Debt Constraints and Employment

Patrick Kehoe, Virgiliu Midrigan and Elena Pastorino
Motivation: U.S. Great Recession

- Large, persistent drop in employment
U.S. Employment-Population, aged 25-54
Motivation: U.S. Great Recession

- Large, persistent drop in employment

- Regions with higher HH debt/income in 2007 experienced
  - larger decline in debt
  - larger decline in consumption
  - larger decline in employment
Household Debt/Income, 2007-2009

source: Midrigan and Philippon (2011)
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△ Employment/Population, 2007-2009

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Motivation: U.S. Great Recession

- Large, persistent drop in employment

- Regions with higher HH debt/income in 2007 experienced
  - larger decline in debt
  - larger decline in consumption
  - larger decline in employment

- Regional employment drop largely due to nontradables
Employment by sector, 2007-2009

This figure presents scatter-plots of county level employment growth from 2007Q1 to 2009Q1 against the debt to income ratio of the county as of 2006. The left panel examines employment in non-tradable industries based on geographical herfindahl index and the right panel focuses on tradable industries based on the same index. The sample includes only counties with more than 50,000 households.

source: Mian and Sufi (2013)
U.S. Great Recession

- Popular interpretation:
  - Tightening of HH credit leads to drop in consumption
  - Drop in consumption leads to drop in employment

- At odds with predictions of standard models
  - Consumption and leisure normal goods
  - Absent relative price changes move together

- Unless prices or wages are sticky
  - Need to assume lots of stickiness
  - Guerrieri-Lorenzoni, Midrigan-Philippon
We study alternative mechanism

• Tighter debt constraints $\rightarrow$ less consumption & less employment

• Idea: large returns to tenure/experience
  • Work is an investment
  • HH debt constraints reduce returns to such investments
  • Make employment less valuable
Alternative mechanism

- Otherwise standard DMP setup

- When debt constraints are tighter
  - Consumers discount returns to experience more
  - So surplus from match is reduced
  \[\Rightarrow\] Firms create fewer vacancies

- Do not explicitly impose wage rigidities
  - But arise endogenously due to debt constraints
Model overview

• Continuum of islands in a closed economy. Labor immobile

• Diamond-Mortensen-Pissarides with
  • on-the-job human capital accumulation
  • idiosyncratic shocks to worker human capital
  • full insurance inside household
  • household debt limit

• No aggregate uncertainty

• Study effect of one-time, unanticipated tightening of debt limit
  1. economy-wide collateral constraint (U.S. recession)
  2. island collateral constraint (predictions for U.S. regions)
Outline

1. Response to economy-wide shock to credit constraint
   - No changes in relative prices
   - No reallocation between tradeable/non-tradeable
   - Identical to those of one good model

2. Island-specific shock to credit constraint
   - Changes in relative prices & terms of trade
   - Labor reallocation from non-tradeable to tradeable
   - More notation, leave for later
One-Good Economy
Household’s problem

- Consists of measure 1 of workers.

- Income of worker $i: y_{it} = \text{wages or home production}$

\[
\max \sum_t \beta^t u(c_t)
\]

s.t.

\[
c_t + q_t a_{t+1} = a_t + \int y_{it} \, di
\]

Borrowing constraint:

\[
q_t a_{t+1} \geq -d_t
\]
Household’s problem

- Debt constraint binds as long as \( u'(c_t)/u'(c_{t+1}) > \beta/q_t \)

- Binds in steady state and our experiments

- Problem reduces to that of choosing employment:

\[
\max_{e_{it} \in \{0,1\}} E_0 \sum_t (\beta \phi)^t Q_t y_{it}
\]

- \( Q_t = u'(c_t) \): multiplier on date \( t \) budget constraint
- \( \phi \): worker survival probability
Financial intermediaries

- Risk-neutral, discount factor $1/(1 + r) > \beta$

- Objective

$$\sum_{t}(1 + r)^{-t}c^f_t$$

s.t.

$$c^f_t + a_t = q_t a_{t+1} + T_t$$

- Own firms. Earn $T_t$: profits net of vacancy posting costs

- Competition implies $q_t = 1/(1 + r)$
  - Debt constraint binds in steady state and our experiments
Technology and Human Capital

- Newborns enter with human capital

\[ \log(z) \sim N\left(0, \frac{\sigma_z^2}{1 - \rho_z^2}\right) \]
Technology and Human Capital

• Newborns enter with human capital

\[ \log(z) \sim N(0, \sigma_z^2/(1 - \rho_z^2)) \]

• On-the-job human capital accumulation/off-the-job depreciation
  
  ◦ employed draw \( z \) from \( F_e(z'|z) \) (drifts up)
    
    \[ \log z' = (1 - \rho_z)\mu_z + \rho_z \log z + \sigma_z \varepsilon' \]
  
  ◦ non-employed draw \( z \) from \( F_u(z'|z) \) (drifts down)
    
    \[ \log z' = \rho_z \log z + \sigma_z \varepsilon' \]
Technology and Human Capital

- Newborns enter with human capital
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  - non-employed draw \( z \) from \( F_u(z'|z) \) (drifts down)
    \[ \log z' = \rho_z \log z + \sigma_z \varepsilon' \]

- Employed: produce \( z \) and receive wage \( w_t(z) \)
- Non-employed: produce \( b \)
Matching technology

\[ M(u_t, v_t) = B u_t^\eta v_t^{1-\eta} \]

- Market tightness: \( \theta_t = v_t/u_t \)

- Probability firm finds worker
  \[ \lambda_{f,t} = \frac{M(u_t, v_t)}{v_t} = \left( \frac{u_t}{v_t} \right)^\eta = B\theta_t^{-\eta} \]

- Probability worker finds firm
  \[ \lambda_{w,t} = \frac{M_t(u_t, v_t)}{u_t} = \left( \frac{v_t}{u_t} \right)^{1-\eta} = B\theta_t^{1-\eta} \]
Worker values

- Match exogenously destroyed with probability $\sigma$

- Discounted lifetime income if currently employed:

$$W_t(z) = \omega_t(z) + \beta \phi \frac{Q_{t+1}}{Q_t} (1 - \sigma) \int \max [W_{t+1}(z'), U_{t+1}(z')] \, dF_e(z'|z)$$

$$+ \beta \phi \frac{Q_{t+1}}{Q_t} \sigma \int U_{t+1}(z') \, dF_e(z'|z)$$

- Discounted lifetime income if currently not employed:

$$U_t(z) = b + \beta \phi \frac{Q_{t+1}}{Q_t} \lambda_{w,t} \int \max [W_{t+1}(z'), U_{t+1}(z')] \, dF_u(z'|z) +$$

$$\beta \phi \frac{Q_{t+1}}{Q_t} (1 - \lambda_{w,t}) \int U_{t+1}(z') \, dF_u(z'|z)$$
Value of filled vacancy

- Firms owned by risk-neutral financial intermediaries

\[ J_t(z) = z - \omega_t(z) + \frac{\phi}{1 + r} (1 - \sigma) \int \max [J_{t+1}(z'), 0] \, dF_e(z'|z) \]
Wages

- Assume wages renegotiated period by period

- Nash bargaining:

\[
\max_{\omega_t(z)} \left[ W_t(z) - U_t(z) \right]^\gamma J_t(z)^{1-\gamma}
\]

\[
\frac{\gamma}{W_t(z) - U_t(z)} = \frac{1 - \gamma}{J_t(z)}
\]
Free entry condition

- Firms pay $\kappa$ units of output to post vacancy

- Let $n_t^u(z)$ measure of unemployed, $\tilde{n}_t^u(z) = \frac{n_t^u(z)}{\int dn_t^u(z)}$

\[
0 = -\kappa + \frac{\phi}{1 + r} \lambda_{f,t} \int \max [J_{t+1}(z'), 0] \ dF_u(z'|z) \ d\tilde{n}_t^u(z)
\]

- pins down $\theta_t$
Parameterization

- Assigned parameters
  - period = 1 quarter
  - $\beta = 0.94^{1/4}$, $1 + r = 0.96^{-1/4}$, $\phi = 1 - 1/160$
  - Probability of separation: $\sigma = 0.10$ (Shimer 2005)
  - Bargaining share and elasticity matching fn: $\eta = \gamma = 1/2$
  - $u(c_t) = \frac{c_t^{1-\alpha}}{1-\alpha}$, $\alpha = 5$ so IES = 0.2
    - Micro-evidence: IES $\approx 0.1 - 0.2$
    - Hall ’88, Attanasio et. al. ’02, Vissing-Jorgensen ’02
• **Calibrated parameters**

• Vacancy posting cost, $\kappa$

• Efficiency matching function: $B$

• Persistence shocks to $z$: $\rho_z$

• Std. dev. of shocks to $z$: $\sigma_z$

• Home production, $b$

• Returns to work: $\mu_z$
- **Calibrated parameters**

  - Vacancy posting cost, $\kappa$
    - Normalize steady-state market tightness $\theta = 1$
  - Efficiency matching function: $B$
  - Persistence shocks to $z$: $\rho_z$
  - Std. dev. of shocks to $z$: $\sigma_z$
  - Home production, $b$
  - Returns to work: $\mu_z$
**Calibrated parameters**

- Vacancy posting cost, $\kappa$

- Efficiency matching function: $B$
  - Employment-populatio ratio $= 0.8$ (U.S. all adults 25-54)

- Persistence shocks to $z$: $\rho_z$

- Std. dev. of shocks to $z$: $\sigma_z$

- Home production, $b$

- Returns to work: $\mu_z$
Calibrated parameters

- Vacancy posting cost, $\kappa$

- Efficiency matching function: $B$

- Persistence shocks to $z$: $\rho_z$
  - std. dev. of log initial wages = 0.94 (PSID)

- Std. dev. of shocks to $z$: $\sigma_z$

- Home production, $b$

- Returns to work: $\mu_z$
**Calibrated parameters**

- Vacancy posting cost, $\kappa$

- Efficiency matching function: $B$

- Persistence shocks to $z$: $\rho_z$

- Std. dev. of shocks to $z$: $\sigma_z$
  - std. dev. changes log wages $= 0.21$ (Floden-Linde 2001)

- Home production, $b$

- Returns to work: $\mu_z$
- **Calibrated parameters**

  - Vacancy posting cost, $\kappa$

  - Efficiency matching function: $B$

  - Persistence shocks to $z$: $\rho_z$

  - Std. dev. of shocks to $z$: $\sigma_z$

  - Home production, $b$
    - $b / \text{median } \omega = 0.4$ (Shimer 2005)

  - Returns to work: $\mu_z$
• **Calibrated parameters**

  • Vacancy posting cost, $\kappa$

  • Efficiency matching function: $B$

  • Persistence shocks to $z$: $\rho_z$

  • Std. dev. of shocks to $z$: $\sigma_z$

  • Home production, $b$

  • Returns to work: $\mu_z$
    - returns to tenure & experience data
Returns to work in the data

- Buchinsky et. al. (2010) estimate

\[ \log(w_{it}) = c_i + x'_{it}\beta + f(\text{experience}_{it}) + g(\text{tenure}_{it}) + J_{it} + \varepsilon_{it} \]

- \( J_{it} \) summarizes history previous jobs \( l = 1 : M_{it} \)

\[ J_{i,t} = \sum_{l=1}^{M_{it}} \sum_{k=1}^{4} (\phi_k^0 + \phi_k^s \text{tenure}_{i}^l + \phi_k^e \text{experience}_{i}^l) d_{k,i}^l \]
Returns to work in the data

\[ \log(w_{it}) = c_i + x'_{it}\beta + f(\text{experience}_{it}) + g(\text{tenure}_{it}) + J_{it} + \varepsilon_{it} \]

<table>
<thead>
<tr>
<th>Cumul. returns to experience:</th>
<th>5 yrs</th>
<th>10 yrs</th>
<th>15 yrs</th>
</tr>
</thead>
<tbody>
<tr>
<td>College graduates</td>
<td>0.43</td>
<td>0.66</td>
<td>0.76</td>
</tr>
<tr>
<td>High School graduates</td>
<td>0.28</td>
<td>0.40</td>
<td>0.44</td>
</tr>
<tr>
<td>High School dropouts</td>
<td>0.24</td>
<td>0.36</td>
<td>0.41</td>
</tr>
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Returns to work in the data

\[
\log(w_{it}) = c_i + x'_{it}\beta + f(\text{experience}_{it}) + g(\text{tenure}_{it}) + J_{it} + \varepsilon_{it}
\]

- Our approach:
  - Simulate paths for experience and tenure for our model
  - Use BFKT estimates (high school grads) to evaluate

\[
\log(\hat{w}_{it}) = f(\text{experience}_{it}) + g(\text{tenure}_{it}) + J_{it}
\]

- Minimize distance mean \( \Delta \log(\hat{w}_{it}) \) & \( \Delta \log(w_{it}) \) model
  - 5.2\% per year
Moments used in calibration

<table>
<thead>
<tr>
<th></th>
<th>Data</th>
<th>Model</th>
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<tbody>
<tr>
<td>fraction employed</td>
<td>0.80</td>
<td>0.80</td>
</tr>
<tr>
<td>mean growth rate wages</td>
<td>0.052</td>
<td>0.052</td>
</tr>
<tr>
<td>home production/ median wage</td>
<td>0.40</td>
<td>0.40</td>
</tr>
<tr>
<td>std. dev. wage changes</td>
<td>0.21</td>
<td>0.21</td>
</tr>
<tr>
<td>std. dev. initial wages</td>
<td>0.94</td>
<td>0.94</td>
</tr>
</tbody>
</table>
Initialize w/ 0 experience, mean $z_{it} | \text{exp} = 0$, no shocks
<table>
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<tr>
<th>Parameter</th>
<th>Value</th>
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<tr>
<td>$B$</td>
<td>0.595</td>
<td>steady state match probability</td>
</tr>
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## Parameter values

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Note: $b$ large relative to mean $z$ of worker with 0 experience: 1.82
Steady state measures

measure workers

log(b)

measure unemployed
Policy and value functions

firm profits

wages

\( J \)

\( W-U \)
## Model implications

<table>
<thead>
<tr>
<th>Fraction</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Workers with $w &lt; b$</td>
<td>0.180</td>
</tr>
<tr>
<td>Probability of job destroyed endogenously</td>
<td>0.002</td>
</tr>
<tr>
<td>Probability of worker matches, $\lambda_w$</td>
<td>0.595</td>
</tr>
<tr>
<td>Fraction of matches with positive surplus</td>
<td>0.722</td>
</tr>
<tr>
<td>Drop in $w$ after non-employment spell</td>
<td>1.9%</td>
</tr>
<tr>
<td>Drop in $w$ if not employed 1 year</td>
<td>6.1%</td>
</tr>
<tr>
<td>Drop in $w$ if not employed 2 years</td>
<td>8.8%</td>
</tr>
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Experiment: economy-wide credit crunch

- Binding debt limit:
  \[ c_t = d_{t+1} - (1 + r)d_t + y_t \]

- Assume unanticipated tightening debt limit \( d_{t+1} \)

- Choose path for \( d_t \) so \( c_t \) falls 5% then mean-reverts
  \[ c_t = 0.90c_{t-1} + 0.10\bar{c} \]

- Implies future discounted more: 
  \[ \frac{Q_{t+1}}{Q_t} = \left( \frac{c_{t+1}}{c_t} \right)^{-\alpha} \downarrow \]
Credit crunch

![Graphs showing consumption and discount factor over quarters.](image)
Employment response

- Maximal drop employment 1.57% vs. 5% drop in $C$
- Employment drop much more persistent
  
  - Cumulative impulse responses:
    - 2 years: $\text{CIR}^E = 0.35 \times \text{CIR}^C$
    - 10 years: $\text{CIR}^E = 0.58 \times \text{CIR}^C$
    - Overall: $\text{CIR}^E = 0.80 \times \text{CIR}^C$
Why does employment drop?

- Drop in $Q_{t+1}/Q_t$ reduces $W_t(z) - U_t(z)$
  - Reduces discounted value of returns to work

- Nash bargaining lowers $J_t(z)$ (increases $\omega_t(z)$) as well
  - Some existing matches endogenously destroyed
  - Value of posting vacancies falls, $\theta_t$ falls
  - Fewer matches positive surplus
Why does employment drop?

![Graph of Job separations](image1)

![Graph of Market tightness](image2)

![Graph of Probability match accepted](image3)

![Graph of Wages and Output](image4)
Role of returns to work

- Results much weaker w/o returns to employment ($\mu_z = 0$)

- Illustrate by setting $\mu_z = 0$ & $\sigma_z = 0$
  - $\sigma = 0.10$, $\bar{e} = 0.8$, $b/w = 0.4$
  - Similar results with heterogeneity: $\sigma_z > 0$
No returns to work

Employment

Job finding rate

Expected value of vacancy

Mean wage

Our model

No returns experience
Comparison with Hall 2014

- Study effect of increase discount rate of both workers and firms

- Additional channel: vacancy creation is an investment

- Results quantitatively modest in standard DMP setting
  - $r$ from 10% to 20%: $U$ up from 5.8% to 5.88%
Intuition from simple model

- First, suppose no learning by doing

\begin{align*}
\rho W(z) &= \omega(z) - \sigma (W(z) - U(z)) \\
\rho U(z) &= b + \lambda_w (W(z) - U(z)) \\
rJ(z) &= z - w(z) - \sigma J(z)
\end{align*}
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\]

- Surplus: \( S(z) = W(z) - U(z) + J(z) \)

\[
S(z) = \frac{z - b}{\tilde{\rho}}
\]

- \( \tilde{\rho} = \frac{1}{2} (\rho + r + \lambda_w + 2\sigma) \)
- not sensitive to \( \Delta \rho \) since \( \lambda_w \) and \( \sigma \) much larger
- e.g. \( \rho = 0.01, \lambda_w = 0.5, \sigma = 0.1 \)
Intuition from simple model

- Next, suppose $dz = gzd\,dt$ if employed, 0 otherwise.

\[
\begin{align*}
\rho W(z) &= \omega(z) - \sigma (W(z) - U(z)) + zgW'(z) \\
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- Surplus: $S(z) = W(z) - U(z) + J(z)$

\[
S(z) = \frac{z - b}{\tilde{\rho}} + \frac{\tilde{g}z}{(\tilde{\rho} - \tilde{g})\tilde{\rho}}
\]

\[
\begin{align*}
\tilde{\rho} &= \frac{1}{2}(\rho + r + \lambda_w + 2\sigma) \\
\tilde{g} &= g \left(1 + \frac{\lambda_w}{2\rho}\right): \text{ sensitive to } \Delta \rho
\end{align*}
\]
Many-Good Economy
Many Good Economy

- Multi-sector economy
- Each island produces tradable and nontradable goods
- Labor cannot move across islands but can switch sectors
- Study response to island specific-shocks
  - evaluate model against Mian and Sufi (2013) evidence
Preferences

Let $s$ index islands. Preferences:

$$\sum_{t=0}^{\infty} (\beta \phi)^t u(c_t(s))$$

Consumption is an aggregate of tradeable (m) and non-tradeables (n):

$$c_t(s) = \left[ \tau \frac{1}{\sigma} (c^n_t(s))^{\frac{\mu-1}{\mu}} + (1 - \tau) \frac{1}{\sigma} (c^m_t(s))^{\frac{\mu-1}{\mu}} \right]$$

Tradeables imported from all other islands, $s'$

$$c^m_t(s) = \left( \int c^m_t(s, s')^{\frac{\nu-1}{\nu}} ds' \right)^{\frac{\nu}{\nu-1}}$$
Prices

• Let \( p_t^n(s) \) and \( p_t^m(s) \): price of goods produced in \( s \)

• Price of composite imported good on \( s \)

\[
P_t^m (s) = \left( \int p_t^m (s')^{1-\nu} ds' \right)^{\frac{1}{1-\gamma}} = \bar{P}^m
\]

• Aggregate price index on \( s \)

\[
P_t (s) = \left[ \tau (p_t^n (s))^{1-\mu} + (1 - \tau) (\bar{P}^m)^{1-\mu} \right]^{\frac{1}{1-\mu}}
\]
Demand for goods

- Assume non-employed produce $b$ units of composite good
  - Let $\bar{b}_t(s) = b(1 - e_t(s))$: total home production
  - Only $c_t(s) - \bar{b}_t(s)$ purchased on the market

- Demand for non-tradeables
  
  $$c^n_t(s) = \tau \left( \frac{p^n_t(s)}{P_t(s)} \right)^{-\mu} (c_t(s) - \bar{b}_t(s))$$

- Demand for variety $s'$ tradeables:
  
  $$c^m_t(s, s') = (1 - \tau) \left( \frac{p^m_t(s')}{\bar{P}_m} \right)^{-\nu} \left( \frac{\bar{P}_m}{P_t(s)} \right)^{-\mu} (c_t(s) - \bar{b}_t(s))$$
Technology

- Two sectors: tradeables (x) and non-tradeables (n)
- \( y = z \) in both sectors
- Matching technology:

\[
M^x_t = B^x(u_t)^\eta(v^x_t)^{1-\eta} \quad \text{and} \quad M^n_t = B^n(u_t)^\eta(v^n_t)^{1-\eta}
\]

\[
\lambda^x_{w,t} = \frac{M^x_t}{u_t} = B^x \left( \frac{v^x_t}{u_t} \right)^{1-\eta} = B^x (\theta^x_t)^{1-\eta}
\]

\[
\lambda^n_{w,t} = \frac{M^n_t}{u_t} = B^n \left( \frac{v^n_t}{u_t} \right)^{1-\eta} = B^n (\theta^n_t)^{1-\eta}
\]
Value functions

• Discount factor: $S_t = \left(\frac{c_{t+1}}{c_t}\right)^{-\alpha} \frac{P_t}{P_{t+1}}$

$W_t^x (z) = \omega_t^x (z) + \beta \phi S_t (1 - \sigma) \int \max [W_{t+1}^x (h', z'), U_{t+1} (z')] \, dF_e (z'|z) + \beta \phi S_t \sigma \int U_{t+1} (z') \, dF_e (z'|z)$

$U_t (z) = P_t b + \beta \phi S_t \lambda_{w,t} \int \max [W_{t+1}^x (1, z'), U_{t+1} (z')] \, dF_u (z'|z) +$ $\beta \phi S_t \lambda_{w,t} \int \max [W_{t+1}^n (1, z'), U_{t+1} (z')] \, dF_u (z'|z) +$ $\beta \phi S_t (1 - \lambda_{w,t} - \lambda_{w,t}^n) \int U_{t+1} (z') \, dF_u (z'|z)$
Firm values:

\[ J^x_t (z) = p^x_t z - \omega^x_t (z) + \frac{\phi}{1 + r} (1 - \sigma) \int \max \left[ J^x_{t+1} (z'), 0 \right] dF_e (z'|z) \]

\[ J^n_t (z) = p^n_t z - \omega^n_t (z) + \frac{\phi}{1 + r} (1 - \sigma) \int \max \left[ J^n_{t+1} (z'), 0 \right] dF_e (z'|z) \]

Free entry:

\[ \bar{P}^n_m \kappa^n = \frac{\phi}{1 + r} \lambda^n_{f,t} \int \max \left[ J^n_{t+1} (z'), 0 \right] dF_u (z'|z) d\tilde{n}_t^u (z) \]

\[ \bar{P}^x_m \kappa^x = \frac{\phi}{1 + r} \lambda^x_{f,t} \int \max \left[ J^x_{t+1} (z'), 0 \right] dF_u (z'|z) d\tilde{n}_t^u (z) \]
Equilibrium prices

- Non-tradeables

\[ \tau \left( \frac{p^n_t}{P_t} \right)^{-\mu} (c_t - b_t) = \int zdn_{t,n}(z) \]

- Tradeables

\[ (1 - \tau) \left( \frac{p^x_t}{P^m} \right)^{-\nu} \left( \frac{P^m}{P} \right)^{-\mu} (\bar{c} - \bar{b}) + \left( \frac{p^x_t}{P^m} \right)^{-\nu} \bar{c}^f = \int zd\tilde{n}_{t,x}(z) \]

- Idea:
  - drop in \( c_t \) reduces \( p^n_t \) (more so when \( \mu \) is low)
  - labor flows to \( x \), reduces \( p^x_t \) (more so when \( \nu \) is low)
Additional parameters

- Preferences:
  - $\tau = 0.82$ (2/3 employment non-traded – Mian-Sufi)
  - $\mu = \nu = 1.5$ (Backus-Kehoe-Kydland)

- Choose $B^x$ and $B^n$ so that:
  - 80% employment-population
  - steady state $p^x = p^n$

- Choose $\kappa_x$ s.t. $\theta^x = 1$, $\kappa_x/B_x = \kappa_n/B_n$
  - Implies $\theta^n = 1$ and $\omega^x(z) = \omega^n(z)$

- Steady state predictions = one-sector model
Responses absent returns to work

Employment

quarters

0 5 10 15 20 25 30

0.03
0.02
0.01
0
0.01
0.02
0.03

quarters

Total
Traded
Non-Traded
Responses absent returns to work

![Graph showing wages over time for different categories: Average, Traded, Non-Traded. The graph illustrates the impact of responses being absent on the return to work and wages over a period of 30 quarters.]
Our model: employment

Employment

quarters

Our model
No returns to experience
Our model: non-traded employment
Our model: traded employment

Our model
No returns to experience
Our model: wages

The graph compares the wages of two scenarios:

- **Our model**: A blue line representing the wages model with returns to experience.
- **No returns to experience**: A red dotted line showing wages without the effect of experience.

The x-axis represents the number of quarters, ranging from 0 to 30, while the y-axis measures the deviation from baseline wages, ranging from -0.14 to 0.02.
Experiment motivated by Mian-Sufi 2013

- Differentially tighten debt constraint on 20 islands
  - Island 1: consumption falls 0.5% after 2 years
  - ... 
  - Island 20: consumption falls 9.5% after 2 years
Employment vs. consumption: data

Slope = 1/2

source: Midrigan and Philippon (2011)
Employment vs. consumption: model

Employment vs. Consumption, 2 years after shock

Slope = 0.55
Summary

- DMP model with returns to work predicts:
  - employment sensitive to $\Delta$ HH debt constraints
  - as debt constraints reduce these returns
  - so reduce match surplus and employment

- Predictions consistent with Mian-Sufi evidence
Ins and outs

\[ E_{t+1} = (1 - s_t)E_t + f_t(1 - E_t) \]

- \( s_t \): separation rate, \( f_t \): job finding rate

- Construct two counterfactual employment series:
  - Fix \( s_t = \bar{s} \), let \( f_t \) vary
  - Fix \( f_t = \bar{f} \), let \( s_t \) vary
Employment

quarters

actual
fix separation rate
fix finding rate
Intuition from simple model

\[ S(z) = -\frac{b}{\tilde{\rho}} + \frac{z}{\tilde{\rho} - \tilde{g}} \]

• Numerical example: \( r = 0.01, \ g = 0.015, \ \sigma = 0.1, \ \lambda_w = 0.775 \)

• Change \( \rho \) from 0.015 to 0.03
  - \( \tilde{\rho} = \frac{\rho + r + \lambda_w + 2\sigma}{2} : \ 0.5000 \rightarrow 0.5075 \)
  - \( \tilde{g} = g \left( 1 + \frac{\lambda_w}{2\rho} \right) : \ 0.4025 \rightarrow 0.2088 \)

• High \( \lambda_w \) key for quantitative effects
Intuition from simple model

No returns to work, $g = 0$

With returns to work, $g = 0.015$
Hall 2014 experiment: employment

Benchmark

With shock to $r$

quarters

Benchmark

With shock to $r$
Hall 2014 experiment: wages
Data: Employment vs. Consumption

Slope = 1/2

source: Midrigan and Philippon (2011)
Employment vs. consumption across islands

Employment vs. Consumption, 2 years after shock

Slope = 0.55
Employment by sector, 2007-2009

This figure presents scatter-plots of county level employment growth from 2007Q1 to 2009Q1 against the debt to income ratio of the county as of 2006. The left panel examines employment in non-tradable industries excluding construction and the right panel focuses on tradable industries. The sample includes only counties with more than 50,000 households. The thin black line in the left panel is the non-parametric plot of non-tradable employment growth against debt to income.

source: Mian and Sufi (2013)
Employment vs. consumption across islands

\[ \Delta \log(C), \text{2 years} \]

\[ \Delta \log(E) \]

**non-tradable**

**tradable**
Our model: non-traded wages
Our model: traded wages

Our model
No returns to experience