Floaters and Swaps

0. Overview

- Swaps as Derivatives
- Floaters and Inverse Floaters
- Swap Analytics
- Swap Engineering
- OTC v Exchange-Traded Products
- Credit Risk
1. The Swap Market

- Market is an outgrowth of the interbank market for currencies and short-term loans/deposits
- Started with international commercial banks, now includes major investment banks
- Market has grown dramatically:

![Graph showing growth in the swap market from 1986 to 1996](image-url)
1. The Swap Market (continued)

- Swaps by category and currency (1995):

<table>
<thead>
<tr>
<th>Product</th>
<th>Notional Principal (US tr)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Interest rate swaps</td>
<td></td>
</tr>
<tr>
<td>US dollar</td>
<td>4.372</td>
</tr>
<tr>
<td>Japanese yen</td>
<td>3.854</td>
</tr>
<tr>
<td>Deutsche mark</td>
<td>2.128</td>
</tr>
<tr>
<td>Pound sterling</td>
<td>0.856</td>
</tr>
<tr>
<td>Cross-currency swaps</td>
<td></td>
</tr>
<tr>
<td>US dollar</td>
<td>0.419</td>
</tr>
<tr>
<td>Japanese yen</td>
<td>0.200</td>
</tr>
<tr>
<td>Deutsche mark</td>
<td>0.110</td>
</tr>
<tr>
<td>Pound sterling</td>
<td>0.046</td>
</tr>
</tbody>
</table>

2. Floating Rate Notes: Examples

- Example 1: Istituto Bancario San Paolo Torino
  - 5-year floating rate notes (FRNs)
  - Maturing June 18, 1997
  - Denominated in ecu’s (European currency units)
  - Semiannual payments of ECU LIBOR + 20 bps
  - Rate set every 6 months, paid 6 months later
  - LIBOR day count convention (reminder):
    \[
    \text{Interest Payment} = \frac{\text{“Actual”}}{360} \times (\text{LIBOR} + 0.20)
    \]
    for LIBOR quoted as a percent.

- Example 2: Citicorp DM notes
  - 5-year floaters
  - Maturing December 15, 1999
  - Quarterly payments of DM LIBOR + 25 bps
2. Floating Rate Notes: Examples (continued)

- Example 3: Daiwa Europe yen notes
  - 5.25-year floaters
  - Maturing March 17, 2000
  - Semiannual payments of 5-year yen swap rate – 90 bps

- Example 4: IBRD (World Bank) inverse floaters
  - 5-year US dollar floaters
  - Maturing October 1, 1997
  - Semiannual payments at a rate of
    \[
    \text{Rate} = 14.5 - 2 \times \text{Dollar LIBOR}
    \]
    (with a minimum of zero)

- Example 5: Deutsche Bank Finance NV notes
  - 10-year C-dollar floaters
  - Maturing September 3, 2002
  - Quarterly rate of 3-month BAs - 30 bps
  - Cap of 8.90, floor of 5-7/8

- Remark: Lots of variety!
3. Floating Rate Note Arithmetic

- Standard floater: 6-month payments of 6-month LIBOR
- Consider a 6-month floater
  - Let $y$ be the bond equivalent of 6-m LIBOR
  - Value in 6 months is principal (100) plus interest
  - Present discounted value is
    \[
    \text{Price} = \frac{100(1 + y/2)}{1 + y/2} = 100.
    \]
- Consider a 12-month floater
  - Value in 6 months is market price (100) plus interest
  - Present discounted value is
    \[
    \text{Price} = \frac{100(1 + y/2)}{1 + y/2} = 100.
    \]
- Consider an 18-month floater ... (you get the idea)
- Remarks:
  - Floaters trade at par on reset dates
  - Despite the uncertain interest payments, this is no more complex than a conventional bond
4. Interest Rate Sensitivity of FRNs

- Standard floaters have short durations

- If fraction $w$ of a period remains until the next payment:

$$\text{Price} = \frac{100 + \text{Interest}}{(1 + \frac{y}{2})^w}$$

(NB: the numerator was set at the start of the period.)

- DV01 computed the usual way (increase $y$ by .01)

- Duration is

$$D = (1 + \frac{y}{2})^{-1}w/2$$
5. Inverse Floaters

- Can be valued by replication
- Example: 5-year, Rate = 20 – LIBOR
  the 10% flat spot rate scenario of previous chapter
- Replication of cash flows:

<table>
<thead>
<tr>
<th>Position</th>
<th>Interest Rate</th>
<th>Principal</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inverse Floater</td>
<td>20 – LIBOR</td>
<td>100</td>
</tr>
<tr>
<td>Long 2 10% Bonds</td>
<td>20</td>
<td>200</td>
</tr>
<tr>
<td>Short FRN</td>
<td>– LIBOR</td>
<td>–100</td>
</tr>
</tbody>
</table>

- Estimate price from components:
  \[ \text{Price} = 2 \times 100 - 100 = 100 \]

- Duration (some numbers from previous set of overheads):
  \[ D = 3.86 \left( \frac{200}{100} \right) + 0.48 \left( \frac{-100}{100} \right) = 7.25 \]

- Remarks
  - Despite the uncertain interest payments,
    this is no more complex than a conventional bond
  - Caveat: floater has floor at zero, replication doesn’t
  - Long duration!
6. Plain Vanilla Interest Rate Swaps

Description

- Swap parties exchange fixed and floating interest payments:

  ![Diagram of swap parties exchanging fixed and floating interest payments]

- Only net payments are made

- Principals not exchanged (net to zero)

- Other parameters:
  - Notional principal (amount interest is applied to)
  - Maturity
  - Payment frequency
  - Index rate (floating rate typically tied to LIBOR)
  - Currency
6. Plain Vanilla Interest Rate Swaps (continued)

Swap arithmetic

- Vanilla swap has semiannual payments of 6-month LIBOR
- Fixed rate typically set to give swap zero value at start
- Valuation: What’s the appropriate fixed rate?
  - Add principal payments ⇒ exchange of bonds
  - Value to Counterparty 1 is

\[
\text{Value of Swap} = \text{Price of FRN} - \text{Price of Fixed Rate Note}
\]

- Since FRN trades at par (100), we choose fixed rate so that the fixed rate note is par, too.

- Swap rate therefore satisfies

\[
100 = (d_1 + \cdots + d_n)(\text{Swap Rate}/2) + d_n 100
\]

\[
\text{Swap Rate} = 2 \times \frac{1 - d_n}{d_1 + \cdots + d_n} \times 100
\]

- Remarks:
  - Par yields again
  - Day count convention follows LIBOR
6. Plain Vanilla Interest Rate Swaps (continued)

Interest sensitivity

- DV01 is difference between DV01’s of the two notes

- Duration not used: percent not defined when price is zero

- Duration of components sometimes used instead
  (we’ll do an example shortly)
6. Plain Vanilla Interest Rate Swaps (continued)

Example

- Infer spot rates from swap quotes, interest rate futures, markup over treasuries, etc.

- Estimates for June 22, 1995:

<table>
<thead>
<tr>
<th>Maturity (Yrs)</th>
<th>Discount Factor</th>
<th>Spot Rate (Annual %)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>0.9707</td>
<td>6.036</td>
</tr>
<tr>
<td>1.0</td>
<td>0.9443</td>
<td>5.809</td>
</tr>
<tr>
<td>1.5</td>
<td>0.9175</td>
<td>5.824</td>
</tr>
<tr>
<td>2.0</td>
<td>0.8913</td>
<td>5.839</td>
</tr>
<tr>
<td>2.5</td>
<td>0.8644</td>
<td>5.914</td>
</tr>
<tr>
<td>3.0</td>
<td>0.8378</td>
<td>5.989</td>
</tr>
</tbody>
</table>

- 3-year swap rate:

\[
\text{Swap Rate} = 2 \times \frac{1 - d_n}{d_1 + \cdots + d_6} \times 100
\]

\[
= 5.980
\]
7. Swap Engineering

- Example: you have a 125m position with duration 2
  How can we use a swap to reduce duration to 1.5?

- Intuit: Swap should pay fixed
  (short the side with the longer duration)

- Swap product: the 3-year swap studied earlier
  - Fixed rate note has duration 2.709 years
    * Coupon is 2.99, price is 100 (trades initially at par), yield is $y = 5.98\%$
    * Standard bond duration calculation
  - Floating rate note has duration 0.485 years
    * It’s like a 6-month zero

- Duration of position plus swap with notional $x$:
  \[
  D = 1.50 = 2 \left( \frac{125}{125} \right) + 0.485 \left( \frac{x}{125} \right) + 2.709 \left( \frac{-x}{125} \right)
  \]
  Answer: $x = 28.1m$.

- Remark: this accomplishes the duration target, but a complete risk analysis would include basis risk (non-parallel shifts again)
8. Non-Vanilla Swaps

There’s no end to the variety:

- Variation over time in the notional principal:
  - Amortizing and accreting swaps

- Variation over time in the fixed payment:
  - Step up and step down swaps

- Basis swaps: two floating rates
  - The TED spread (treasury for LIBOR)
  - LIBOR for the prime rate

- Other index rates:
  - Constant Maturity Treasury (CMT)
  - Swap rates

- Total return swaps
  - Pay LIBOR, receive return on basket of Aa corporates
  - Pay LIBOR, receive return on basket of Bradies
  - Pay LIBOR, receive return on S&P 500
8. Non-Vanilla Swaps (continued)

Example 1: Step-up swap

- **Description**
  - Maturity is 2 years
  - Notional principal is 100
  - Semianual payments
  - Swap rate is $4$ the first year, $C$ the second

- **Valuation**
  - Floating rate leg: value is 100
  - Fixed rate leg:
    
    $$
    \text{Value} = (d_1 + d_2) \times 2 + (d_3 + d_4) \times C/2 + d_4 \times 100
    $$

- **Market rate $C$**:

    $$
    100 = (d_1 + d_2) \times 2 + (d_3 + d_4) \times C/2 + d_4 \times 100
    \Rightarrow C/2 = \frac{100(1 - d_4) - (d_1 + d_2) \times 2}{d_3 + d_4} = 7.788
    $$

    (high to compensate for the initial low rate)
8. Non-Vanilla Swaps (continued)

Example 2: Amortizing swap

- **Description**
  - Maturity is 2 years
  - Notional principal is 100 the first year, 50 the second
  - Semiannual payments
  - Swap rate is C throughout

- **Valuation:** add principals of 50 after year one, another 50 after year two
  - Floating rate leg: sum of 1-year and 2-year FRNs, each with notional principal of 50
  - Fixed rate leg:
    \[
    \text{Value} = (d_1+d_2) \times C/2 + d_2 \times 50 + (d_3+d_4) \times C/4 + d_4 \times 50
    \]

- **Market rate C:**
  \[
  100 = (d_1 + d_2) \times C/2 + d_2 \times 50 + (d_3 + d_4) \times C/4 + d_4 \times 50
  \]

  \[\Rightarrow C/2 = \frac{100 - (d_2 + d_4)50}{d_1 + d_2 + (d_3 + d_4)/2} = 5.830\]

  (mixture of 1- and 2-year rates)
8. Non-Vanilla Swaps (continued)

Example 3: Forward-starting swap

- Description
  - Maturity is 2 years, starting in one year
  - Notional principal is 100
  - Semiannual payments
    (in 18, 24, 30, and 36 months)
  - Swap rate is $C$

- Valuation
  - Floating rate leg: in one year, this is an FRN
    \[
    \text{Value} = d_2 \times 100 = 94.435
    \]
  - Fixed rate leg:
    \[
    \text{Value} = (d_3 + d_4 + d_5 + d_6) \times C/2 + d_6 \times 100
    \]

- Market rate $C'$:
  \[
  d_2 \times 100 = (d_3 + d_4 + d_5 + d_6) \times C/2 + d_6 \times 100
  \]
  \[
  \Rightarrow C/2 = \frac{(d_2 - d_6) \times 100}{d_3 + d_4 + d_5 + d_6} = 6.072
  \]

- This is the underlying for a swaption
9. Cross-Currency Swaps

Description

- Exchange interest and principal in two currencies, as in:

  Counterparty 1 \[\text{DM Payments} \rightarrow \text{Counterparty 2}\]
  \[\text{Dollar Payments}\]

- Only net payments are made

- Principals *are* exchanged (they don’t net to zero)

- Other parameters:
  - Notional principal (amount interest applied to)
  - Maturity
  - Payment frequency
  - Types of interest payments (fixed or floating)
  - Index rates (for floating rates)
  - Currencies
9. Cross-Currency Swaps (continued)

Risk assessment

- Value (in dollars) to Counterparty 1:

\[
\text{Value of Swap} = \text{Price of Dollar Note} - \text{Price of DM Note} = p_1 - sp_2
\]

where \( s \) is the exchange rate (dollar price of one DM).

- Change in value is approximately:

\[
\Delta v \approx \Delta p_1 - s\Delta p_2 - sp_2 \frac{\Delta s}{s}
\]

\[
\approx -D_1 p \Delta y_1 + D_2 sp_2 \Delta y_2 - sp_2 \frac{\Delta s}{s}
\]

(the usual linear approximation that underlies duration, with an extra term due to changes in the exchange rate)

- Sources of change in value:
  - Change in dollar yields (monthly std dev about 0.6%)
  - Change in DM yields (monthly std dev about 0.5%)
  - Change in exchange rate (monthly std dev about 3%)

- Except for very long durations, currency risk larger

- Statistical risk systems incorporate correlations among these three components
10. OTC v Exchange-Trade Derivatives

Relative to exchange-traded instruments, OTC products

- Can be custom made

- Generally have less liquidity

- Generally have greater credit risk

- Sometimes get different accounting treatment
11. Credit Risk

Methods used to control credit risk:

- Netting built into ISDA master agreement and US law

- Diversification across counterparties (standard practice)

- Collateral

- Mark-to-market (analogous to futures)

- Credit guarantees

- Termination triggers for downgrades

- Aaa-rated derivatives subsidiaries
Summary

- Floaters and swaps: no big deal!

- Floating rate notes make interest payments tied to market rates, typically LIBOR

- Inverse floaters often have long durations

- In a plain vanilla interest rate swap, two parties exchange the difference between fixed and floating interest payments

- In a cross-currency swap, two parties exchange the difference between interest and principal payments in two different currencies

- The OTC swap market has unlimited variety

- Swap contracts, like futures contracts, are designed to minimize the impact of credit risk