Monetary business cycle accounting

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ABSTRACT

This paper investigates the quantitative importance of various types of distortions for inflation and nominal interest rate dynamics by extending business cycle accounting to monetary models. Representing various classes of real and nominal distortions as 'wedges' in standard equilibrium conditions allows a quantitative assessment of those distortions. Decomposing the data into movements due to these wedges shows that distortions generating movements in TFP and wedges in equilibrium conditions for asset markets are essential. In contrast, wedges capturing the effects of sticky prices play less important role. These results are robust to alternative implementations of the accounting method.

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1. Introduction

Chari et al. (2007a) propose a method, labeled 'business cycle accounting' (BCA), intended to determine what types of distortions are behind the main features of aggregate fluctuations and thus to guide development of structural models of the business cycle. Chari et al. (2007a), henceforth CKM, focus on fluctuations in four key real variables: output, hours, investment, and consumption. Given the growing interest in the interaction between the real and nominal sides of the economy over the business cycle, this paper extends the method to two key nominal variables, inflation and the short-term nominal interest rate—an interest rate broadly under the control of the Federal Reserve. Our aim is to investigate what types of distortions play essential role in the observed dynamics of these two nominal variables at business cycle frequencies. In particular, in their lead–lag relationships with output.

The general idea behind BCA is that detailed model economies with frictions and primitive shocks can be mapped into a parsimonious, perfectly competitive ‘prototype’ economy with a number of time-varying ‘wedges’. The wedges distort the equilibrium conditions of the prototype economy in the same way as the propagation of primitive shocks through frictions distorts those conditions in detailed economies. Thus, the equilibrium allocations and prices in the prototype economy are the same as in the detailed economies.1

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1 The idea that wedges in equilibrium conditions of a competitive economy reflect the distortionary effects of some underlying frictions not modeled explicitly was previously proposed by Hall (1997) and Mulligan (2002).

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In the business cycle literature, many dynamic general equilibrium models with frictions and primitive shocks can be viewed as extensions of the neoclassical growth model with labor-leisure choice. In these extensions the underlyng frictions and shocks distort one or more equilibrium conditions of the growth model: the production function, the intratemporal optimality condition for labor-leisure choice, the intertemporal optimality condition for a consumption–investment decision, and the aggregate resource constraint. CKM show that at a theoretical level it is possible to replicate equilibrium output, labor, consumption, and investment of the detailed economies with frictions within a growth model augmented with four wedges. We will refer to this model as the ‘CKM economy’. At face value, these wedges look like total factor productivity (TFP), a labor income tax, an investment tax, and the sum of government consumption and net exports. CKM label these wedges efficiency, labor, investment, and government consumption wedges, respectively. As a result of this theoretical equivalence, the propagation of primitive shocks through frictions in detailed economies shows up in the prototype as variations in one or more wedges.

Assuming that observed data are the result of decisions by economic agents operating in an environment characterized by the presence of frictions and shocks, the prototype economy can then be used to measure how much of the observed variation in output, consumption, investment, and hours is due to different wedges, and thus due to different classes of underlying distortions (by construction all four wedges account for 100% of the movements in the four data series). Applying the method to the Great Depression and the U.S. post-Korean War business cycle, CKM find that most of the movements in the four data series are due to fluctuations in efficiency and labor wedges, calling thus for models that explain a bulk of the variation in these two wedges, rather than for mechanisms that lead to movements only in investment and government consumption wedges.\(^2\)

In order to extend the method to inflation and the nominal interest rate dynamics, we construct a prototype monetary economy, which is based on a neoclassical growth model (with labor-leisure choice) augmented to include nominal bonds and a parsimonious description of monetary policy. The usefulness of this framework lies in the fact that it underlies a large class of monetary business cycle models (e.g., McGrattan, 1999; Ireland, 2004; Smets and Wouters, 2007, to name a few). In line with this literature, monetary policy is described by a Taylor (1993)-type rule. In addition to the four wedges of the CKM economy, the prototype monetary economy has two additional wedges: an asset market wedge, which acts like a tax on nominal bonds and distorts a standard Euler equation for bonds, and a monetary policy wedge, which distorts the Taylor rule.

The prototype economy is general enough to capture the distortionary effects of the most common frictions in the literature. We show that sticky nominal prices (including their extensions with backward inflation indexation) are equivalent to equal investment and labor wedges, limited participation in asset markets to asset market wedges, and sticky nominal wages to labor wedges.\(^3\) In addition, we show that some detailed monetary policy rules, such as those with regime changes, can be mapped into a prototype Taylor rule with monetary policy wedges. We also show how the effects of various distortions on inflation can be viewed through the lenses of a simple asset-pricing-like equation derived from the equilibrium conditions of the prototype economy.

We apply the method to the dynamics of inflation and the nominal interest rate in the U.S. post-Korean War business cycle with the aim to shed light on two well-documented stylized facts: the correlations at various leads and lags between output and the short-term nominal interest rate, and between output and inflation. In the U.S. business cycle both variables are negatively correlated with future output and positively correlated with past output (although the actual correlations have changed, qualitatively this is the case both before and after the 1979 monetary policy change; see Gavín and Kydland, 2000).\(^4\) Accounting for these features of the business cycle is important for, among other things, enhancing our understanding of the transmission mechanism of monetary policy and the sources of inflation persistence.

Previously, the lead–lag pattern of inflation has been usually studied separately from the lead–lag pattern of the nominal interest rate.\(^5\) Our decomposition, however, shows that the observed dynamics of the two variables over the business cycle are largely driven by the same factors. Specifically, the efficiency and asset market wedges are both necessary, and to some extent also sufficient, for generating the observed lead–lag pattern of the two nominal variables. The other four wedges are less important. Especially the investment and government consumption wedges have little effects. These findings suggest that sticky prices, a friction often invoked in the study of inflation dynamics, are not of a first-order importance for the lead–lag pattern. This is because distortions due to sticky prices manifest themselves as investment and labor wedges. Rather, promising models of the cyclical behavior of inflation and the nominal interest rate need to have, first and foremost, mechanisms that explain the bulk of the variation in efficiency and asset market wedges.

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\(^2\) Other applications in the literature, covering different periods and countries, include, among others, Crucini and Kahn (2003), Ahearne et al. (2005), Chakraborty (2005), Kobayashi and Inaba (2006), Kersting (2008), Simonovska and Soderlind (2008), and Cho and Doblas-Madrid (2009). While the importance of different wedges varies across countries and periods, the efficiency wedge is usually important. Smith and Zin (1997) use a model similar to that of CKM to investigate how much of U.S. output fluctuations is due to the realizations of TFP and government expenditures, while Otsu (2009) extends BCA to a two-country setup.

\(^3\) Sustek (2008) also shows that a model of inflation dynamics based on energy price shocks is equivalent to the prototype model with efficiency wedges.

\(^4\) The ‘inverted leading indicator’ property of the nominal interest rate has been pointed out by, among others, King and Watson (1996), and more recently Backus et al. (2007), while the lagging characteristic of inflation has been highlighted by, among others, Fuhrer and Moore (1995), Gali and Gertler (1999), and more recently Wang and Wen (2007).

\(^5\) For example, Wang and Wen (2007) study only inflation dynamics, while Backus et al. (2007) focus only on the dynamics of yields.
BCA has been critiqued by Christiano and Davis (2006). Their critique—related to potential sensitivity of the decomposition to the exact implementation of the method (whether the investment wedge is a tax on investment or on capital income)—applies to our analysis too (and it also applies to our asset market wedge). We therefore use both specifications of the two wedges. The main quantitative findings are, however, largely unaffected. We also investigate the sensitivity of our findings to different parameterizations of the prototype economy and assumptions about the stochastic process for the wedges. The findings again hold.

The quantitative results are intuitive. As in the CKM economy, the efficiency wedge is crucial for helping the model produce the observed movements in output (and other real variables), while the asset market wedge makes up for the systematic failure of the standard Euler equation to correctly price nominal bonds over the business cycle. Given that in the data inflation and the short-term nominal interest rate have similar dynamics in relation to output (and given that the monetary policy wedge is little correlated with the cycle), helping price bonds correctly, the asset market wedge also brings the model closer to the observed inflation dynamics.

Our results are related to two recent papers. Canzoneri et al. (2007) show that errors in estimated Euler equations for government bonds move systematically with the stance of monetary policy for a wide range of utility functions. Atkeson and Kehoe (2008) show that yield curve data imply systematic cyclical movements in the conditional variance of a pricing kernel for government bonds, something most existing business cycle models (including those with internal habits) fail to generate. Both of these findings are broadly consistent with our result that the asset market wedge is crucial in accounting for the observed inflation and nominal interest rate dynamics at business cycle frequencies.

It is important to be aware of the limits of BCA when drawing conclusions from our quantitative findings. At a theoretical level, more than one friction can be mapped into a given wedge and some frictions can distort more than one equilibrium. The method thus does not identify from the data these two sources of variation separately. For example, persistent movements in a wedge might be due to either iid primitive shocks being propagated through frictions or due to persistent shocks that are at magnified by frictions. In the extreme case, there might be no frictions whatsoever in the data-generating economy and a wedge is just a convex combination of a number of primitive (persistent) shocks. BCA is only a first step, a diagnostic tool, in understanding which frictions need to be distorted if a detailed model is to stand any chance of accounting for movements in the data. In other words, the method points out which wedges are worth studying in more detail and which we can abstract from.

For understanding the cyclical behavior of the two nominal variables, our findings call for further investigation of factors (frictions/shocks/mechanisms) driving the observed movements in efficiency and asset market wedges.

The rest of the paper proceeds as follows. Section 2 describes the prototype monetary economy, Section 3 provides examples of mappings from frictions to wedges, Section 4 uncovers the wedges from the data and characterizes their cyclical behavior, Section 5 carries out the decomposition, and Section 6 addresses the Christiano–Davis critique and carries out additional sensitivity analysis. Section 7 concludes. Appendix A provides proofs of the theorems in Section 3 while Appendix B further elaborates on the mapping from sticky prices to wedges.

2. The prototype economy

The prototype economy is a neoclassical growth model with labor-leisure choice, augmented to include nominal bonds, a parsimonious description of monetary policy, and six wedges.

2.1. Description of the economy

The prototype economy consists of an infinitely lived representative consumer and a representative producer. Both are price takers in all markets. A government taxes the consumer and sets the nominal rate of return on a one-period bond. In period $t$, the economy experiences one of finitely many events $z_t$ (underlying primitive shocks). Let $z^t = (z_0, \ldots, z_t)$ denote the history of events up through and including period $t$, $\mu_t(z^t)$ its probability, and $\mu_t(z^{t+1}|z^t)$ its conditional probability $\mu_{t+1}(z^{t+1})/\mu_t(z^t)$. The economy has six exogenous random variables, all of which are measurable functions of the history of events $z^t$: an efficiency wedge $\epsilon_t(z^t)$, a labor wedge $\tau_l(z^t)$, an investment wedge $\tau_i(z^t)$, a government consumption wedge $g_c(z^t)$, an asset market wedge $\tau_m(z^t)$, and a monetary policy wedge $\tau_p(z^t)$. The first four wedges are the same as those in the CKM economy (in a sense made precise below) and will be therefore referred to as CKM wedges.

Defining all quantities in per-capita terms, the consumer maximizes expected utility over stochastic paths of consumption $c_t(z^t)$ and labor $l_t(z^t)$.

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6 Charl et al. (2007b) provide a response to the critique.

7 It is also possible that a friction that produces movements in the empirically important wedge ends up distorting other equilibrium conditions too. In this case, including the friction into a structural model can still produce the main features of the data as long as the distortions in the other equilibrium conditions are in line with the empirical behavior of the other wedges or, if not, are quantitatively small. Otherwise, frictions that undo those effects need to be included.
\[
\sum_{t=0}^{\infty} \sum_{z'} \beta^t \mu_t(z') u(c_t(z'), 1 - l_t(z'))(1 + \gamma_n)^t,
\]

where \( \beta \) is a discount factor, \( u(\ldots) \) has the standard properties, and \( \gamma_n \) is a population growth rate, subject to a budget constraint

\[
c_t(z') + [1 + \tau_{c}t(z')] x_t(z') + [1 + \tau_{bt}(z')] \left[ (1 + \gamma_n) \frac{b_t(z')}{(1 + R_t(z')} \right] p_t(z') - \frac{b_{t-1}(z'^{-1})}{p_t(z')}
= [1 - \tau_{t}(z')] w_t(z') l_t(z') + r_t(z') k_t(z'^{-1}) + T_t(z').
\]

(2)

Here, \( x_t(z') \) is investment, \( p_t(z') \) is a nominal price of goods in terms of a unit of account, \( b_t(z') \) is holdings of a bond that pays a net nominal rate of return \( R_t(z') \) in all states of the world in period \( t + 1 \) (and is in net zero supply), \( w_t(z') \) is the real wage rate, \( r_t(z') \) is the real rental rate for capital, \( k_t(z'^{-1}) \) is capital held at the start of period \( t \), and \( T_t(z') \) is a lump-sum transfer from the government. Capital evolves as

\[
(1 + \gamma_n) k_{t+1}(z') = (1 - \delta) k_t(z'^{-1}) + x_t(z'),
\]

(3)

where \( \delta \) is a depreciation rate. Notice that the investment and asset market wedges act like taxes on capital and nominal bond accumulation, respectively, while the labor wedge acts like a tax on labor income.

The producer operates an aggregate constant-returns-to-scale production function

\[
y_t(z') = A_t(z') F(k_t(z'^{-1})), (1 + \gamma_A)^l_t(z'),
\]

(4)

where \( \gamma_A \) is the growth rate of labor-augmenting technological progress and \( F(\ldots) \) has the standard properties. The producer maximizes profits \( y_t(z') - w_t(z') l_t(z') - r_t(z') k_t(z'^{-1}) \) by setting the marginal products of capital and labor equal to \( r_t(z') \) and \( w_t(z') \), respectively. The aggregate resource constraint requires that

\[
c_t(z') + x_t(z') + g_t(z') = y_t(z').
\]

(5)

Notice that the efficiency wedge is equal to TFP and the government consumption wedge acts like government consumption.

Following Taylor (1993), and a large empirical literature on monetary policy (surveyed, for instance, by Woodford (2003, Chapter 1), most existing monetary business cycle models describe monetary policy as following a parsimonious feedback rule like that proposed by Taylor. In order to preserve the structure of this class of models, we also assume such a rule. Specifically,

\[
R_t(z') = (1 - \rho_R)[R + \omega_T \left( \ln y_t(z') - \ln y \right) + \omega_T (\pi_t(z') - \pi)] + \rho_R R_{t-1}(z'^{-1}) + \tilde{R}_t(z'),
\]

(6)

where \( \rho_R \in [0, 1] \), \( \pi_t(z') = \ln p_t(z') - \ln p_{t-1}(z'^{-1}) \) is the inflation rate, and a variable's symbol without a time subscript denotes the variable’s steady-state (or balanced growth path) value. In addition, in line with much of the literature, we assume that \( \omega_T > 1 \), thus eliminating explosive inflation paths.\(^8\)

2.2. Equilibrium and the distortionary effects of wedges

An equilibrium of the prototype economy is characterized by the production function (4), the resource constraint (5), the monetary policy rule (6), and the consumer’s first-order conditions for labor, capital, and bonds, respectively,

\[
[1 - \tau_{ct}(z')] A_t(z') (1 + \gamma_A) F_t(z') u_{ct}(z') = u_{bt}(z'),
\]

(7)

\[
\sum_{z_{t+1}} Q_t(z_{t+1} | z_t) \left[ (1 + \tau_{s,t+1}(z_{t+1})) (1 - \delta) + A_{t+1}(z_{t+1}) F_{s,t+1}(z_{t+1}) \right] = 1,
\]

(8)

\[
\sum_{z_{t+1}} Q_t(z_{t+1} | z_t) \left[ \frac{1 + \tau_{b,t+1}(z_{t+1})}{1 + \tau_{bt}(z_t)} \right] p_t(z') \frac{p_{t+1}(z'^{+1})}{p_{t+1}(z'^{-1})} \left[ 1 + R_t(z') \right] = 1.
\]

(9)

where

\(^8\) Most existing monetary business cycle models with centralized markets and an interest rate policy rule abstract from money. This is because in such environments nominal money balances are determined residually, after all other equilibrium allocations and prices have been determined (see, for instance, Woodford, 2003). In Sustek (2008) we allow for money in the prototype and detailed economies through a ‘shopping time’ specification. In this setup real money balances generally affect equilibrium allocations and prices. However, for U.S. post-Korean War calibration these effects are small and thus do not change the main quantitative results of this paper. In addition, the theoretical results established in the next section are unaffected by the presence of money in the prototype.
\[ Q_t(z^{t+1}|z^t) \equiv \beta \mu_t(z^{t+1}|z^t) \frac{u_{ct+1}(z^{t+1})}{u_{ct}(z^t)} \]

is a stochastic discount factor, with \( \beta^\ast \equiv (1 + \gamma_t)^{-1} \beta \). Here, and throughout the paper, \( u_{ct}, u_{bt}, F_t, \) and \( F_t \) denote the derivatives of the utility and production functions with respect to their arguments. Once we substitute for capital from the law of motion (3), these six equilibrium conditions determine equilibrium \((c_t(z^t), x_t(z^t), y_t(z^t), l_t(z^t), p_t(z^t), R_t(z^t))\).

The equilibrium conditions (4), (5), (7), and (8) are the same as those in the CKM economy. The equilibrium conditions (6) and (9) are new. As in the CKM economy, the efficiency wedge affects the productivity of the economy, the government consumption wedge determines the amount of output available for consumption and investment, the labor wedge distorts the intratemporal optimality condition for labor, and the investment wedge distorts the intertemporal optimality condition for investment (Chari et al., 2007a, provide examples of frictions for these four wedges). In addition, the asset market wedge distorts the intertemporal optimality condition for bonds and the monetary policy wedge distorts the Taylor rule.

Notice that the economy is block recursive. First, (4), (5), (7), and (8) determine the equilibrium allocations \((c_t(z^t), x_t(z^t), y_t(z^t), l_t(z^t))\). Then the monetary policy rule (6) and the optimality condition for bonds (9) determine equilibrium \(p_t(z^t)\) and \(R_t(z^t)\). As a result of this recursive structure, CKM wedges affect both real and nominal variables, whereas the asset market and monetary policy wedges affect only the two nominal variables. The propagation of primitive shocks due to underlying frictions, including nominal rigidities, will manifest itself as movements in various wedges in the prototype economy. If these frictions manifest themselves as CKM wedges, they will affect all variables. If, instead, they manifest themselves as asset market or monetary policy wedges, they will only affect the two nominal variables. If the underlying economy has a friction (or host of frictions) that distort all six equilibrium conditions, the propagation of a shock(s) through these frictions will generate movements in all six wedges. No orthogonality conditions are therefore imposed on the joint stochastic process for the wedges.

### 2.3. Inflation dynamics through the lenses of the prototype economy

For the purposes of the following discussion it is instructive to combine Eqs. (8) and (9) and to log-linearize the resulting equation around the model’s non-stochastic steady state. We obtain

\[-a_2 \hat{\tau}_t + E_t(a_1 \hat{\tau}_{t+1} + a_3 \hat{A}_{t+1} + a_7 \hat{R}_{t+1} - a_5 \hat{k}_{t+1}) = a_6 E_t \hat{a}_{b,t+1} + a_8 \hat{R}_t + a_9 E_t \hat{a}_{t+1}.\]

Similarly, we log-linearize the Taylor rule (6)

\[ \hat{R}_t = (1 - \rho_R) \omega_R \hat{y}_t + (1 - \rho_R) \omega_\pi \hat{\pi}_t + \rho_R \hat{R}_{t-1} + \hat{r}_t. \]

In these two equations variables with a ‘hat’ denote percentage deviations from steady state and the coefficients in (11) are defined as: \( a_1 \equiv (1 - \delta)/(1 + \tau_x) \), \( a_2 \equiv [(1 - \delta)(1 + \tau_x) + A \hat{F}_t]/(1 + \tau_x)^2 \), \( a_3 \equiv F_t A/(1 + \tau_x) \), \( a_4 \equiv A \hat{F}_t/(1 + \tau_x) \), \( a_5 \equiv -A \hat{F}_t k/(1 + \tau_x) \), \( a_6 \equiv (1 + R)/(1 + \pi)(1 + \tau_b) \), \( a_7 \equiv (1 + R)/(1 + \pi)(1 + \tau_b) \), \( a_8 \equiv 1, a_9 \equiv (1 + R)/(1 + \pi)^2.\) Notice that for \( \tau_x, \tau_b > -1 \), which we assume here (and which is the case in the actual application of the method to the U.S. data), all these coefficients are positive.

Assuming, for illustration, that each wedge follows an AR(1) process (in the actual application the wedges will follow a joint VAR(m) process), and eliminating \( \hat{R}_t \) from Eqs. (11) and (12), inflation in period \( t \) can be expressed as

\[ \hat{\pi}_t = \frac{1}{(1 - \rho_R) \omega_\pi} \left\{ -(a_2 - a_1 \rho_\pi) \hat{\tau}_t + a_3 \rho_A \hat{A}_t + a_4 \hat{E}_t \hat{\pi}_{t+1} - a_5 E_t \hat{a}_{t+1} \right. \]

\[ + \left. (a_7 - a_6 \rho_R) \hat{a}_{b,t} - (1 - \rho_R) \omega_R \hat{y}_t - \rho_R \hat{R}_{t-1} - \hat{r}_t + a_9 E_t \hat{a}_{t+1} \right\}. \]

where \( \rho_A, \rho_\pi, \rho_\pi \) are the autocorrelation coefficients of the AR(1) processes for the investment, efficiency, and asset market wedges, respectively. It can be shown that the terms \( (a_2 - a_1 \rho_\pi) \) and \( (a_7 - a_6 \rho_R) \) are positive, for \( \rho_\pi, \rho_R \in (0, 1). \) The difference equation (13) can be solved by forward substitution to obtain a particular solution for inflation. Notice that by appearing in the difference equation, investment, efficiency, asset market, and monetary policy wedges have direct effects on inflation. In addition, investment, efficiency, labor, and government consumption wedges have an indirect effect on inflation by affecting equilibrium output, labor, and capital. By substituting (13) into (12), we can in a similar way also characterize the nominal interest rate as a function of the wedges. In all detailed economies with frictions that can be mapped into our prototype, inflation satisfies Eq. (13). As a result, we can view the qualitative effects of the underlying frictions on equilibrium inflation dynamics in such detailed economies through the lenses of this equation.\(^9\)

\(^9\) In the case of the investment, asset market, and monetary policy wedges, the inflation rate, and the nominal interest rate, the variables are expressed as percentage point deviations from steady state.

\(^\ast\) Although Eq. (13) is not a particular solution for inflation, we can still use it to discuss the qualitative effects of wedges on inflation in an equilibrium that excludes explosive paths of inflation. In such an equilibrium, the term \( a_9 (1 - \rho_R) \omega_R \hat{a}_{t+1} \) drops out of the particular solution as \( i \to 10\), while (since \( a_9 > 0 \)) all other terms have the same qualitative effects on inflation as in the difference equation (13).
2.3.1. Example with sticky prices

As an example, consider an economy in which sticky prices are the only friction (e.g., Ireland, 2004). As the next section shows, sticky prices are equivalent to equal investment and labor wedges. In a sticky-price economy a negative ‘demand’ shock typically leads to a fall in both output and inflation; see Ireland (2004), Fig. 1. The propagation of such a shock through sticky prices shows up in our prototype economy as an equal increase in labor and investment wedges. An increase in these two wedges has two effects on inflation. First, there is an indirect effect working through allocations: when the substitution effect in the choice between current and future leisure is sufficiently strong (as is usually the case with standard calibrations), an increase in the labor wedge (a tax on labor) in the current period causes a decline in labor supply and a fall in output. A fall in output working through Eq. (13) increases inflation. Second, there is a direct effect. Sticky prices are equivalent to equal investment and labor wedges. In a sticky-price economy a negative ‘demand’ shock that affects the degree of substitutability between intermediate goods (as discussed in Section 3.1.3, the choice of ϕ in Eq. (13) is the same across the economies). This ensures consistency of expectations across the two economies. In addition, we assume that u(.,.) and F(.,.) are the same across the economies.

3. Equivalence results

This section provides three examples of mappings from detailed economies with frictions to the prototype. Throughout we retain the notation of Section 2. For new variables, notation will be introduced as we go. For brevity, we abstract from population and technology growth. In each example, we assume that the underlying probability space of the detailed economy is the same as that of the prototype. This ensures consistency of expectations across the two economies. In addition, we assume that u(.,.) and F(.,.) are the same across the economies.

3.1. Sticky prices

3.1.1. Detailed economy

There are two types of producers: identical final good producers and intermediate good producers indexed by \( j \in [0, 1] \). Final good producers take all prices as given and solve

\[
\max_{\alpha_t(z'), \alpha_t(j, z')} p_t(z') y_t(z') - \int p_t(j, z') y_t(j, z') \, dj
\]

subject to a production function

\[
y_t(z') = \left[ \int y_t(j, z') \phi_t(z') \, dj \right]^{1/\phi_t(z')}.
\]

Here, \( y_t(z') \) is aggregate output, \( y_t(j, z') \) is input of an intermediate good \( j \), \( p_t(j, z') \) is its price, and \( \phi_t(z') \) is an exogenous shock that affects the degree of substitutability between intermediate goods (as discussed in Section 3.1.3, the choice of the shock is not crucial for the main result). The solution to this problem is characterized by a demand function for an intermediate good \( j \)

\[
y_t(j, z') = \left( \frac{p_t(z')}{p_t(j, z')} \right)^{1/1-\phi_t(z')} y_t(z'), \quad j \in [0, 1].
\]

and a price aggregator

\[
p_t(z') = \left[ \int p_t(j, z') \phi_t(z') \, dj \right]^{\phi_t(z')^{-1}/\phi_t(z')}.
\]

The problem of a producer of an intermediate good \( j \) can be split into two sub-problems. First, for a given level of output \( y_t(j, z') \) the producer solves

\[
\min_{l_t(j, z'), k_t(j, z')} w_t(z') l_t(j, z') + r_t(z') k_t(j, z')
\]

subject to \( F(k_t(j, z'), l_t(j, z')) = y_t(j, z') \). Denoting the value function of this cost minimization problem by \( \vartheta(y_t(j, z'), w_t(z'), r_t(z')) \), in the second step the producer chooses its price \( p_t(j, z') \) in order to maximize the present value of profits

\[
\sum_{t=0}^{\infty} \sum_{z'} Q_t(z') \left[ \frac{p_t(j, z')}{p_t(z')} y_t(j, z') - \vartheta(y_t(j, z'), w_t(z'), r_t(z')) - \frac{\phi}{2} \left( \frac{p_t(j, z')}{\pi p_t-1(j, z'-1) - 1} \right)^2 \right]
\]

In order to simplify our discussion, we abstract here from the effects of the increase in the two wedges on the consumption-saving decision.

When \( u(.,.) \) and \( F(.,.) \) in detailed economies differ from those in the prototype, the deviations show up as wedges (Chari et al., 2007b).
subject to the demand function (14). Here, \( Q_t(z^t) \) is an appropriate discount factor and the last term in the square brackets is a price adjustment cost as in Rotemberg (1982). Given a symmetry across producers, all of them choose the same price, capital, and labor. Rotemberg’s specification of price stickiness is used here for its simplicity. Appendix B shows that the main result also holds for Calvo-style price setting (Calvo, 1983).

The consumer maximizes the utility function (1), subject to the budget constraint
\[
c_t(z^t) + x_t(z^t) + \frac{b_t(z^t)}{p_t(z^t)(1 + R_t(z^t))} = w_t(z^t)l_t(z^t) + r_t(z^t)k_t(z^t) + \frac{b_{t-1}(z^{t-1})}{p_t(z^t)} + T_t(z^t) + \psi_t(z^t),
\]
where \( \psi_t(z^t) \) is profits from intermediate goods producers, and the law of motion for capital (3). The government follows the monetary policy feedback rule (6), but without the monetary policy wedge. Its budget constraint is \( T_t(z^t) = 0.5\phi(p_t(z^t)/\pi p_{t-1}(z^{t-1}) - 1)^2 \); i.e., we assume that the price adjustment cost acts like a tax that is rebated to the consumer.

An equilibrium of this sticky-price economy is a set of allocations \((c_t(z^t), x_t(z^t), y_t(z^t), l_t(z^t), k_{t+1}(z^t))\) and a set of prices \((p_t(z^t), R_t(z^t), r_t(z^t), w_t(z^t))\) that satisfy: (i) a set of consumer’s first-order conditions for labor, capital, and bonds, respectively,
\[
u_{zt}(z^t)w_t(z^t) = u_{ht}(z^t), \tag{15}
\]
\[
\sum_{z_{t+1}} Q_t(z^{t+1}|z^t)[1 + r_{t+1}(z^{t+1}) - \delta] = 1, \tag{16}
\]
\[
\sum_{z_{t+1}} Q_t(z^{t+1}|z^t)[1 + R_t(z^t)]\frac{p_t(z^t)}{p_{t+1}(z^{t+1})} = 1, \tag{17}
\]
where \( Q_t(z^{t+1}|z^t) \) is given by (10); (ii) a set of optimality conditions for the cost minimization problem of intermediate goods producers
\[
\frac{F_{ht}(z^t)}{F_{lt}(z^t)} = \frac{r_t(z^t)}{w_t(z^t)}, \tag{18}
\]
\[
y_{t}(z^t) = F(k_t(z^{t-1}), l_t(z^t)); \tag{19}
\]
(iii) a first-order condition for the profit maximization problem of intermediate goods producers, the so-called ‘New-Keynesian Phillips curve’
\[
\Phi(\pi_t(z^t), y_t(z^t), \eta_t(z^t), \xi_t(z^t)) + \sum_{z_{t+1}} Q_t(z^{t+1}|z^t)\Psi(\pi_{t+1}(z^{t+1}), y_{t+1}(z^{t+1}), \xi_{t+1}(z^{t+1})) = 0, \tag{20}
\]
where \( \eta_t(z^t) = \partial \psi_t(z^t)/\partial y_t(z^t) \) is the marginal cost and \( \Phi(\ldots, \ldots) \) and \( \Psi(\ldots, \ldots) \) are smooth functions; (iv) the resource constraint \( c_t(z^t) + x_t(z^t) = y_t(z^t) \); (v) the capital accumulation law (3); and (vi) the monetary policy rule (6) without the monetary policy wedge. Notice that in this economy equilibrium \( r_t(z^t) = F_{ht}(z^t) \) is replaced by the New-Keynesian Phillips curve.

3.1.2. The mapping
Consider now a version of the prototype economy of Section 2 that has an investment wedge that acts like a tax on capital income rather than a tax on investment.\(^{13}\) The equilibrium condition (8) then becomes
\[
\sum_{z_{t+1}} Q_t(z^{t+1}|z^t)[1 - \tau_{k,t+1}(z^{t+1})]A_{t+1}(z^{t+1})F_{k,t+1}(z^{t+1}) + (1 - \delta) = 1, \tag{21}
\]
where \( \tau_{ht}(z^t) \) is the capital income tax and \( Q_t(z^{t+1}|z^t) \) is given as before by (10).

**Proposition 1.** Consider equilibrium allocations of the economy with sticky prices \((c_t^*(z^t), x_t^*(z^t), y_t^*(z^t), l_t^*(z^t), k_{t+1}^*(z^t))\) and prices \((p_t^*(z^t), R_t^*(z^t))\). Let the wedges in the prototype economy satisfy: \( A_t(z^t) = 1, \tau_{ht}(z^t) = g_t(z^t) = R_t(z^t) = 0, \) and
\[
\tau_{kt}(z^t) = \tau_{ht}(z^t) = 1 - \frac{r_t^*(z^t)}{F_{kt}^*(z^t)}, \tag{22}
\]
for all \( z^t \), where \( F_{ht}^*(z^t) \) and \( r_t^*(z^t) \) are evaluated at the equilibrium of the sticky-price economy. Then \((c_t^*(z^t), x_t^*(z^t), y_t^*(z^t), l_t^*(z^t), k_{t+1}^*(z^t))\) and \((p_t^*(z^t), R_t^*(z^t))\) are also equilibrium allocations and prices of the prototype economy.

\(^{13}\) Both types of taxes distort the optimality condition for capital accumulation, but the proof is more straightforward in the case of a capital income tax. On a relation between the two representations of the investment wedge see Chari et al. (2007b).
This proposition shows that sticky prices act like equal labor and investment wedges. Intuitively, imperfect competition in product markets leads to mark-ups that create a distortion in factor markets as marginal products of capital and labor are no longer set equal to factor prices. These distortions can be replicated in the prototype economy by choosing \( \tau_F \) and \( \tau_R \) according to (22).

3.1.3. Alternative specifications of the sticky-price economy

We assumed that the price adjustment cost in the sticky-price economy acts like an implicit tax on firms that is rebated in a lump-sum way to the consumer. If instead we assume that the cost is a pure resource loss, this loss shows up in the prototype economy as a government consumption wedge. Notice also that various extensions of the price-setting behavior, such as backward inflation indexation, that only show up in the New-Keynesian Phillips curve (20) will not change Proposition 1. This is because they do not generate any additional distortions above and beyond preventing \( \tau_R \) from being equalized to \( F_{Rt} \). Often, Calvo-style price setting is used instead of Rotemberg’s cost of adjustment. In such a case, aggregation issues lead to an efficiency wedge, in addition to the investment and labor wedges given by Eq. (22). The efficiency wedge, however, disappears in a log-linearized equilibrium (see Appendix B). Finally, the assumption that the exogenous shocks causing fluctuations in the detailed economy are shocks to the elasticity of substitution between intermediate goods is not crucial for Eq. (22) to hold. If, for example, we instead assume that the source of impulses are standard monetary policy shocks \( \xi_t(z') \) in the Taylor rule, we only change Proposition 1 by setting \( R_t(z') = \xi_t(z') \).

To summarize, regardless of whether Rotemberg- or Calvo-style price setting is used (with or without inflation indexation), or the sources of impulses, a common feature of sticky prices is that they manifest themselves as equal investment and labor wedges.

3.2. Limited participation in asset markets

3.2.1. Detailed economy

Consider now a simple limited participation economy due to Christiano and Eichenbaum (1992). In this economy some agents are excluded from the money market at the time of a central bank’s money injection. At the beginning of a period the utility function (1) subject to a ‘transactions constraint’

\[
P_t(z') c_t(z') = \left[ m_{t-1}(z'^{-1}) - q_t(z'^{-1}) \right] + p_t(z') w_t(z') l_t(z') + \psi_t(z'),
\]

where \( \psi_t(z') \) is dividends from firms, and a law of motion for nominal wealth

\[
m_t(z') = \left[ 1 + R_t(z') \right] q_t(z'^{-1}) + \psi_t(z'),
\]

where \( \psi_t(z') \) is profits from financial intermediaries.

The intermediaries take deposits from consumers and make loans to firms. They operate in a perfectly competitive market, which implies that the interest rate on deposits is equal to the interest rate on loans. Firms have access to an aggregate production function \( y_t(z') = F(k_t(z'^{-1}), l_t(z')) \). They need to finance a fraction \( \phi_t \) of their wage bill \( w_t(z') l_t(z') \) through loans from financial intermediaries, which they repay at the end of the period. Using the consumers’ stochastic discount factor, firms maximize a discounted sum of per-period dividends \( F(k_t(z'^{-1}), l_t(z')) + (1 - \beta) k_t(z'^{-1}) - k_{t+1}(z') - [1 + \phi_t(z') R_t(z')] w_t(z') l_t(z') \) by choosing \( k_{t+1}(z') \) and \( l_t(z') \). Notice that the marginal cost of labor to firms is \( [1 + \phi_t(z') R_t(z')] w_t(z') l_t(z') \).

The central bank sets the nominal interest rate according to the feedback rule

\[
R_t(z') = (1 - \rho_R) \left[ R + \omega_y (\ln y_t(z') - \ln y) + \omega_{\pi_t} (\pi_t(z') - \pi) \right] + \rho_R R_{t-1}(z'^{-1}) + \xi_t(z').
\]

where \( \xi_t(z') \) is a monetary policy shock. The central bank implements \( R_t(z') \) through money transfers \( \eta_t \) to financial intermediaries. Notice that as \( R_t \) is a function of \( z' \), this happens only after consumers have made their deposits. The total funds at the disposal of the intermediaries are therefore \( q_t(z'^{-1}) + \eta_t(z') \) and they are lent to firms at the rate \( R_t(z') \). Clearing the money market requires

\[
q_t(z'^{-1}) + \eta_t(z') = \phi_t(z') p_t(z') w_t(z') l_t(z')
\]

and the money injection \( \eta_t(z') \) is such that this condition holds at \( R_t(z') \). Profits of the intermediaries \( \psi_t(z') \) are given by the interest they earn on the extra money balances injected by the central bank, i.e., \( \psi_t(z') = R_t(z') \eta_t(z') \).
An equilibrium of this economy with limited participation is a set of allocations \((c_t(z'), x_t(z'), y_t(z'), l_t(z'), k_{t+1}(z'), m_t(z'), q_t(z', \eta_t(z')))\) and a set of prices \((p_t(z'), R_t(z'), w_t(z'))\) that satisfy: (i) the consumers' first-order conditions for deposits and labor, respectively,

\[
\sum_{z_t} \mu_{t-1}(z'_t | z_{t-1}) \frac{u_{ct}(z'_t)}{p_t(z'_t)} = \beta \sum_{z_t} \mu_{t-1}(z'_{t+1}) \frac{u_{ct+1}(z'_{t+1})}{p_{t+1}(z'_{t+1})} (1 + R_t(z'_t)),
\]

(ii) the firms' first-order conditions for capital

\[
u_{ut}(z'_t) \frac{1 + \phi_t(z'_t) R_t(z'_t)}{F_{lt}(z'_t)} = u_{ct}(z'_t);
\]

and labor \(w_t(z'_t) = F_{lt}(z'_t) / [1 + \phi_t(z'_t) R_t(z'_t)]\); (iii) the transactions constraint (23); (iv) the law of motion for nominal wealth (24); (v) the money market clearing condition (26); (vi) the aggregate resource constraint \(c_t(z') + x_t(z') = y_t(z')\), where \(y_t(z') = F(k_t(z'_{t-1}), l_t(z'))\); (vii) the capital accumulation law (3); and (viii) the monetary policy rule (25).

3.2.2. The mapping

**Proposition 2.** Consider equilibrium allocations of the economy with limited participation \((c^*_t(z'), x^*_t(z'), y^*_t(z'), l^*_t(z'), k^*_{t+1}(z'))\) and prices \((p^*_t(z'), R^*_t(z'))\). Let the wedges in the prototype economy satisfy: \(A_t(z') = 1, R_t(z') = g_t(z') = 0, k_t(z') = \xi_t(z')\), and

\[
[1 - \tau_{lt}(z')] = \frac{1}{1 + \phi_t(z') R^*_t(z')},
\]

\[
[1 + \tau_{lt}(z')] \frac{u^*_{ct}(z'_t)}{p^*_t(z'_t)} = \sum_{z_t} \mu_t(z'_{t+1} | z'_t) \frac{u^*_{ct+1}(z'_{t+1})}{p^*_{t+1}(z'_{t+1})} \frac{\beta u^*_{ct+1}(z'_{t+1})}{p^*_{t+1}(z'_{t+1})} [1 + \tau_{0,t+1}(z'_{t+1})][1 + R^*_t(z'_t)]
\]

for all \(z'_t\), where \(u^*_{ct}(z'_t)\) is evaluated at the equilibrium of the detailed economy. Then \((c^*_t(z'), x^*_t(z'), y^*_t(z'), l^*_t(z'), k^*_{t+1}(z'))\) and \((p^*_t(z'), R^*_t(z'))\) are also equilibrium allocations and prices of the prototype economy.

**Proof.** See Appendix A. "

Consider now a special case of Proposition 2. Suppose that the fraction of the wage bill financed through loans \(\phi_t\) fluctuates in response to changes in the interest rate so as to leave the effective wage rate \((1 + \phi_t R_t) w_t\) unchanged. In this case, monetary policy shocks lead to movements in \(\tau_{lt}\) but not \(\xi_t\). The main point here is that limited participation in the money market acts like a tax on nominal bonds in the prototype economy that distorts the standard Euler equation for bonds. Fuerst (1992) calls this distortion a 'liquidity effect'.

3.3. Sticky wages

3.3.1. Detailed economy

As a final example consider an economy populated by a continuum of infinitely lived consumers differentiated by a labor type \(j \in [0, 1]\). Consumers of type \(j\) are organized in a labor union \(j\). A representative producer has access to an aggregate production function \(y_t(z') = F(l_t(z'_{t-1}), l_t(z'))\), where

\[
l_t(z') = \int l_t(j, z') \delta_z(j, z') \, dz'
\]

is a labor aggregate and \(\delta_z(j, z')\) is a shock to the degree of substitutability between labor types (for reasons similar to those in Section 3.1.3, the main result of this section does not depend on the source of impulses). The representative producer's problem can be described in two steps. First, for a given \(l_t(z')\), the producer solves

\[
\min_{l_t(j, z') | j \in [0, 1]} \int W_t(j, z'_{t-1}) l_t(j, z') \, dz'
\]

subject to (32), where \(W_t(j, z'_{t-1})\) is a nominal wage rate for labor type \(j\), set by union \(j\) before the realization of \(z_t\). The solution to this problem gives the producer's demand function for each labor type

\[
l_t(j, z') = \frac{W_t(j, z'_{t-1})}{\sum_{j'} W_t(j', z'_{t-1})} l_t(z'),
\]
where
\[ W_t(z^t) = \left[ \int W_t(j, z^t) \frac{\rho_t(z^t)(z^t-1)}{\rho_t(z^t)} \, dj \right]^{\frac{z^t}{z^t-1}} \]
is the aggregate nominal wage rate. In the second step, the producer chooses \( k_t \) and \( l_t \) to maximize profits \( F(k_t(z^t-1), l_t(z^t-1)) - r_t(z^t)k_t(z^t-1) - [W_t(z^t-1)/p_t(z^t)]l_t(z^t) \) by setting the marginal products of capital and labor equal to factor prices. When setting the wage rate, the union agrees to supply in period \( t \) whatever amount of labor is demanded at the real wage rate \( W_t(j, z^t-1)/p_t(z^t) \).

The preferences of consumer \( j \) are characterized by the utility function (1), where \( c \) and \( l \) are indexed by \( j \). The consumer/union’s problem is to choose plans for \( c_t(j, z^t), x_t(j, z^t), k_{t+1}(j, z^t), l_t(j, z^t), b_t(j, z^t) \), and \( W_{t+1}(j, z^t) \) in order to maximize the utility function (1), subject to the labor demand function (33), the budget constraint (2) (without wedges), and the capital accumulation law (3), where the appropriate quantities are indexed by \( j \). Assuming that \( k_0 \) and \( b_0 \) are the same for all types, the solution to this problem is symmetric across all \( j \)’s. The government sets the nominal interest rate according to the policy rule (6) (without the monetary policy wedge).

An equilibrium of this economy with sticky nominal wages is a set of allocations \( (c_t(z^t), x_t(z^t), y_t(z^t), l_t(z^t), k_{t+1}(z^t)) \) and a set of prices \( (p_t(z^t), R_t(z^t), r_t(z^t), W_{t+1}(z^t)) \) that satisfy: (i) the consumer’s first-order conditions for wages, capital, and bonds, respectively,
\[ \sum_{z_t=1} \mu_t(z_t+1|z_t)u_{h_t+1}(z_t+1)W_{t+1}(z_t) = W_{t+1}(z_t), \]
\[ \sum_{z_t=1} Q_t(z_t+1|z_t)(1 + r_{t+1}(z_t+1) - \delta) = 1, \]
\[ \sum_{z_t=1} Q_t(z_t+1|z_t)(1 + R_t(z_t)) \frac{p_t(z_t)}{p_t(z_t+1)} = 1, \]
where \( Q_t(z_t+1|z_t) \) is given by (10); (ii) the producer’s first-order conditions \( r_t(z^t) = F_{k_t}(z^t) \) and \( W_t(z^t-1)/p_t(z^t) = F_{l_t}(z^t) \); (iii) the resource constraint \( c_t(z^t) + x_t(z^t) = y_t(z^t) \), where \( y_t(z^t) = F(k_t(z^t-1), l_t(z^t)) \); (iv) the capital accumulation law (3); and (v) the monetary policy rule.

3.3.2. The mapping

CKM show that when \( \tau_t(z^t) = 1 - u_{c_t}^*(z^t)/[u_{c_t}^*(z^t)F_{k_t}^*(z^t)] \), where \( u_{c_t}^*(z^t), u_{c_t}^*(z^t), \) and \( F_{k_t}^*(z^t) \) are evaluated at the equilibrium of the detailed economy, their real prototype economy is equivalent (in terms of allocations) to the sticky-wage economy just described (see their Proposition 2). This condition also implies equivalence between the detailed economy and our prototype. Notice that setting \( \tau_t(z^t) \) and \( R_t(z^t) \) equal to zero ensures that the equilibrium condition for bonds and the monetary policy rule in the prototype are the same as their counterparts in the sticky-wage economy. And as the other equilibrium conditions in our prototype are the same as those in the CKM economy, by setting \( \tau_t(z^t) \) according to the above condition, our prototype reproduces the equilibrium allocations, as well as \( p_t^* \) and \( R_t^* \), of the sticky-wage economy.

An implication of this result for inflation and nominal interest rate dynamics is that, viewed through the lenses of the pricing function (13), sticky wages have only an indirect effect on inflation and the nominal interest rate by affecting equilibrium allocations. This is because the labor wedge does not enter directly the pricing function (13).

3.4. The monetary policy wedge

The monetary policy wedge represents all aspects of monetary policy above and beyond the responses of the monetary authority to output and inflation as summarized by a standard Taylor rule usually assumed in macroeconomic models. As an example, consider a Taylor-type rule with fluctuations in the inflation target (due to, for example, appointments of different central bankers) considered by Gavin et al. (2007). Their monetary policy rule has the form
\[ R_t(z^t) = R + \omega_y (\ln y_t(z^t) - \ln y) + \omega_{\pi_t} (\pi_t(z^t) - \bar{\pi}_t(z^t)) + \rho_R R_{t-1}(z^t-1), \]
where \( \bar{\pi}_t(z^t) \) is a time-varying inflation target. This policy rule is equivalent to the prototype rule (6), where the inflation target is constant and the monetary policy wedge is given by \( \bar{R}_t(z^t) = -\omega_{\pi_t} (\bar{\pi}_t(z^t) - \pi) \).

4. Data-implied wedges

This section uses an operational version of the prototype economy together with data on the economy’s endogenous variables to uncover the six wedges from the data.
4.1. Overview of the procedure

Our procedure follows that of CKM (see their paper for details). We assume that, given a history $z^t$, there is a one-to-one and onto mapping between $z_{t+1}$ and a vector of wedges, $\omega_{t+1} \equiv (\log A_{t+1}, t_1, t_{x, t+1}, t_{r, t+1}, \log g_{t+1}, t_{b, t+1}, \hat{R}_{t+1})$; i.e., conditionally on a history $z^t$ the agents in the prototype can uniquely infer $z_{t+1}$ from $\omega_{t+1}$. We can therefore replace the probability space for the events for one with events and change the expectation operator in the utility function (1) accordingly. However, as the wedges depend on the entire history of events, generally the stochastic process for the wedges is not Markov, even when the process for the events is—as is commonly assumed in dynamic general equilibrium models. Therefore, in order to make the method operational, we need to assume that the process for the wedges can be approximated by an $m$th-order Markov process. In particular, we assume a VAR($m$) specification

$$\omega_t = P_0 + P_1 \omega_{t-1} + \cdots + P_m \omega_{t-m} + \epsilon_t,$$

(35)

where $\epsilon_{t+1}$ is iid over time and distributed normally with mean zero and a covariance matrix $V = BB'$. There are no restrictions imposed on this stochastic process besides stationarity. In particular, in line with our discussion in Section 2.2, the off-diagonal elements of $P$ and $V$ are allowed to be non-zero.

Using the approximate prototype economy and data series on $c_t, x_t, y_t, b_t, p_t,$ and $R_t$ we can uncover the realized wedges that exactly reproduce the six data series. The procedure consists of three steps. First, we choose functional forms of the utility and production functions, and calibrate their parameter values, as well as the parameter values of the monetary policy rule. Second, we estimate the parameters of the stochastic process (35). And third, we use the equilibrium decision rules and pricing functions of the parameterized, approximate prototype economy to back out the realized wedges from the data.

Notice that in the second and third steps we need to compute the equilibrium decision rules and pricing functions of the prototype. As the state space is large—there are nine state variables in the model ($\omega_t, p_{t-1}, R_{t-1},$ and $k_t$)—the prototype is further approximated by a linear-quadratic economy and the equilibrium is computed using the method described by Hansen and Prescott (1995). Before computing the equilibrium, the model is transformed so that the price level is stationary. The output of this solution method is a set of decision rules and pricing functions that express the deviations of the prototype. As the state space is large—there are nine state variables in the model ($\omega_t, p_{t-1}, R_{t-1},$ and $k_t$)—the prototype is further approximated by a linear-quadratic economy and the equilibrium is computed using the method described by Hansen and Prescott (1995). Before computing the equilibrium, the model is transformed so that the price level is stationary. The output of this solution method is a set of decision rules and pricing functions that express the deviations of the utility and production functions, and calibrate their parameter values, as well as the parameter values of the monetary policy rule. Second, we estimate the parameters of the stochastic process (35). And third, we use the equilibrium decision rules and pricing functions of the parameterized, approximate prototype economy to back out the realized wedges from the data.

4.2. Calibration

We use the same functional forms and parameter values for the utility and production functions as CKM: $u(\ldots) = \lambda \log c_t + (1 - \lambda) \log h_t, F(\ldots) = k^2 ((1 + \gamma_A)^2 l_t)^{1-\delta}, \gamma_A = 0.0037, \beta = 0.99, \lambda = 0.31, \gamma_A = 0.004, \delta = 0.0118,$ and $\alpha = 0.35$. These functional forms and parameter values are standard in the business cycle literature. The parameters of the monetary policy rule are also set equal to fairly standard values (Woodford, 2003, Chapter 1): $\omega_{yt} = 0.125$ (which corresponds to 0.5 when inflation and the nominal interest rate are expressed at annualized rates), $\omega_{k_t} = 1.5$, and $\rho_R = 0.75$. In Section 5 we also consider alternative parameterizations of the monetary policy rule.

4.3. Estimation of the stochastic process

As in CKM, the parameters ($P_0, \ldots, P_m, B$) of the process (35) are estimated by maximum likelihood estimation. In our baseline specification, we assume $m = 1$ (in Section 6 we consider $m = 2$). The likelihood function is based on a state-space representation consisting of the process (35) and the aforementioned linear approximations to the equilibrium decision rules and pricing functions for $(y_t, l_t, x_t, c_t, p_t, R_t)$. The estimation is carried out for the period 1959.Q1–2004.Q4 using quarterly data on output (GDP plus imputed services from consumer durable goods), investment (which includes household expenditures on durable goods), consumption (the sum of household expenditures on nondurable goods, services, and imputed flow of services from durable goods), hours from the Establishment Survey, the GDP deflator, and the yield on 3-month Treasury bills; capital is computed recursively using the law of motion (3), data on investment, and an initial capital stock.14 The search for the maximum is implemented using simulated annealing with a conservative temperature reduction factor of 0.95 (Goffe et al., 1994) in order to thoroughly explore the surface of the likelihood function. The estimates are reported in Table 1.

---

14 The data on output, investment, consumption, and hours are in per-capita terms and are transformed by taking logarithms. In addition, a common trend of 0.4% (at a quarterly rate) is removed from per-capita output, investment, and consumption, and a trend of 0.91% (the average quarterly inflation rate) is removed from the price level (in logarithm).
Table 1
Stochastic process for wedges—baseline prototype economy.a.

| $P_0 = [-0.0811, 0.0074, -0.0336, 0.0476, -0.012, -0.012]_t$ | $0.8581$ | $-0.0965$ | $0.1732$ | $-0.0064$ | $-0.0435$ | $0.5188$ |
| $P_1 = $ | $0.0675$ | $1.0610$ | $0.0019$ | $0.0110$ | $0.0467$ | $-0.7241$ |
| $P_2 = $ | $-0.0860$ | $-0.0220$ | $1.0890$ | $0.0026$ | $-0.0121$ | $0.4020$ |
| $P_3 = $ | $0.0821$ | $0.0587$ | $-0.0987$ | $1.0061$ | $0.0254$ | $0.367$ |
| $P_4 = $ | $0.0985$ | $-0.3110$ | $0.0870$ | $-0.0101$ | $0.8260$ | $0.1200$ |
| $\mathbf{B} = $ | $0.0073$ | $0$ | $0$ | $0$ | $0$ | $0$ |
| | $0.0038$ | $0.0011$ | $0$ | $0$ | $0$ | $0$ |
| | $0.0058$ | $-0.0009$ | $0.0031$ | $0$ | $0$ | $0$ |
| | $0.0009$ | $0.0051$ | $0.0119$ | $0.0087$ | $0$ | $0$ |
| | $0.0005$ | $-0.0175$ | $-0.0014$ | $0.0015$ | $0.0219$ | $0$ |
| | $0.0003$ | $9.5e-6$ | $0.0001$ | $-0.0004$ | $0.0038$ | $0.0011$ |

\(a\) In the baseline prototype economy investment and asset market wedges are specified as taxes on investment and changes in bond holdings, respectively, and the six wedges follow a VAR(1) process.

Table 2

<table>
<thead>
<tr>
<th>Wedges</th>
<th>Rel.</th>
<th>Correlations of output in period $t$ with wedges in $t + j$:</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>std.b</td>
<td>$j = -4$</td>
</tr>
<tr>
<td>Baseline prototype economy(c)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\log A_{t+j}$</td>
<td>0.63</td>
<td>0.33</td>
</tr>
<tr>
<td>$\tau_{t+j}$</td>
<td>0.92</td>
<td>$-0.17$</td>
</tr>
<tr>
<td>$\tau d_{t+j}$</td>
<td>0.50</td>
<td>0.16</td>
</tr>
<tr>
<td>$\log b_{t+j}$</td>
<td>1.51</td>
<td>$-0.40$</td>
</tr>
<tr>
<td>$\tau d_{t+j}$</td>
<td>2.59</td>
<td>0.06</td>
</tr>
<tr>
<td>$\hat{R}_{t+j}$</td>
<td>0.12</td>
<td>0.11</td>
</tr>
<tr>
<td>VAR(2) specification</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\tau_{t+j}$</td>
<td>0.40</td>
<td>0.02</td>
</tr>
<tr>
<td>$\tau d_{t+j}$</td>
<td>2.25</td>
<td>0.11</td>
</tr>
<tr>
<td>Investment wedge as a tax on capital returns</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\tau_{t+j}$</td>
<td>0.64</td>
<td>0.24</td>
</tr>
<tr>
<td>$\tau d_{t+j}$</td>
<td>3.10</td>
<td>0.11</td>
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<tr>
<td>Asset market wedge as a tax on bond returns</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\tau_{t+j}$</td>
<td>0.61</td>
<td>0.30</td>
</tr>
<tr>
<td>$\tau d_{t+j}$</td>
<td>1.19</td>
<td>0.20</td>
</tr>
</tbody>
</table>

\(a\) The statistics are for wedges and per-capita output filtered with the HP-filter.
\(b\) The standard deviations are measured relative to that of per-capita output, which is 1.58.
\(c\) In the baseline prototype economy investment and asset market wedges are specified as taxes on investment and changes in bond holdings, respectively, and the six wedges follow a VAR(1) process.

4.4. Uncovering the realized wedges

The estimated process is then used to compute the equilibrium of the model and to uncover the realized wedges from the data. We denote the vector of realized wedges by $\omega^d = (\log A^d_t, \tau^d_t, \tau^d_t, \log g^d_t, \tau^d_t, \hat{R}^d_t)$. From the resource constraint (5) and our measurement of output, consumption, and investment follows that $g^d_t$ is simply the sum of government expenditure and net exports. The government consumption wedge can thus be obtained directly from the National Income and Product Accounts. The remaining five wedges are obtained from the linear approximations to the equilibrium decision rules and pricing functions for $y_t, l_t, x_t, p_t$, and $R_t$—a linear system of five equations that in each period can be solved for the five wedges ($\log A^d_t, \tau^d_t, \tau^d_t, \tau^d_t, \hat{R}^d_t$), using data on ($\log y_t, \log x_t, \log l_t, \log p_t, R_t$) and ($\log g^d_t, \log p_t-1, R_{t-1}, \log k_t$). Notice that as a result of this procedure, all six wedges together account for 100% of the movements in the six data series on $y_t, l_t, x_t, p_t, R_t$, and $R^d_t$. Notice also that this procedure is essentially equivalent to obtaining $\log A^d_t$ from a linear approximation to the production function (4), $\tau^d_t, \tau^d_t, \tau^d_t, \tau^d_t, \tau^d_t$, and $\tau^d_t$ from linear approximations to the first-order conditions (7), (8), and (9), respectively, and $\hat{R}^d_t$ from the monetary policy rule (6). Thus, only $\tau^d_t$ and $\tau^d_t$ depend on expectations and thus on the estimated stochastic process for the wedges.

4.5. Business cycle properties of the realized wedges

Table 2 characterizes the cyclical behavior of the realized wedges—filtered with the HP-filter (Hodrick and Prescott, 1997)—by reporting their correlations, at various leads and lags, with HP-filtered (logged) output. The table also reports
Fig. 1. The asset market wedge in the 1960 and 1990 recessions. The plots are for deviations of log per-capita consumption from a linear trend, and for inflation (annualized), the nominal interest rate (annualized), and the asset market wedge from their respective mean values. The deviations are normalized to be zero at the start of each recession.

their standard deviations relative to that of output, which is 1.58. Here we focus on the top panel of the table, which is for the baseline specification of the prototype economy—VAR(1) specification and investment and asset market wedges as in the budget constraint (2).

As the four equilibrium conditions of the ‘real’ block of the prototype economy are the same as those of the CKM economy, and given the block-recursive structure of the prototype, the realized efficiency, labor, and government consumption wedges are exactly the same as those obtained by CKM, while the realized investment wedge differs slightly from theirs because our stochastic process is different (it contains two additional wedges). We therefore do not provide a detailed discussion of the cyclical behavior of the CKM wedges. For our future discussion, we just point out that the efficiency wedge tends to lead the cycle—it is more positively correlated with future output than past output. Turning to the two new wedges, we see that the asset market wedge is the most volatile of the six wedges, moving 2.59 times as much as output. It is also strongly procyclical, with a contemporaneous correlation of 0.82, and tends to lag output—it is more positively correlated with past output than future output. In contrast, the monetary policy wedge is the least volatile and only weakly correlated with the cycle.

The high volatility of the asset market wedge reflects the well-known failure of asset pricing models based on standard Euler equations to account for the volatility of asset prices (e.g., Hansen and Singleton, 1983). The strong positive comovement of the asset market wedge with output reported in Table 2 further reveals that the extent of this failure, here in the case of 3-month T-bills, varies systematically over the business cycle.15 The strong positive comovement of the asset market wedge with output can be broadly understood by inspecting the equilibrium condition (9). When log-linearized, for a CRRA utility function (separable in leisure) the equation becomes

\[ -\log \beta + \gamma E_1 \Delta c_{t+1} \approx -(1 - \rho_b) \tau_{bt} + R_t - E_1 \pi_{t+1}, \]

where \( \gamma \) is the inverse of the intertemporal elasticity of substitution and \( \Delta c_{t+1} \) is a consumption growth rate. As in Section 2.3, we assume (for expositional purposes) that the asset market wedge follows an AR(1) process with an autocorrelation coefficient \( \rho_b \in [0, 1] \). Fig. 1 plots the movements in the level of consumption, inflation, the nominal interest rate, and the asset market wedge during the 1960 and 1990 recessions, which we use as examples. In both periods we see a persistent fall in the level of consumption and thus a negative \( \Delta c_{t+1} \) for the first couple of periods of the downturns. Further, \( R_t \) falls generally by more than \( \pi_t \), leading to a fall in the real interest rate. (These movements are representative of other postwar downturns, with the exception of the 1982 recession.) The fall in the real interest rate, however, is larger than the fall in \( \gamma \Delta c_{t+1} \) for reasonable values of \( \gamma \) (we use \( \gamma = 1 \)). Essentially, this figure shows that demand for government bonds is greater in downturns (perhaps due to ‘flight to quality’) than can be justified by standard preferences. This drives the real return on T-bills below that predicted by standard Euler equations. Thus, in order to compensate for this ‘excessive’ fall in the real return, the asset market wedge in (36) has to decline. Notice also that in line with the cyclical behavior of the asset market wedge reported in Table 2, the wedge begins to recover after the business cycle trough (here proxied by the trough in consumption).

\[ \text{Fig. 1. The asset market wedge in the 1960 and 1990 recessions. The plots are for deviations of log per-capita consumption from a linear trend, and for inflation (annualized), the nominal interest rate (annualized), and the asset market wedge from their respective mean values. The deviations are normalized to be zero at the start of each recession.} \]

15 Canzoneri et al. (2007) find that common utility functions produce errors in estimated Euler equations that are strongly correlated with the stance of monetary policy, while Atkeson and Kehoe (2008) discuss the failure of CRRA utility functions and those with internal habits to price government bonds.
5. Accounting for the stylized facts

This section decomposes the observed movements in output, inflation, and the nominal interest rate into movements due to each wedge. Our aim is to investigate to what extent different wedges generate the observed dynamics of inflation and the nominal interest rate over the business cycle described in the Introduction.

5.1. The procedure

The decomposition follows the procedure proposed by CKM. Recall that in our approximate prototype economy the probability space for the events is replaced with a probability space for the wedges, which follow an $m$th-order Markov process. As a result, in the approximate prototype economy the wedges play two roles: (i) a distortionary role—they distort the equilibrium conditions like in the theoretical prototype economy of Section 2; and (ii) a forecasting role—they form the probability space of the economy (a role played by events in Section 2). In our decompositions, like in Sections 2 and 3, we are interested in the distortionary role of the wedges. Thus, when we want to isolate the distortionary effects of, for example, the labor wedge, we solve a version of the approximate prototype economy in which: (i) the agents form expectations using the stochastic process (35), with the parameter values reported in Table 1 (top panel); and (ii) in the budget and resource constraints, as well as in the monetary policy rule, all wedges except the labor wedge are held constant at their steady-state values. As a result all wedges are used to form expectations but only the labor wedge distorts the equilibrium. Let $y^{t}(\omega_{t}, \log p_{t-1}, R_{t-1}, \log k_{t})$, $x^{t}(\omega_{t}, \log p_{t-1}, R_{t-1}, \log k_{t})$, $c^{t}(\omega_{t}, \log p_{t-1}, R_{t-1}, \log k_{t})$, $I^{t}(\omega_{t}, \log p_{t-1}, R_{t-1}, \log k_{t})$, $P^{t}(\omega_{t}, \log p_{t-1}, R_{t-1}, \log k_{t})$, and $R^{t}(\omega_{t}, \log p_{t-1}, R_{t-1}, \log k_{t})$ be the equilibrium (linear) decision rules and pricing functions of this modified approximate prototype economy (inflation is calculated as $\log p_{t} - \log p_{t-1}$). Starting from $\log p_{t-1}$, $R_{t-1}$ for 1959.Q1 and $\log k_{t}$ for 1959.Q2, these decision rules and pricing functions are used, together with $\omega^{t}$ (the vector of realized wedges obtained in Section 4), to compute the labor wedge component of the data. In a similar way we also obtain the components of the data due to the fluctuations in the other wedges, and in their various combinations. By construction, when we allow all six wedges to distort the equilibrium, we exactly reproduce the data.

5.2. The stylized facts

Figs. 2 and 3 contain plots, respectively, of the correlations between output in period $t$ and the nominal interest rate in period $t + j$, and between output in period $t$ and inflation in period $t + j$, for $j = [-5, \ldots, 0, \ldots, 5]$, for the period 1959.Q1–2004.Q3 (see plots labeled ‘data’). Here, output is transformed by taking logarithm and all three series are filtered with the HP-filter, though similar results are obtained for the Christiano and Fitzgerald (2003) filter and for demeaned inflation and the nominal interest rate. We see that both nominal variables are negatively correlated with future output and positively correlated with past output. This pattern is somewhat more pronounced for the nominal interest rate than for inflation. As mentioned in the Introduction, in the case of the nominal interest rate, the focus of the literature has been on the negative lead, whereas in the case of inflation it has been on the positive lag. Looking at these two variables together, however, reveals that the two aspects of their dynamics pointed out by the literature are a part of the same pattern.

5.3. Results of the decomposition

The results of the decomposition are presented in Figs. 2–4. In each panel of Figs. 2 and 3 we plot against the correlations in the data the correlations obtained from the approximate prototype economy when we hold, in the sense described above, one wedge at a time fixed. Fig. 3 is for the nominal interest rate, while Fig. 4 is for inflation. As all six wedges exactly reproduce the data, holding one wedge fixed at a time assesses the necessity of the particular wedge for reproducing the observed dynamics.

In the top-left panel of Fig. 2 we see that holding fixed the efficiency wedge generates the opposite dynamics to those in the data: the nominal interest rate becomes strongly positively correlated with future output and only little correlated with past output. Holding fixed the labor wedge also deteriorates the observed dynamics, but somewhat preserves its general pattern, leaving the interest rate negatively correlated at leads and slightly positively correlated at most lags. Holding fixed the investment wedge has hardly any effect at all, while holding fixed the government consumption wedge has only a small effect on the pattern of the dynamics. However, when we hold fixed the asset market wedge, the nominal interest rate becomes strongly countercyclical with no apparent lead–lag relationship with output. Holding fixed the monetary policy wedge in contrast does not change the overall pattern of the observed dynamics, although it makes the lead–lag pattern less pronounced than in the data.

The results of the decomposition are essentially the same for inflation. As we can see in Fig. 3, holding fixed the efficiency wedge again turns the lead–lag pattern the other way around, while holding fixed the asset market wedge makes

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16 The shape of the lead–lag pattern of inflation is also robust to alternative measures of the aggregate price level, such as the consumer price index (CPI) and CPI excluding food and energy.
inflation strongly negatively correlated with output contemporaneously without any clear lead–lag pattern. In contrast, without any of the other four wedges, the model still produces the right shape of the lead–lag relationship between output and inflation.

So far we have investigated the necessity of the individual wedges. Given the importance of the efficiency and asset market wedges, we now investigate to what extent these two wedges alone (i.e., holding the other four wedges fixed) can reproduce the observed cross-correlations. We carry out this experiment for the baseline parameterization of the Taylor rule, and for four alternative parameterizations: \( \omega_y = [0.05, 0.175] \) (which correspond to 0.2 and 0.7, respectively, for annualized rates), and \( \omega_\pi = [1.3, 1.8] \). These values fall within the range of estimates found in different empirical studies (Woodford, 2003, Chapter 1). In each case we re-estimate the stochastic process for the wedges, back out the wedges from the re-estimated model (only \( \tau_{xt}, \tau_{bt}, \) and \( \tilde{R}_t \) are affected), and use the re-estimated economy and the wedges to carry out the
The results are reported in Fig. 4. We see that for all five parameterizations these two wedges alone produce the right shape of the lead–lag empirical relationship between output and the two nominal variables, though not the same correlations.

The reason for why the efficiency and asset market wedges play such an important role is this. As in a standard real business cycle model the movements in the efficiency wedge account reasonably well for fluctuations in output, while the asset market wedge—which is strongly positively correlated with the efficiency wedge but somewhat lags output—helps the model price the government bond over the business cycle. And because the monetary policy wedge is relatively small (and largely uncorrelated with output over the business cycle) the efficiency and asset market wedges alone can to some extent produce the observed cross-correlations of both inflation and the nominal interest rate.
6. Alternative specifications of the prototype economy

In this section we consider alternative specifications of the prototype economy. For each specification we report the estimated stochastic process for the wedges in Table 3, the cyclical properties of the two intertemporal wedges in Table 2, and the results of the decompositions in Table 4 (for the nominal interest rate) and in Table 5 (for inflation).

6.1. VAR(2) process for wedges

First we investigate the sensitivity of our results to including additional lags in the stochastic process for wedges. Namely, we consider a VAR(2) specification. Although including more lags increases the accuracy of the approximation of the stochastic process (recall that the wedges in period $t$ generally depend on the entire history of wedges up through and including period $t-1$), it considerably increases computational requirements of the estimation. While for VAR(1) process we need to estimate 61 parameters (the elements of $P_0$, $P_1$, and $B$), for VAR(2) we need to estimate 97 parameters (the elements of $P_2$ in addition to those above). The estimates of $P_1$ and $P_2$, which are used in forming the agents’ expectations, are reported in the top panel of Table 3. In Table 2 we see that the cyclical behavior of the investment and asset market wedges is similar to that under VAR(1) specification, although the investment wedge is now less correlated with output at all leads and lags. In Tables 4 and 5 we then see that the results of the decomposition are also similar to those under VAR(1) specification.

6.2. Christiano–Davis critique

Christiano and Davis (2006) critique BCA on the grounds that the results of the decomposition can be sensitive to the exact specification of the investment wedge—whether it is a tax on investment, as assumed in our economy, or a tax on capital income. In the latter case, the wedge—denoted by $\tau^I_{t,t}$—enters the right-hand side of the budget constraint (2) as $(1 - \tau^I_{t,t})r_t + \tau^I_{t,t}\delta$. While in theory the choice of the investment wedge cannot matter (Chari et al., 2007b), in practice it can. This is because in practice we work only with an approximate prototype economy, in which the probability space for events is replaced with a probability space for wedges approximated by an $m$th-order Markov process. In the case of our prototype monetary economy the Christiano–Davis critique applies not only to the investment wedge, but also to the asset market wedge. When specified as a tax on bonds income, the wedge—denoted by $\tau^B_{t,t}$—enters the right-hand side of the budget constraint as $(1 - \tau^B_{t,t})b_{t-1}/p_t$. However, as Tables 2, 4, and 5 show, neither the cyclical behavior of the two wedges, nor the results of the decomposition, are particularly sensitive to the exact specification of the two wedges.

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17 Another critique of Christiano and Davis concerns the decomposition itself. Christiano and Davis argue in favor of SVAR decompositions over BCA decompositions as a tool for identifying promising models of the business cycle. As discussed in detail by Chari et al. (2007a, 2007b), the two decompositions ask very different questions and the two methods are not necessarily inconsistent with each other. Essentially, BCA asks how much of the variation in the data is due to a particular type of frictions, regardless of what primitive shocks are propagated through them. In contrast, SVAR decompositions ask how much of the variation is due to a particular primitive shock, regardless of through what frictions it is propagated.
The transition matrix in the stochastic process for wedges—alternative specifications of the prototype economy.

<table>
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<tr>
<th>Model version</th>
<th>Correlations of output in period $t$ with the nominal interest rate in $t + j$:</th>
</tr>
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<td></td>
<td>$j = -5$</td>
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<tr>
<td>Baseline</td>
<td>0.39</td>
</tr>
<tr>
<td>VAR(2)</td>
<td>-0.14</td>
</tr>
<tr>
<td>$r^*_t$</td>
<td>-0.16</td>
</tr>
<tr>
<td>$r^*_t$</td>
<td>-0.31</td>
</tr>
<tr>
<td>$r^*_t$</td>
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<tr>
<td>No investment wedge</td>
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<tr>
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<tr>
<td>$r^*_t$</td>
<td>0.01</td>
</tr>
<tr>
<td>$r^*_t$</td>
<td>0.16</td>
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<tr>
<td>No asset market wedge</td>
<td>-0.36</td>
</tr>
<tr>
<td>Baseline</td>
<td>-0.41</td>
</tr>
<tr>
<td>VAR(2)</td>
<td>-0.37</td>
</tr>
<tr>
<td>$r^*_t$</td>
<td>-0.42</td>
</tr>
</tbody>
</table>

*Baseline = investment and asset market wedges are specified as taxes on investment and changes in bond holdings and the six wedges follow a VAR(1) process; VAR(2) = same as baseline, expect that wedges follow a VAR(2) process; $r^*_t$ = same as baseline, except that the investment wedge is specified as a tax on capital returns; $r^*_t$ = same as baseline except that the asset market wedge is specified as a tax on bond returns.*
the asset market wedge based on a simple model of limited participation. Indeed, other asset market frictions might prove to lead output, while the asset market wedge tends to lag output. We have provided an example of a possible interpretation of our findings suggest that they should, first and foremost, include distortions that account for bulk of the movements in generally, distortions equivalent to the labor wedge need to be included too. And although the labor wedge is not as crucial implies that the labor wedge is as important for real variables as in the CKM economy. If the goal is to construct a model that accounts not only for the empirical regularity targeted in this paper, but for the behavior of the U.S. economy more crucial in accounting for the lead–lag relationship of the two nominal variables with output, rather than to assess the relationship between output and inflation and between output and the short-term nominal interest rate. Using different process; VAR(2)

<table>
<thead>
<tr>
<th>Model versiona</th>
<th>Correlations of output in period t with the nominal interest rate in t + j: j = −5 −4 −3 −2 −1 0 1 2 3 4 5</th>
</tr>
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<tr>
<td>No efficiency wedge</td>
<td>Baseline</td>
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<tr>
<td>No labor wedge</td>
<td>Baseline</td>
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<tr>
<td>No investment wedge</td>
<td>Baseline</td>
</tr>
<tr>
<td>No asset market wedge</td>
<td>Baseline</td>
</tr>
<tr>
<td>Efficiency and asset market wedges only</td>
<td>Baseline</td>
</tr>
<tr>
<td>a Baseline = investment and asset market wedges are specified as taxes on investment and changes in bond holdings and the six wedges follow a VAR(1) process; VAR(2) = same as baseline, except that wedges follow a VAR(2) process; r^<em>_j = same as baseline, except that the investment wedge is specified as a tax on capital returns; r^</em>_j = same as baseline except that the asset market wedge is specified as a tax on bond returns.</td>
<td></td>
</tr>
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</table>

7. Conclusions

This paper extended BCA with the aim to shed light on two stylized facts in the nominal business cycle: the lead–lag relationship between output and inflation and between output and the short-term nominal interest rate. Using different specifications of the prototype economy, data decompositions show that efficiency and asset market wedges are necessary for generating the observed dynamics. And at least qualitatively, they are to some extent also sufficient. The other wedges play less important roles. This finding is particularly interesting as sticky prices—a friction usually invoked in the study of inflation dynamics—show up in the prototype economy as (equal) labor and investment wedges, rather than efficiency and/or asset market wedges. Although this finding does not imply that sticky prices play no role in the propagation of shocks or in the transmission mechanism of monetary policy, it suggests that other mechanisms—those that lead to movements in efficiency and asset market wedges—are more important in accounting for the observed lead–lag relationship between output and inflation and between output and the nominal interest rate.

CKM find that the labor wedge plays an important role in the dynamics of real variables, in particular hours worked and slow recoveries. Our findings are not inconsistent with their result. The block recursive structure of our prototype economy implies that the labor wedge is as important for real variables as in the CKM economy. If the goal is to construct a model that accounts not only for the empirical regularity targeted in this paper, but for the behavior of the U.S. economy more generally, distortions equivalent to the labor wedge need to be included too. And although the labor wedge is not as crucial as efficiency and asset market wedges for the issue studied here, we saw that its contribution to the dynamics of nominal variables, especially the nominal interest rate, is not insignificant. Our goal was to highlight which wedges are absolutely crucial in accounting for the lead–lag relationship of the two nominal variables with output, rather than to assess the importance of different wedges more generally.

We hope that our findings will provide useful information to researchers constructing models to analyze the business cycle and monetary policy. To the extent that such models should be consistent with basic nominal business cycle facts, our findings suggest that they should, first and foremost, include distortions that account for bulk of the movements in efficiency and asset market wedges. Especially the latter wedge has been mostly omitted in the study of inflation dynamics. We have documented that these two wedges are strongly positively correlated with output. The efficiency wedge tends to lead output, while the asset market wedge tends to lag output. We have provided an example of a possible interpretation of the asset market wedge based on a simple model of limited participation. Indeed, other asset market frictions might prove...
promising in generating the observed movements in the asset market wedge, as might a theory of cyclical movements in risk premia. We leave such investigations for future research.

Appendix A

A.1. Proof of Proposition 1

The proof proceeds by comparing the equilibrium conditions of the detailed economy with those of the prototype. Notice that when in the prototype \( A_t(z^*) = 1 \) and \( \tau_{kt}(z^*) = \tau_{kt}(z^*) = \gamma_t(z^*) = g_t(z^*) = R_t(z^*) = 0 \), the equilibrium conditions in the two economies are the same except that in the prototype the capital rental rate is set equal to the marginal product of capital, whereas in the detailed economy this equilibrium condition is replaced by the optimal price-setting condition (20). Since in the detailed economy \( F^*_t(z^*) \neq F^*_t(z^*) \), it follows from the equilibrium condition (18) that also \( w^*_t(z^*) \neq F^*_t(z^*) \).

The two economies thus differ only in terms of the prices of capital and labor that consumers face. We can, however, eliminate these differences by appropriately choosing \( \tau_{kt}(z^*) \) and \( \tau_{kt}(z^*) \) in the prototype. In particular, let \( \tau_{kt}(z^*) \) satisfy \( r^*_t(z^*) = (1 - \tau_{kt}(z^*))F^*_t(z^*) \) and let \( \tau_{kt}(z^*) \) satisfy \( w^*_t(z^*) = (1 - \tau_{kt}(z^*))F^*_t(z^*) \) for every history \( z^* \), where \( F^*_t \) and \( F^*_t \) are evaluated at the equilibrium allocations of the detailed economy. Then the first-order conditions for capital and labor in the two economies are the same and the equilibrium allocations \( (c^*_t(z^*), x^*_t(z^*), y^*_t(z^*), l^*_t(z^*), k^*_t(z^*), k^*_t(z^*)) \) and prices \( (p^*_t(z^*), R^*_t(z^*)) \) of the detailed economy are also equilibrium allocations and prices of the prototype economy. In addition, since in the detailed economy \( w^*_t(z^*) = (1 - \tau_{kt}(z^*))F^*_t(z^*) \) and therefore \( \tau_{kt}(z^*) = \tau_{kt}(z^*) \).

A.2. Proof of Proposition 2

The proof of (30) is based on a similar argument as that of Proposition 1. We therefore concentrate on the proof of (31). Notice that by using the law of iterated expectations, Eq. (27) can be written as \( E_{t-1}[\Lambda_t] = 0 \), where

\[
\Lambda_t \equiv u_{ct}/p_t - (1 + R_t)\beta E_t[u_{c,t+1}/p_{t+1}],
\]

which is generally non-zero. By setting \( \tau_{kt} \) in the equilibrium condition for bonds (9) in the prototype economy equal to zero in all states of the world, the condition becomes

\[
0 = u_{ct}/p_t - (1 + R_t)\beta E_t[u_{c,t+1}/p_{t+1}].
\]

Notice that (38) differs from (37) only in terms of the left-hand side. Choosing \( \tau_{kt} \) according to (31) implies that the right-hand side of (38), when evaluated at the equilibrium allocations and prices of the detailed economy, is equal to \( \Lambda_t \).

Appendix B. Calvo-style price setting

An economy with Calvo-style price setting differs from the one in Section 3.1 only in the price-setting behavior of intermediate good producers. In order to simplify the exposition, we assume that the production function \( F(\ldots) \) is Cobb-Douglas.

With probability \( \varphi \) an intermediate good producer \( j \) is allowed to set its price optimally in period \( t \). Otherwise it has to charge the price chosen last time it was allowed to change it. The shock that determines whether a producer can change its price is iid across producers and time. Producers that are allowed to change price choose its price \( p_t(j) \) to maximize expected discounted profits \( \varepsilon_t \sum_{i=0}^{\infty}(1 - \varphi)^i Q_{t+i}(|p_t(j)/p_{t+i}|y_{t+i}(j) - \kappa_t y_{t+i}(j)) \), subject to the demand function (14). In the profit function \( \kappa_t = (r_t/\varphi)^F[w_t/(1 - \varphi)]^{1-\varphi} \) is a marginal cost obtained from the solution of the cost minimization problem described in Section 3.1. The solution to the profit maximization problem is

\[
p_t^* = \left[ \frac{E_t \sum_{i=0}^{\infty}(1 - \varphi)^i Q_{t+i} \kappa_t y_{t+i}^{1/(1-\varphi)} p_{t+i}^{(1-\varphi)/\varphi}}{E_t \sum_{i=0}^{\infty}(1 - \varphi)^i Q_{t+i} p_{t+i}^{(1-\varphi)/\varphi} y_{t+i}} \right]^{(\varphi-1)/\varphi}.
\]

(39)

After substituting for \( y_{t+i} \) in the profit function of final good producers from Eq. (14), a zero-profit condition for these producers implies that the aggregate price level is given by

\[
p_t = \left[ \int p_t(j)^{\varphi/(1-\varphi)} \, dj \right]^{(\varphi-1)/\varphi} = \left[ \varphi(p_t^*)^{\varphi/(1-\varphi)} + (1 - \varphi)p_{t-1}^{\varphi/(1-\varphi)} \right]^{(\varphi-1)/\varphi}.
\]

(40)

Notice that in Rotemberg's model, \( p_t(k) = p_t(j) \) for all \( j \neq k \) as the producers only differ by their index. Here, however, \( p_t(k) = p_t(j) \) only for those \( k \) and \( j \) that are allowed to change price in period \( t \). Eq. (40), with \( p_t^* \) given by (39), constitutes a New-Keynesian Phillips curve in a model with Calvo-style price setting. It replaces the New-Keynesian Phillips curve (20) in the model of Section 3.1 and thus creates the same distortion as (20), namely \( r_t \neq F_{kt} \). Finally, aggregating output across intermediate good producers by integrating (14) leads to
where

$$y_t = k_t \hat{p}_t^{1/\alpha}$$

$$d_j = \left( \frac{p_t}{p_t(j)} \right)^{1/(\alpha-1)}$$

(41)

Notice that in the function for aggregate output (41), \(p_t/\hat{p}_t\) is an aggregation bias, which shows up in our prototype economy as an efficiency wedge. This bias is not present in a model with Rotemberg price setting as \(p_t(k) = p_t(j) = p_t\) for all \(j \neq k\). In a log-linear approximation to the model, commonly employed in the study of the postwar U.S. business cycle, the aggregation bias, however, disappears and Calvo-style price setting ends up distorting the equilibrium only through labor and investment wedges. To show this, we rewrite Eqs. (40)-(42) in their log-linearized forms:

$$\hat{y}_t = \frac{1}{\epsilon - 1} (\hat{p}_t - \hat{p}_t) + \alpha \hat{k}_t + (1 - \alpha) \hat{y}_t.$$

$$\hat{p}_t = \frac{\varphi(p_t^*)}{\epsilon^{1/\epsilon}} \hat{p}_t^* + \frac{(1 - \varphi)p_t^*}{\epsilon^{1/\epsilon}} \hat{p}_t - 1.$$

$$\hat{p}_t = \frac{\varphi(p_t^*)}{\epsilon^{1/\epsilon}} \hat{p}_t^* + \frac{(1 - \varphi)p_t^*}{\epsilon^{1/\epsilon}} \hat{p}_t - 1.$$

(42)

where variables with a 'hat' denote percentage deviations from steady state. Notice that in steady state Eq. (40) becomes \(p_t = \varphi \hat{p}_t + (1 - \varphi) \hat{p}_t - 1\) and \(\hat{p}_t = \varphi \hat{p}_t + (1 - \varphi) \hat{p}_t - 1\), which implies \(\hat{p}_t - \hat{p}_t = 0\).

References


