generated by selling one additional unit of output) is equal to price. Why? Competitive firms are “price takers.” They are too small to affect the market price through their own actions. A competitive firm can’t raise its price above the industry price, because it would then lose all its customers to competitors. And while it could set a lower price, there would be no point: it could sell the same quantity at the (higher) market price.

Now consider a monopolist. You can set your price however you like, but you sell only what the market demands at each price. You can either price low and sell a lot, or price low and sell a little. (You might guess that which you choose depends on how sensitive demand is to price, something we’ll return to shortly.) The key insight is that the advantage of selling an additional unit (marginal revenue) is less than for a competitive firm. If a competitive firm sells an additional unit, its marginal revenue is just the price. For a monopolist to sell an additional unit, it must lower the price on that unit, and also on all other units. Thus the benefits are less than the price. Since the advantage of selling more is less than for a competitive firm, the monopolist sells less and charges more.

If this went by too fast, consider an example. Suppose you sell 80 CDs for $15.10, and to sell 81 you must reduce the price to $15. Is the marginal revenue $15? No! Your revenue rises from 1208 (=15.10x80) to 1215 (=15x81), an increase of only $7. This is less than the $15 price, because we had to reduce the price on the first 80 CDs.
(0.10x80=$8, the difference). Thus the marginal revenue of an extra unit is less than the sales price.

Now how much should we produce? We produce until the extra benefit of an additional unit is balanced by the extra cost: when marginal revenue equals marginal cost. For a competitive firm, this is our competitive supply curve: \( p=MC \). For a monopolist, the analogous condition is \( MR=MC \). Since \( MR \) is less than price, and marginal cost is level or increasing, this results in less output under monopoly.

**Monopoly Pricing: Calculus**

We can see the same thing mathematically. If the inverse demand curve (price as a function of quantity) is \( p=D(q) \), then revenue (expressed as a function of output, \( q \)) is

\[ R(q) = pq = D(q)q. \]

Marginal revenue (the extra revenue from an additional unit) is

\[ MR(q) = \frac{dR(q)}{dq} = D(q) + D'(q)q = p + D'(q)q < p. \]

The last step follows from our standard assumption that demand declines with output (hence \( D'<0 \)). We find output by setting \( MR(q)=MC(q) \) and solving for \( q \).

Note that output is lower under monopoly. Why? Because \( MR \) is lower. Since \( MC \) is increasing, the only way to equate it to \( MR \) is to reduce output.

(Skip this paragraph unless you’re brave.) There are some fine points that you can see if you work through the problem of maximizing profit. Expressed as a function of output, profit is

\[ \text{Profit}(q) = R(q) – C(q). \]

We maximize by setting the derivative equal to zero, then verifying that we have a maximum. The derivative is

\[ \frac{d\text{Profit}(q)}{dq} = MR(q) – MC(q) = 0, \]

as stated. To check to see that we have a maximum, we look at the second derivative:

\[ \frac{d^2\text{Profit}(q)}{dq^2} = MR'(q) – MC'(q). \]

If this is negative, \( MR=MC \) gives us a max. It’s sufficient that \( MC \) increase with \( q \) (\( MC'>0 \)) and \( MR \) decrease with \( q \) (\( MR'<0 \)). We’ll assume \( MR'<0 \) and \( MC'\geq0 \), which is enough to cover us without ruling out constant marginal cost. The only difficulty comes
with inverse demand curves whose second derivative is strongly positive, which won’t happen in this course.

**Elasticity and Margins**

The math is useful for some people because it’s clear (if you like that kind of thing) and concise. It’s also useful for demonstrating an interesting relationship between seller margin and elasticity. Define the margin as \( m = (p - MC)/p \). For a competitive firm, the margin is zero. Price equals marginal cost, period. For a monopolist, the margin is generally positive. But how large is it? The answer is

\[
m = -1/\varepsilon,
\]

where \( \varepsilon \) is the price elasticity of demand and is negative. We’ll refer to this as “the elasticity condition” in the rest of the course. Note that the margin depends on how sensitive demand is to price. If it’s very sensitive (|\( \varepsilon \)| large), then the margin is small. Even for a monopolist it’s hard to squeeze out a large margin, since customers will simply decline to buy the product. But if the elasticity is small, the margin can be quite large. (There’s a limit of one, of course, for reasons related to the fine points above).

Where did this come from? Recall that

\[
MR = p + D'(q)q = MC.
\]

Then

\[
(p - MC)/p = -D'(q) q/p.
\]

Except for the minus sign, this is just the inverse of the elasticity of demand.

Examples. Local telephone service is a must for most people, so demand is inelastic (|\( \varepsilon \)| small). An unfettered monopolist would presumably choose a high markup and price. The classic role of telecomm regulation, therefore, is to keep the local operator from exploiting its position. Similarly, demand for prescription drugs is inelastic, both because they’re necessities (sorry, that drug’s too expensive, I’d rather be sick) and because in most cases the customer isn’t paying anyway.

Conversely, a competitive firm might be regarded as one facing an infinitely elastic demand curve (horizontal), which produces a markup of zero. Thus the key issue of a monopoly is how elastic their demand is. If it’s very elastic, the cost of monopoly is low.

**Numerical Example**

Consider a monopoly facing linear demand and constant marginal cost. Demand is
\[ q = a - b \cdot p, \]

with \( a=12 \) and \( b=2 \). Costs are \( C(q)=cq \), with \( c=1 \), so \( MC=c=1 \). What’s the optimal price for the monopolist?

Answer. The inverse demand curve (we need to solve for \( p \)) is \( p=(a-q)/b \). Then profits are

\[
\text{Profit}(q) = pq - cq = [(a-q)/b]q - c \cdot q = (a/b)q - q^2/b - c \cdot q.
\]

To maximize, we differentiate with respect to \( q \) and set the result equal to zero, implying \( MR=MC \):

\[
(a/b) - 2q/b = c.
\]

Optimal output is therefore \( q=(a-cb)/2=5 \), price is \( p=(a-q)/b=3.5 \), and profit is 12.5.

As for the elasticity rule: At the optimal price, the elasticity is

\[
\varepsilon = \frac{dq}{dp} \cdot \frac{p}{q} = -b \cdot \frac{p}{q} = -2 \cdot \frac{3.5}{5} = -7/5.
\]

Thus the markup is \( m=(p-MC)/p=2.5/3.5=5/7=-1/\varepsilon \), as promised.

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