Interest-Rate Forwards and Futures

0. Overview

• Leading Futures Contracts

• Forward Contracts

• Futures Contracts

• Bond Futures

• Eurocurrency Futures
## 1. Leading Futures Contracts

- Contracts ranked by dollar volume:

<table>
<thead>
<tr>
<th>Contract</th>
<th>Open Interest Contracts (mm)</th>
<th>Monthly Volume Contracts (mm)</th>
<th>Dollars (bb)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Eurodollar</td>
<td>1.325</td>
<td>5.044</td>
<td>5,044</td>
</tr>
<tr>
<td>Euroyen</td>
<td>0.439</td>
<td>1.247</td>
<td>1,122</td>
</tr>
<tr>
<td>10-yr JGB</td>
<td>0.132</td>
<td>0.989</td>
<td>890</td>
</tr>
<tr>
<td>3-m sterling</td>
<td>0.212</td>
<td>0.941</td>
<td>724</td>
</tr>
<tr>
<td>Euromark</td>
<td>0.370</td>
<td>1.014</td>
<td>628</td>
</tr>
<tr>
<td>30-yr US T-bond</td>
<td>0.305</td>
<td>5.834</td>
<td>583</td>
</tr>
<tr>
<td>PIBOR</td>
<td>0.146</td>
<td>0.536</td>
<td>492</td>
</tr>
<tr>
<td>10-yr Notionnel</td>
<td>0.231</td>
<td>2.584</td>
<td>237</td>
</tr>
<tr>
<td>S&amp;P 500</td>
<td>0.157</td>
<td>1.035</td>
<td>232</td>
</tr>
<tr>
<td>German Bund</td>
<td>0.139</td>
<td>1.134</td>
<td>176</td>
</tr>
<tr>
<td>Nikkei 225</td>
<td>0.149</td>
<td>0.950</td>
<td>172</td>
</tr>
<tr>
<td>10-yr US T-note</td>
<td>0.177</td>
<td>0.935</td>
<td>93</td>
</tr>
</tbody>
</table>


- Remarks
  - Fixed income contracts dominate
  - Eurocurrencies first, then government bonds
  - Bond contracts have greater volume to open interest than euros: short-term trading v buy-and-hold
2. Forward Contracts

- A forward contract is an agreement to exchange assets at a future date for a price arranged now.

- Terminology: trade date is when trade is made, settlement date is when assets are exchanged:

<table>
<thead>
<tr>
<th>trade</th>
<th>settlement</th>
</tr>
</thead>
<tbody>
<tr>
<td>t</td>
<td>t + n</td>
</tr>
</tbody>
</table>

- With interest rate contracts, a third date is the maturity of the asset being exchanged (typically for cash):

<table>
<thead>
<tr>
<th>trade</th>
<th>settlement</th>
<th>maturity</th>
</tr>
</thead>
<tbody>
<tr>
<td>t</td>
<td>t + n</td>
<td>t + n + m</td>
</tr>
</tbody>
</table>

Convention (usually): m is maturity at settlement.

- Interest rate forwards (futures, too) differ in the magnitudes of n and m:
  - Forward rate agreements: n and m are typically single-digit months.
  - Bond futures: short settlement and long maturity (eg, n = 3 months and m of 10 years).
  - Eurocurrency futures: long settlement and short maturity (n out to 10 years and beyond, m = 3 months).
2. Forward Contracts (continued)

- Forward contract on a zero:
  - Terms: agree at $t$ to buy $m$-period zero at $t + n$ for price $F$.
  - This should look familiar!
  - No cash flows on trade date (like a swap)

- Timing of cash flows:

<table>
<thead>
<tr>
<th>$t$</th>
<th>$t + n$</th>
<th>$t + n + m$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>$-F$</td>
<td>100</td>
</tr>
</tbody>
</table>

- Replication with zeros ($p_n =$ price of $n$-period zero):

<table>
<thead>
<tr>
<th>$t$</th>
<th>$t + n$</th>
<th>$t + n + m$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$-p_{n+m}$</td>
<td></td>
<td>100</td>
</tr>
<tr>
<td>$xp_n$</td>
<td>$-100x$</td>
<td></td>
</tr>
</tbody>
</table>

Choose $x$ to replicate cash flows of forward contract:

$$0 = -p_{n+m} + xp_n$$

- Solution:

$$x = \frac{p_{n+m}}{p_n} \implies F = 100 \times \frac{p_{n+m}}{p_n}$$
2. Forward Contracts (continued)

- Forward contract on a coupon bond
  - Terms: agree at $t$ to buy an $m$-period bond at $t+n$ for price $F$

  - Cash flows:

    | $t$   | $t+n$ |
    |-------|-------|
    | 0     | $-F_t$ |
    | 0     | $P_{t+n}$ |

  I.e., price of bond at $t+n$ is $P_{t+n}$ (unknown at $t$), but contract locks in price of $F_t$ (set at $t$).

  - Coupons complicate the analysis, but the idea is the same: replicate forward with long position in the bond and short positions sufficient to offset the purchase price and (in this case) the coupons between now and settlement.

  - Key ingredient: replication includes a long position in the bond, so you’re indirectly gaining exposure to the bond.
2. Forward Contracts (continued)

- Using forwards to modify interest sensitivity

  - Duration not defined: duration is the proportional change in price and is not defined for contracts (like swaps, forwards, and futures) that have no net value.

  - Quantify interest sensitivity as we did with swaps:
    * Use the DV01
    * Compute duration for long and short positions separately
    * Short cut: ignore the short position
      (this approach is common for bond futures)

  - Bottom line:
    * Forward contracts on long-maturity bonds are useful tools for modifying duration: you add (say) a long position in a long bond, and short a position of equal value in a short bond.
    * Forward contracts on short bonds — FRAs, for example — are useful protection against near-term changes in short rates.
3. Forward Rate Agreements (FRAs)

- Contract terms
  - In an “$n \times m$” (both quoted in months), $m$ is what we’ve called $m + n$

  - Fixed “contract” rate ($C$)

  - Floating “reference” rate ($r$) (fixed at settlement) (typically $(m - n)$-month LIBOR)

  - Notional principal

  - Cash flow at settlement:
    \[
    \text{Payment} = \frac{(C - r)(m - n)/12}{1 + r(m - n)/12} \times \text{Principal}
    \]
    (plus the usual eurocurrency day count adjustments)

  - Equivalent to paying this in $m$ months:
    \[
    \text{Payment} = [(C - r)(m - n)/12] \times \text{Principal}
    \]

  - Remark: swap with one payment!
3. FRAs (continued)

- Example: 6 × 12, 1mm notional
  - Contract rate = 6%
  - Reference rate = 6-month LIBOR
  - If 6-month LIBOR is 5% at settlement,
    \[
    \text{Payment} = \frac{(.06 - .05)/2}{1 + .05/2} \times 1,000,000 = 4,878
    \]

- Contract rate is the forward rate
  - Approach like swaps
    * Consider payments in 12 months
    * Add principal to both sides
  - Value now of fixed payment in 12 months:
    \[
    d_2 \times (1 + C/2) \times 100
    \]
  - Value now of floating payment in 12 months:
    \[
    d_1 \times \frac{(1 + r/2)100}{1 + r/2} = d_1 \times 100
    \]
  - Equate values to find \( C \):
    \[
    d_1 \times 100 = d_2 \times (1 + C/2) \times 100 \Rightarrow 1 + C/2 = d_1/d_2
    \]
    \( (C \) is the first forward rate)
4. Futures Overview

- Features of futures
  - Standardized contracts
  - Liquid, low transactions costs
  - Easy to short
  - Low credit risk
  - Trades public: good source of market information
  - Differ from forwards in daily “mark-to-market”

- Cash flows on futures
  - No payment due on trade date
  - . . . but money is set aside in margin account
  - Margin account varies due to
    * Daily changes in contract price
    * Margin calls
    * Interest on the account

- Daily “mark to market”
  - Reduces credit exposure of exchange
  - Complicates cash flows and valuation (slightly)
5. Bond Futures

- US treasury bond/note contracts (CBOT)  
  (foreign government bond contracts are similar)

- Standard features
  - Contract size: $100,000 face value
  - Contracts expire quarterly (Mar/Jun/Sep/Dec)
  - Delivery controlled by short position, which
    * Can deliver any time in the contract month
    * Delivers $100,000 face value of bonds, gets cash.

- Eligible (“contract grade”) bonds:
  * 30-year bond contract: US treasury bonds with maturity at least 15 years from first delivery date
  * 10-year note contract: US treasury notes with maturity 6.5 to 10 years from first delivery date
  * 5-year note contract: US treasury notes with original maturity no more than 5.25 years and maturity on first day of delivery month of at least 4.25 years

- Wild-card option: price at close (2pm) good till 8pm
- Timing option: futures price fixed on last trading day
5. Bond Futures (continued)

Settlement for delivered bonds

- Invoice price (what long position pays):
  \[ \text{Invoice Price} = F \times \text{Conv Factor} + \text{Acc Interest} \]
  \[(F \text{ is quoted futures price)}\]

- Conversion factor:
  - Why? Make more bonds deliverable, avoid squeezes
  - Problem: people would always deliver the low-coupon bonds, since they’re cheaper
  - Solution: scale up price by conversion factor, the ratio of the quoted price of a bond to the price of an 8% bond, using yields of 8%
  - Computed this way:
    * Compute maturity at first delivery date, rounded down to nearest 3-month interval \( \Rightarrow \)
      \[ n \text{ (number of coupons remaining)} \text{ and } w \text{ (length of initial period, 0.5 or 1)} \]
    * Compute
      \[ \text{Conversion Factor} = \frac{\text{Quoted Price at } y=8\%}{\text{Quoted Price of 8\% Bond}} \]
      (Denominator is 100)
5. Bond Futures (continued)

Calculating conversion factors

- Formula:

\[
\text{Quoted Price} = d^w \left(\frac{1 - d^n}{1 - d}\right) \text{Coupon} + d^{w+n-1}100 - (1 - w)\text{Coupon}
\]

(Just like our earlier work on bond yields, except that we have rounded maturity down to the nearest 3 months.)

- Examples for the Dec 98 long bond contract:
  - 11 1/4 of 2/15/15: \( n = 32 \) (Feb 99 less than 3 months), \( w = 1 \), Quoted Price = 129.04, CF = 1.2904.
  - 10 5/8 of 8/15/15: \( n = 33 \), \( w = 1 \), CF = 1.2382.
  - 9 7/8 of 11/15/15: \( n = 34 \), \( w = 0.5 \), CF = 1.1711.
  - 9 1/8 of 5/15/18: \( n = 39 \), \( w = 0.5 \), CF = 1.1093.
  - 5 1/2 of 8/15/28: \( n = 59 \), \( w = 1 \), CF = 1.1093.

- Remarks
  - Crude adjustment
  - Effects vary with yields: if we used yields higher (lower) than 8%, we would tend to choose bonds with higher (lower) duration
5. Bond Futures (continued)

The Basis

- Basis is difference between cash price of bond and price through the futures contract:

$$\text{Basis} = \text{Bond Price} - F \times \text{Conversion Factor}$$

- Question: do we use invoice or quoted price? Answer: doesn’t matter (accrued interest in both)

- Analysis for 11/2/98 quotes for the Dec 98 long bond contract ($F = 128.656$, 33 contract grade bonds in all):

<table>
<thead>
<tr>
<th>Bond</th>
<th>Quoted Price</th>
<th>Conv Fac</th>
<th>Basis</th>
</tr>
</thead>
<tbody>
<tr>
<td>11 1/4 of 2/15/15</td>
<td>166.297</td>
<td>1.290</td>
<td>0.280</td>
</tr>
<tr>
<td>10 5/8 of 8/15/15</td>
<td>159.563</td>
<td>1.238</td>
<td>0.260</td>
</tr>
<tr>
<td>9 7/8 of 11/15/15</td>
<td>151.000</td>
<td>1.171</td>
<td>0.331</td>
</tr>
<tr>
<td>9 1/8 of 5/15/18</td>
<td>144.688</td>
<td>1.109</td>
<td>1.969</td>
</tr>
<tr>
<td>5 1/2 of 8/15/28</td>
<td>104.891</td>
<td>0.718</td>
<td>12.46</td>
</tr>
</tbody>
</table>

Remark: try to reproduce these numbers, to make sure you understand where they come from.

- The basis is zero at delivery: the short position delivers the cheapest bond, which should sell for the same price in the cash and futures markets.
5. Bond Futures (continued)

Interest sensitivity

- Rule-of-thumb:

\[
\text{DV01 of Futures} = \frac{\text{DV01 of Bond}}{\text{Conversion Factor}}
\]

- Assumption: basis doesn’t change

  - Ignores short position in forward replication (small, but not zero)
  
  - Also ignores daily mark to market, which induces a sensitivity of the basis to changes in yields and volatility
  
  - Basis risk!

- Convexity: since the cheapest to deliver varies with yields, bond futures tend to have negative convexity

  (In case you ask: as rates fall, cheapest bond shifts to lower duration.)
Debt Instruments

6. Eurocurrency Futures

- Contracts on 3-month LIBOR in major currencies (CME, LIFFE, SIMEX)
- Standard features
  - Contract size: interest on $1,000,000
  - Contracts expire quarterly (Mar/Jun/Sep/Dec) out ten years or more (third Wednesday)
  - Quoted index:
    \[ \text{Index} = 100 \times (1 - \text{Yield}) \]
  - Effective price of a contract:
    \[ \text{Price} = 1mm \times (1 - \text{Yield}/4) \]
  - Cash settlement at
    \[ \text{Settlement Price} = 1mm \times (1 - 3\text{-m LIBOR}/4) \]
    Note: no strange delivery options!
- Strips, bundles, and packs: combinations of contracts with different maturities
- Uses
  - Helpful for hedging FRNs, FRAs, swaps, etc
  - Source of market information on forward rates
  - In earlier terminology: \( n \) can be large, but \( m \) is small
Summary

- Forward contracts for bonds are equivalent to a combination of a long position in a long bond and a short position of equal value in a short bond.

- Futures contracts differ from forwards in the daily mark to market.

- Fixed income futures include government bond and eurocurrency contracts.

- Bond futures are truly ugly contracts, but useful tools for managing interest rate risk: liquid, easy to short, low transaction costs.

- Like other derivatives, forwards and futures offer leverage: you can arrange great exposure with less money down than buying the underlying instrument.