Interest-Rate Options

0. Overview

- Fixed Income Options

- Option Fundamentals

- Caps and Floors

- Options on Bonds

- Options on Futures

- Swaptions
1. Fixed Income Options

- Options imbedded in bonds:
  - Callable bonds
  - Putable bonds
  - Convertible bonds

- Options on futures
  - Bond futures
  - Eurocurrency futures

- OTC options
  - Caps, floors, and collars
  - Swaptions
2. Option Basics

- Big picture
  - Options are everywhere
    * Stock options for CEOs and others
    * Corporate equity: call option on a firm
    * Mortgages: the option to refinance
  - Options are like insurance
    * Premiums cover the down side, keep the up side
    * Customers like this combination
    * Insurer bears risk or shares it
      (diversification or reinsurance)
  - Managing cost of insurance
    * Out-of-the-money options are cheaper
      (insurance with a big deductible)
    * Collars: sell some of the up side
    * Aggregate: basket option cheaper than basket of options
  - Managing option books
    * Customer demands may result in exposed position
    * Particular exposure to volatility:
      puts and calls both rise with volatility
    * Hedging through replication is another route
2. Option Basics (continued)

- Option terminology
  - Basic terms
    * Options are the right to buy (a call) or sell (a put) at a fixed price (strike price)
    * The thing being bought or sold is the underlying
    * This right typically has a fixed expiration date
    * A short position is said to have written an option
  
- Kinds of options
  * European options can be exercised only at expiration
  * American options can be exercised any time
  * Bermuda options can be exercised at specific dates (eg, bonds callable only on coupon dates)
2. Option Basics (continued)

- Features of options
  - Leverage (cheap source of exposure)
  - Nonlinear payoffs
    * Payoffs vary with underlying
      (in- and out-of-the money)
    * Translates into variable duration
      (convexity rears its ugly head)
    * Creates risk management hazards
  - Volatility has positive effect on both puts and calls
    * Another risk management hazard!
  - They’re state-contingent claims
    (no way around it, but nothing new either)
3. Approaches to Valuation

- Why use a pricing model? No choice — the instruments demand that we value uncertain cash flows (state-contingent claims).

- What pricing model? Good question.

- Interest rate trees
  - Been there... (and it hasn’t changed)
  - We’ll return to them shortly

- The Black-Scholes benchmark (Black’s formula)
  - Underlying: an arbitrary bond with (say) maturity $m$
  - Parameters: $n$-period European call with strike price $k$
  - Formula:
    \[
    \text{Call Price} = pN(x) - d_n k N(x - n^{1/2} \sigma)
    \]
    with
    \[
    p = \text{current price of underlying} \\
    f = \text{forward price of underlying} \\
    d_n = \text{$n$-period discount factor} \\
    N = \text{normal cdf} \\
    x = \frac{\log(f/k) + n\sigma^2/2}{n^{1/2}\sigma}
    \]
3. Approaches to Valuation (continued)

- Remarks on Black-Scholes for fixed income
  - Formula based on log-normal price of underlying
    $\Rightarrow$ normal (continuously compounded) spot rates
    $\Rightarrow$ possibility of negative spot rates

- Volatility $\sigma$ varies systematically with maturities of option and underlying ("term structure of volatility")

Sample swaption volatility matrix (%):

<table>
<thead>
<tr>
<th>Option Maturity</th>
<th>1 yr</th>
<th>2 yr</th>
<th>5 yr</th>
<th>10 yr</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 m</td>
<td>15.50</td>
<td>16.00</td>
<td>16.75</td>
<td>15.25</td>
</tr>
<tr>
<td>3 m</td>
<td>17.50</td>
<td>18.50</td>
<td>18.25</td>
<td>18.25</td>
</tr>
<tr>
<td>1 yr</td>
<td>21.50</td>
<td>21.25</td>
<td>19.25</td>
<td>16.50</td>
</tr>
<tr>
<td>5 yr</td>
<td>20.00</td>
<td>19.00</td>
<td>17.50</td>
<td>15.50</td>
</tr>
</tbody>
</table>

Remark: "hump" is typical

- Despite problems, a common benchmark (dealers often quote volatility instead of price)
3. Approaches to Valuation (continued)

- Properties of Black-Scholes option prices (most of these generalize to other settings)
  - The Delta:
    \[
    \Delta = \frac{\Delta \text{Call Price}}{\Delta p} = N(x),
    \]
    which varies between zero and one (nonlinear).

- Volatility is the only unobservable (we use call prices to “imply” it)

- If volatility rises, so does the call price (puts, too)
4. Caps, Floors, and Collars

- **Terminology:**
  - A *cap* pays the difference between a reference rate and the cap rate, if positive. (Series of call options on an interest rate)
  - A *floor* pays the difference between the floor rate and a reference rate, if positive. (Series of put options on an interest rate)
  - A *collar* is a long position in a cap plus a short position in a floor.

- **Contract terms:**
  - Cap and/or floor rate
  - Reference rate (typically LIBOR)
  - Frequency of payment
  - Notional principal (amount on which interest is paid)

- **Approaches:**
  - Apply Black’s formula
  - Interest rate tree (we did this earlier)
  - An uncountable number of other models
4. Caps, Floors, and Collars (continued)

- Example 1: two-year semiannual 7% cap on 6-m LIBOR

Payments shifted back one period:

\[
\frac{(r - 7\%)^+ / 2}{1 + r/2} \times \text{Notional Principal}
\]

(three such semi-annual payments, excluding the first)

Short rate tree (same as before):

\[
\begin{array}{ccc}
6.036 & 6.587 & 7.869 \\
6.587 & 4.587 & 5.869 \\
5.869 & 3.869 & 2.913 \\
\end{array}
\]

Price path for cap (for 100 notional):

\[
\begin{array}{ccc}
0.202 & 0.416 & 0.858 \\
0.416 & 0.000 & 0.000 \\
0.000 & 0.000 & 0.000 \\
\end{array}
\]
4. Caps, Floors, and Collars (continued)

- Example 1 (continued): the effects of volatility

  Short rate tree ($\sigma = 2\%$, same implied spot rates):

  \[
  \begin{array}{c}
  6.036 \\
  3.602 \\
  \end{array}
  \begin{array}{c}
  7.602 \\
  5.912 \\
  \end{array}
  \begin{array}{c}
  9.912 \\
  8.001 \\
  \end{array}
  \begin{array}{c}
  12.001 \\
  4.001 \\
  0.001 \\
  \end{array}
  \]

  Price path for cap (for 100 notional):

  \[
  \begin{array}{c}
  0.892 \\
  0.115 \\
  \end{array}
  \begin{array}{c}
  1.712 \\
  0.234 \\
  \end{array}
  \begin{array}{c}
  2.740 \\
  0.481 \\
  \end{array}
  \begin{array}{c}
  2.359 \\
  0.000 \\
  0.000 \\
  \end{array}
  \]

  Remarks:

  - Volatility increase cap prices (they’re bets on extreme events, and higher $\sigma$ makes them more likely)
  - Similar in this respect to Black-Scholes
4. Caps, Floors, and Collars (continued)

- Example 2: 2-Year FRN with collar (7% cap, 4% floor)
  Effective interest rates (boxes indicate cap/floor binds):

\[
\begin{align*}
\text{6.036} & \quad 6.587 & \quad 7.000 \\
\quad & \quad 4.587 & \quad 5.869 \\
& \quad & \quad 4.000 \\
\end{align*}
\]

Price path for note:

\[
\begin{align*}
99.88 & \quad 99.58 & \quad 99.14 & \quad 99.08 \\
& \quad 100.16 & \quad 100.00 & \quad 100.00 \\
& \quad & \quad 100.33 & \quad 100.54 \\
\end{align*}
\]

Remarks:
- Differences from 100 indicate impact of collar
- Giving up low rates partially offsets cost of cap
  \((0.12 = 100 - 99.88 < 0.20)\)
- Issuers would generally adjust cap and floor to get a price of 100
5. Options on Bonds

- Earliest and most common interest-rate option?

- Examples of callable corporate bonds


  - Intel Overseas Corporation’s 8-1/8s, issued April 1, 1987 (really), due March 15, 1997, callable at par. Par in this situation means par plus accrued interest: the firm pays the relevant interest as well as the face value. The bonds were called March 15, 1994, at 100.

5. Options on Bonds (continued)

- Example: call option on 2-year 5% bond
  (the usual rate tree)

Price path of bond:

\[
\begin{align*}
100.94 & \leftarrow 100.00 & 99.77 & \leftarrow 100.63 & \leftarrow 102.50 \\
102.81 & \leftarrow 101.65 & 101.58 & \leftarrow 102.50 & \leftarrow 102.50 \\
& \leftarrow 103.58 & 102.54 & \leftarrow 102.50 & \leftarrow 102.50 \\
& \leftarrow 103.53 & 102.50 & \leftarrow 102.50 & \leftarrow 102.50
\end{align*}
\]

- 18-month European option (callable at 102.5 — "par")

Price path is:

\[
\begin{align*}
0.134 & \leftarrow 0.010 & 0.000 & \leftarrow 0.000 & \leftarrow 0.000 \\
0.227 & \leftarrow 0.021 & 0.000 & \leftarrow 0.000 & \leftarrow 0.000 \\
& \leftarrow 0.525 & 0.042 & \leftarrow 0.000 & \leftarrow 0.000 \\
& \leftarrow 1.028 & & \leftarrow 0.000 & \leftarrow 0.000
\end{align*}
\]

Nodes in boxes indicate cash flows from exercise, other nodes indicate value in earlier periods.
5. Options on Bonds (continued)

- American option has greater value
  (can exercise either at expiration, or earlier if better)

- Approach:
  - Start at expiration, work backwards
  - At each node, choose better of “exercise” or “hold”

- Cash flows from immediate exercise:

  \[
  \begin{array}{cccccc}
  0.000 & 0.000 & 0.000 & 0.000 & 0.000 \\
  0.314 & 0.000 & 0.000 & 0.000 & 0.000 \\
  1.080 & 0.042 & 0.028 & 0.000 & 0.000 \\
  \end{array}
  \]

- Node with box:
  - If hold:
    \[
    \text{Value} = \frac{0.5}{1 + 0.03869/2} \times (0.042 + 1.028) = 0.525
    \]
  - If exercise:
    \[
    \text{Value} = 1.080
    \]
    This is better: we take it.
5. Options on Bonds (continued)

- American option (continued)

- Node (0,1) (one down move from start):
  - If hold:
    \[
    \text{Value} = \frac{0.5}{1 + 0.04587/2} \times (0.000 + 1.080) = 0.538
    \]
  - If exercise:
    \[
    \text{Value} = 0.314
    \]
    Hold is better in this case, so we write 0.538 here.

- Complete price path:

Boxes indicate nodes where option is exercised.

- Summary
  - Worth more than European call
  - Valued recursively (as usual)
6. Callable Bonds

- Example: Callable bond based on previous example:
  2-year 5% bond with 18-month American call

  Price path:

  \[
  \begin{array}{cccccc}
  100.67 & 99.99 & 99.77 & 100.63 & 102.50 \\
  102.28 & 101.63 & 101.58 & 102.50 & 102.50 \\
  102.50 & 102.50 & 102.50 & 102.50 & 102.50 \\
  \end{array}
  \]

- Interest-sensitivity 1: replication with \((x_a, x_b)\) units, resp., of underlying bond and one-period zero

  \[
  \begin{array}{cccc}
  .81,.19 & .99,.10 & 1.00,.00 \\
  .56,.45 & .96,.04 & 0,1.025 \\
  \end{array}
  \]

  Eg, the callable bond is equivalent, in the initial node, to 0.81 units of the underlying noncallable bond and 0.19 units of a one-period zero.

Remarks:

- The callable has shorter duration than the noncallable
- How much shorter varies throughout the tree
6. Callable Bonds (continued)

- Interest-sensitivity 2: price-yield relation
  - How does price vary if we shift the whole short rate tree up and down?
  - Below we graph price against initial short rate
  - Slope used to compute “effective duration”

![Graph of Callable Bonds](image-url)
6. Callable Bonds (continued)

- Dumb ideas
  - Yield to first call date for bonds in the money
  - Yield to worst: find call date with highest yield
  - Remarks:
    * These approaches ignore the intrinsic difficulties of valuing uncertain cash flows
    * They’re dumb for exactly that reason
    * Our approach: call decision varies through the tree

- Option-Adjusted Spread (OAS)
  - Consider the valuation of a callable bond
  - Suppose market price is $p$
  - Compute spread $s$ added to the short rate tree required to reproduce the market price
  - Positive spread means the market values the bond more highly than the model
7. Options on Futures

- Options available on major futures contracts
  - Government bond contracts
  - Eurocurrency contracts
  - Brady bond futures

- Same strengths as the underlying futures
  - Highly liquid markets
  - Low transaction costs
8. Swaptions

- Swaptions: options on swaps
  - Option to enter a swap
  - Option to extend a swap
  - Option to terminate a swap
  - European, American, and Bermuda

- Properties
  - Similar to bond options (swap = bond - FRN, or reverse)
  - Exposure to long-dated volatility
  - Currently the OTC option standard
Summary

- Options are ubiquitous.

- Their nonlinear payoffs pose challenges to valuation and risk management.

- Nonlinearity translates in this context into nonlinear price-yield relations — convexity, in other words.

- Black-Scholes is less well suited for fixed income than other securities, but remains a common benchmark nonetheless.

- American options are valued recursively: at each node, we decide whether to exercise or hold.

- Shortcuts don’t work: yield-to-call is meaningless.