Identifying Taylor Rules in Macro-Finance Models

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Identifying the Taylor rule

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- In the context of a new-Keynesian model, ... I show that the parameters of the Fed’s policy rule are not identified.

- [The Taylor rule parameter] is not identified from data on \( \{i_t, \pi_t\} \) [the short rate and inflation] in the equilibrium of this model.

- The crucial Taylor rule parameter is not identified in the new-Keynesian model.
The parameters of a Taylor rule are not econometrically identified within [affine macro-finance models].

Several recent studies interpret the short-rate equation as a Taylor-style rule. ... However, without imposing additional economic structure, ... the parameters are not meaningfully interpretable as the reaction coefficients of a central bank.

Absent additional economic structure, there appears to be no basis for interpreting any one of these equivalent rotations as the Taylor rule of a structural model.
Identifying the Taylor rule

What we do

- Illustrate the problem
- Characterize its solution
Identifying the Taylor rule

What we do

- Illustrate the problem
- Characterize its solution
- Monetary policy and asset pricing can once more safely coexist
Two examples
Cochrane’s example

Model

\[ i_t = r + E_t \pi_{t+1} \quad \text{(Euler equation)} \]
\[ i_t = r + \tau \pi_t + s_t \quad \text{(Taylor rule)} \]

State and shock

\[ x_{t+1} = Ax_t + Bw_{t+1}, \quad \{w_t\} \sim \text{NID}(0, I) \]
\[ s_t = d^\top x_t \]
Cochrane’s example

Model

\[ i_t = r + E_t \pi_{t+1} \]  \hspace{1cm} \text{(Euler equation)}

\[ i_t = r + \tau \pi_t + s_t \]  \hspace{1cm} \text{(Taylor rule)}

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\[ s_t = d^\top x_t \]

**Information structure**

- Agents observe everything
- We (economists) observe \((\pi, i, x)\) — but not the shock \(s\)
Cochrane’s example: solution

Equate EE and TR

\[ E_t \pi_{t+1} = \tau \pi_t + s_t \]

Solution: guess \( \pi_t = b^\top x_t \)

\[
\begin{align*}
    b^\top A x_t &= \tau b^\top x_t + d^\top x_t \\
    b^\top A &= \tau b^\top + d^\top 
\end{align*}
\]

\[ \Rightarrow b^\top = -d^\top (\tau I - A)^{-1} \]

Unique stationary solution if \( \tau > 1 \), \( A \) stable
Cochrane’s example: identification

What do we observe?

- State $x$: so we can estimate $A$
- Inflation rate $\pi$: so we can estimate $b$
- **Not the shock** $s$: so we **cannot** estimate $d$

Can we infer Taylor rule parameter $\tau$?

$$
\begin{align*}
    b^\top A &= \begin{cases}
        \tau b^\top + d^\top & \text{(TR)} \\
        & \text{(EE)}
    \end{cases}
\end{align*}
$$
Affine example

Model

\[ i_t = - \log E_t \exp(m_{t+1}^s) = \delta^\top x_t \quad \text{(Euler equation)} \]
\[ m_{t+1}^s = -\lambda^\top \lambda - \delta x_t + \lambda^\top w_{t+1} \quad \text{(pricing kernel)} \]
\[ x_{t+1} = Ax_t + Bw_{t+1} \quad \text{(state transition)} \]

We observe \( x \) and \( i \), so can estimate \( \delta \)

Suppose first two elements of \( x \) are inflation and GDP growth

Can we interpret short rate equation as a Taylor rule?
Thinking out loud

Would an extra shock help? (Gertler)

\[
\begin{align*}
    i_t &= E_t \pi_{t+1} + s_{1t} \\
    i_t &= \tau \pi_t + s_{2t} \\
    s_{it} &= d_i^\top x_t
\end{align*}
\]

(Fisher equation)  (Taylor rule)  (shocks)

Let’s say we observe \( s_1 \) but not \( s_2 \)

If shocks are orthogonal, use \( s_1 \) as an instrument

Why does this work?
Representative agent model
Representative agent model

- Model

\[ i_t = - \log E_t \exp(m_{t+1} - \pi_{t+1}) \]  \hspace{1cm} \text{(Euler equation)}

\[ i_t = r + \tau \pi_t + s_{2t} \]  \hspace{1cm} \text{(Taylor rule)}

\[ m_{t+1} = -\rho - \alpha \log g_{t+1} \]  \hspace{1cm} \text{(real pricing kernel)}

\[ \log g_t = g + s_{1t} \]  \hspace{1cm} \text{(consumption growth)}

\[ x_{t+1} = Ax_t + Bw_{t+1} \]  \hspace{1cm} \text{(state transition)}

\[ s_{it} = d_i^\top x_t \]  \hspace{1cm} \text{(shocks)}

- Solution: guess \( \pi_t = b^\top x_t \)

\[
\underbrace{b^\top A + \alpha d_1^\top}_{\text{EE}} = \underbrace{\tau b^\top + d_2^\top}_{\text{TR}} \implies b^\top = (\alpha d_1 - d_2)^\top (\tau I - A)^{-1}
\]
Representative agent model: identification

What do we observe?

- State $x$: so we can estimate $A$
- Inflation rate $\pi$: so we can estimate $b$
- Consumption growth $g$: so we can estimate $d_1$
- **Not the Taylor rule shock $s_2$: so we cannot estimate $d_2$**

Can we infer Taylor rule parameter $\tau$?

\[
b^\top A + \alpha d_1^\top = \tau b^\top + d_2^\top
\]
Representative agent model: identification

Reminder

\[ b^\top A + \alpha d_1^\top = \tau b^\top + d_2^\top \]

What if we set one element of \( d_2 \) equal to zero?

Other linear restrictions: \( d_2^\top e = 0 \)

What about Gertler’s suggestion that \( s_1 \) and \( s_2 \) are orthogonal?
Representative agent model: identification

Reminder

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\[ d_2^\top \text{Var}(x)d_1 = 0 \]

\[ = e \]
Where does that leave us?
Where does that leave us?

If shock isn’t observed, we need one restriction to infer TR (identification is never free)

Stan: “If a rule depends on everything in an arbitrary way, it’s not a rule.”

More generally, we need one restriction for every TR parameter (ditto other equations)

How do we observe the state?

- Use Kalman filter, replace $x$ with $\hat{x}$
- Harder than it sounds
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It’s now safe again to link finance and macro, charge ahead