Debt Instruments

Set 1

Backus/February 3, 1998

Propectus

1. Fixed income is a fascinating part of finance . . .

2. . . . but it’s quantitative

3. . . . and takes time and effort to master

4. Assignments are critical learning experiences

5. Do them in groups (it’s easier)

6. We’ll emphasize international markets

7. Home page has “text” and “overheads” (like these)

8. Useful references:
   - Garbade, *Fixed Income Analytics*
   - Fabozzi, *Bond Markets, Analysis, and Strategy*

9. Read the syllabus!
Theme 1: Debt Markets are Global

- Bond Markets
  (amounts outstanding, billions of US dollars, 1995)

<table>
<thead>
<tr>
<th>Category</th>
<th>Outstanding</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total</td>
<td>24,110.0</td>
</tr>
<tr>
<td>Private</td>
<td>8,776.7</td>
</tr>
<tr>
<td>Public</td>
<td>14,197.5</td>
</tr>
<tr>
<td>US</td>
<td>10,726.0</td>
</tr>
<tr>
<td>Japan</td>
<td>4,958.6</td>
</tr>
<tr>
<td>Germany</td>
<td>1,906.4</td>
</tr>
</tbody>
</table>

Source: IMF.

- International capital flows of all kinds are booming
**Theme 2: Debt Markets are Derivatives Markets (and vice versa)**

Exchange-Traded Derivatives  
(annual turnover, millions of contracts traded, 1995)

<table>
<thead>
<tr>
<th>Category</th>
<th>Turnover (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Interest rate futures</td>
<td>561.0</td>
</tr>
<tr>
<td>Interest rate options</td>
<td>225.5</td>
</tr>
<tr>
<td>Currency futures</td>
<td>98.3</td>
</tr>
<tr>
<td>Currency options</td>
<td>23.2</td>
</tr>
<tr>
<td>Stock market index futures</td>
<td>114.8</td>
</tr>
<tr>
<td>Stock market index options</td>
<td>187.3</td>
</tr>
<tr>
<td>North America</td>
<td>455.0</td>
</tr>
<tr>
<td>Europe</td>
<td>353.3</td>
</tr>
<tr>
<td>Asia-Pacific</td>
<td>126.5</td>
</tr>
<tr>
<td>Other</td>
<td>275.4</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>1,210.1</strong></td>
</tr>
</tbody>
</table>

Source: BIS.
Theme 2: Derivatives (continued)

All Derivatives
(notional outstandings, billions of US dollars, 1995)

<table>
<thead>
<tr>
<th>Category</th>
<th>Over-the-Counter</th>
<th>Exchanges</th>
</tr>
</thead>
<tbody>
<tr>
<td>Interest rate</td>
<td>26,645</td>
<td>15,669</td>
</tr>
<tr>
<td>Currency</td>
<td>13,095</td>
<td>120</td>
</tr>
<tr>
<td>Equity and stock indexes</td>
<td>579</td>
<td>442</td>
</tr>
<tr>
<td>Commodities</td>
<td>318</td>
<td>142</td>
</tr>
</tbody>
</table>

Tell Figlewski: futures and options = fixed income!

Remark: OTC derivatives tied to global interbank market
Theme 3: Debt Markets are Emerging

- Net Capital Flows to Emerging Markets
  (billions of US dollars, 1995)

  Total
  Direct Investment
  Portfolio (Debt and Equity)
  Loans

- Gross Private Issues of Debt and Equity
  (billions of US dollars, 1995)

  Debt
  Total amount
  Share of emerging markets (%)  
  Equity
  Total amount
  Share of emerging markets (%)  

- Summary of emerging markets:
  - Significant and growing share
  - Increasing use of public markets
Fixed Income Analytics at Work

Example 1: Bell Atlantic

- Stylized balance sheet (typical of nonfinancial corps) (year-end 1996, billions of dollars)

<table>
<thead>
<tr>
<th>Assets</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>PP&amp;E</td>
<td>16</td>
</tr>
</tbody>
</table>

Liabilities and Shareholders’ Equity

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Debt</td>
<td>6</td>
</tr>
<tr>
<td>Shareholders’ Equity</td>
<td>8</td>
</tr>
<tr>
<td>Stuff</td>
<td>2</td>
</tr>
</tbody>
</table>

- Debt notes:
  - Primarily fixed rate, with maturities through 2033
  - Accounting: coupons charged against income
  - 1.5b callable, and some putable
  - Derivatives: 0.2b interest rate swaps (receive fixed)

- Question: Is long debt less risky than short?

  Answer 1: Yes, interest expense is predictable
  Answer 2: No, market value varies more
Fixed Income Analytics at Work (continued)

Example 2: Intel

- Balance sheet summary
  (year-end 1996, billions of dollars)

  Assets
  - Cash and Securities 7.9
  - PP&E 8.5
  - Stuff 1.2

  Liabilities and Shareholders’ Equity
  - Debt 0.7
  - Shareholders’ Equity 16.9

- Securities and debt notes:
  - Everything swapped into dollar-LIBOR (floating rate)
  - Accounting at market value: interest and changes in market value included in revenue and expense

- Question: Are floating rate (short) securities less risky?
  Answer 1: Yes, market value is stable
  Answer 2: No, interest income/expense unpredictable
Fixed Income Analytics at Work (continued)

Example 3: Banc One

- Stylized balance sheet
  (year-end 1996, billions of dollars)

<table>
<thead>
<tr>
<th>Assets</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Loans</td>
<td>73</td>
</tr>
<tr>
<td>Cash and Securities</td>
<td>29</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Liabilities and Shareholders’ Equity</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Deposits</td>
<td>72</td>
</tr>
<tr>
<td>Debt</td>
<td>21</td>
</tr>
<tr>
<td>Shareholders’ Equity</td>
<td>8</td>
</tr>
</tbody>
</table>

- Summary and comments:
  - Highly levered (like all commercial banks)
  - Assets shorter than liabilities
    \[\Rightarrow\] vulnerable to fall in rates
  - Swaps used to moderate interest sensitivity
  - Accounting: mixture of market value ("available for sale") and historical cost ("held to maturity")
Fixed Income Analytics at Work (continued)

Example 4: Emerging Markets

- General characteristics of emerging markets
  - Debt easier to issue than equity
  - Often comes with more stringent disclosure requirements than domestic issues
  - Typically denominated in major currency (dollars, say)
  - Borrowers are sovereigns and firms with strong credit, hard-currency revenues
  - Foreign-currency denomination adds currency risk to the usual credit risk (economies and currencies often implode together)

- Examples:
  - Par Bonds, Mexico (Bradies)
  - Globals, Mexico (eurobonds)
  - Grupo Carso SA, Mexico, floating rate eurobonds
  - Brazilian “C” Bonds (Bradies)
  - Argentinian FRB’s
  - ICICI, India, eurobonds (144A)
  - Ministry of Finance, Russia (144A)
Outline

Part I: Bonds and Close Relatives

1. Fixed Income Securities
   assets whose value depends on interest rates

2. Bond Arithmetic
   calculating spot rates, yields, etc

3. Macrofoundations of Interest Rates
   monetary policy and other factors

4. Quantifying Interest Rate Risk
   duration and beyond, activist investment strategies

5. Interest Rate Swaps
   also floaters and inverse floaters

6. Risk Management, Accounting, and Control
   market and book value, disasters and their sources
Outline (continued)

Part II: Interest Rate Derivatives

7. State-Contingent Claims
   analytical framework for derivative valuation

8. Forwards and Futures
   bond and interest rate futures

9. Options
   analytics of options, callable bonds, caps and floors

10. Corporate Bonds
    introduction to credit risk

11. Emerging Market Debt
    Brady bonds, eurobonds, trends

12. Mortgages
    Mortgages, mortgage-backed securities, structured notes
Bond Arithmetic

0. Overview

- Zeros and coupon bonds
- Spot rates and yields
- Day count conventions
- Replication and arbitrage
- Forward rates
- Yields and returns
1. Zeros or STRIPS

- A zero is a claim to $100 in $n$ periods (price = $p_n$)

\[
\begin{array}{cc}
\text{Pay } p_n & \text{Get } $100 \\
\hline
\end{array}
\]

\[t\quad t+n\]

- A spot rate is a yield on a zero:

\[p_n = \frac{100}{(1 + y_n/2)^n}\]

- US treasury conventions:
  - price quoted for principal of 100
  - time measured in half-years
  - semi-annual compounding
1. Zeros (continued)

- A *discount factor* is a price of a claim to one dollar:

\[
d_n = \frac{p_n}{100} = \frac{1}{(1 + y_n/2)^n}
\]

- Examples (US treasury STRIPS, May 1995)

<table>
<thead>
<tr>
<th>Maturity (Yrs)</th>
<th>Price ($)</th>
<th>Discount Factor</th>
<th>Spot Rate (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>97.09</td>
<td>0.9709</td>
<td>5.99</td>
</tr>
<tr>
<td>1.0</td>
<td>94.22</td>
<td>0.9422</td>
<td>6.05</td>
</tr>
<tr>
<td>1.5</td>
<td>91.39</td>
<td>0.9139</td>
<td></td>
</tr>
<tr>
<td>2.0</td>
<td>88.60</td>
<td>0.8860</td>
<td>6.15</td>
</tr>
</tbody>
</table>
2. Yield Conventions

- A yield convention is an arbitrary set of rules for computing yields (like spot rates) from discount factors.

- US Treasuries use semiannual compounding:
  \[ d_n = \frac{1}{(1 + y_n/2)^n} \]
  with \( n \) measured in half-years.

- Other conventions with \( n \) measured in years:
  \[
  d_n = \begin{cases} 
  (1 + y_n)^{-n} & \text{annual compounding} \\
  (1 + y_n/k)^{-kn} & \text{"k" compounding} \\
  e^{-ny_n} & \text{continuous compounding (}k \rightarrow \infty) \\
  (1 + ny_n)^{-1} & \text{"simple interest"} \\
  (1 - ny_n) & \text{"discount basis"}
  \end{cases}
  \]

- All of these formulas define rules for computing the yield \( y_n \) from the discount factor \( d_n \), but of course they’re all different and the choice among them is arbitrary. That’s one reason discount factors are easier to think about.
3. Coupon Bonds

- Coupon bonds are claims to fixed future payments \( (c_n, \text{say}) \)

- They’re collections of zeros and can be valued that way:

\[
\text{Price} = d_1c_1 + d_2c_2 + \cdots + d_nc_n
\]

\[
= \frac{c_1}{(1 + y_1/2)} + \frac{c_2}{(1 + y_2/2)^2} + \cdots + \frac{c_n}{(1 + y_n/2)^n}
\]

- Example: Two-year “8-1/2s”

Four coupons remaining of 4.25 each

\[
\text{Price} = 0.9709 \times 4.25 + 0.9422 \times 4.25 \\
+ 0.9139 \times 4.25 + 0.8860 \times 104.25 \\
= 104.38.
\]

- Two fundamental principles of asset pricing:
  - Replication: two ways to generate same cash flows
  - Arbitrage: equivalent cash flows should have same price
4. Spot Rates from Coupon Bonds

- We computed the price of a coupon bond from prices of zeros

- Now reverse the process with these coupon bonds:

<table>
<thead>
<tr>
<th>Bond</th>
<th>Maturity (Yrs)</th>
<th>Coupon</th>
<th>Price</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0.5</td>
<td>8.00</td>
<td>100.97</td>
</tr>
<tr>
<td>B</td>
<td>1.0</td>
<td>6.00</td>
<td>99.96</td>
</tr>
</tbody>
</table>

- Compute discount factors “recursively”:

\[
100.97 = d_1 \times 104 \quad \Rightarrow \quad d_1 = 0.9709
\]

\[
99.96 = d_1 \times 3 + d_2 \times 103 \\
= 0.9709 \times 3 + d_2 \times 103 \quad \Rightarrow \quad d_2 = 0.9422
\]

- Spot rates follow from discount factors:

\[
d_n = \frac{1}{(1 + y_n/2)^n}
\]
4. Spot Rates from Coupon Bonds (continued)

- Do zeros and coupon bonds imply the same discount factors and spot rates?

- Example: Suppose Bond B sells for 99.50, implying $d_2 = 0.9377$ (B seems cheap)
  
  - Replication of B’s cash flows with zeros:
    \[
    3 = x_1 \times 100 \quad \Rightarrow \quad x_1 = 0.03 \\
    103 = x_2 \times 100 \quad \Rightarrow \quad x_2 = 1.03
    \]
  
  - Cost of replication is
    \[
    \text{Cost} = 97.09 \times 0.03 + 94.22 \times 1.03 = 99.96
    \]

  - Arbitrage strategy: buy B and sell its replication
    Riskfree profit is $99.96 - 99.50 = 0.46$

- **Proposition.** If (and only if) there are no arbitrage opportunities, then zeros and coupon bonds imply the same discount factors and spot rates.

- Presumption: markets are approximately “arbitrage-free”

- Practical considerations: bid/ask spreads, hard to short
5. Yields on Coupon Bonds

- Spot rates apply to specific maturities

- The *yield-to-maturity* on a coupon bond satisfies

  \[
  \text{Price} = \frac{c_1}{(1 + y/2)} + \frac{c_2}{(1 + y/2)^2} + \cdots + \frac{c_n}{(1 + y/2)^n}
  \]

- Example: Two-year 8-1/2s

  \[
  104.38 = \frac{4.25}{(1 + y/2)} + \frac{4.25}{(1 + y/2)^2} + \frac{4.25}{(1 + y/2)^3} + \frac{104.25}{(1 + y/2)^4}
  \]

  The yield is \( y = 6.15\% \).

- Comments:
  
  - Yield depends on the coupon
  
  - Computation: guess \( y \) until price is right
6. Par Yields

- We’ve found prices and yields for given coupons

- Find the coupon that delivers a price of 100 (par)

\[
\text{Price} = 100 = (d_1 + \cdots + d_n)\text{Coupon} + d_n100
\]

The annualized coupon rate is

\[
\text{Par Yield} = 2 \times \text{Coupon} = 2 \times \frac{1 - d_n}{d_1 + \cdots + d_n} \times 100
\]

- This obscure calculation underlies the initial pricing of bonds and swaps
7. Yield Curves

- A yield curve is a graph of yield $y_n$ against maturity $n$

- May 1995, from US Treasury STRIPS:
8. Estimating Bond Yields

- Standard practice is to estimate spot rates by fitting a smooth function of $n$ to spot rates or discount factors.

- “Noise”: bid/ask spread, stale quotes, liquidity, coupons, special features.

- Reminder that the frictionless world of the proposition is an approximation.
9. Day Counts for US Treasuries

- **Overview**
  - Bonds typically have fractional first periods
  - You pay the quoted price plus a pro-rated share of the first coupon (accrued interest)
  - Day count conventions govern how prices are quoted and yields are computed

- **Details**
  - Invoice price calculation (what you pay):
    \[
    \text{Invoice Price} = \text{Quoted Price} + \frac{u}{u + v} \text{Coupon}
    \]
    \[
    \text{Accrued Interest} = \frac{u}{u + v} \text{Coupon}
    \]
    \[
    u = \text{Days Since Last Coupon}
    \]
    \[
    v = \text{Days Until Next Coupon}
    \]
  - Yield calculation (“street convention”):
    \[
    \text{Invoice Price} = \frac{\text{Coupon}}{(1 + y/2)^w} + \frac{\text{Coupon}}{(1 + y/2)^{w+1}}
    \]
    \[
    + \cdots + \frac{\text{Coupon} + 100}{(1 + y/2)^{w+n-1}}
    \]
    \[
    w = \frac{v}{u + v}
    \]
    \[
    n = \text{Number of Coupons Remaining}
    \]
9. Day Counts for US Treasuries (continued)

- US Treasuries use “actual/actual” day counts for \( u \) and \( v \) (ie, we actually count up the days)

- Example: 8-1/2s of April 97 (as of May 95)

| Issued       | April 16, 1990 |
| Settlemenent | May 18, 1995  |
| Matures      | April 15, 1997|
| Coupon Freq  | Semiannual    |
| Coupon Dates | 15th of Apr and Oct |
| Coupon Rate  | 8.50          |
| Coupon       | 4.25          |
| Quoted Price | 104.19        |

Time line:

<table>
<thead>
<tr>
<th>Previous Coupon Date</th>
<th>Settlement Date</th>
<th>Next Coupon Date</th>
</tr>
</thead>
<tbody>
<tr>
<td>4/15/95</td>
<td>5/18/95</td>
<td>10/15/95</td>
</tr>
</tbody>
</table>

\[ u = \]
\[ v = \]
\[ n = \]
9. Day Counts for US Treasuries (continued)

Price calculations:

Accrued Interest  =
Invoice Price  =

Yield calculations:

\[ w = \]
\[ d = \frac{1}{1 + y/2} \quad \text{(to save typing)} \]
\[ 104.95 = d^w(1 + d + d^2 + d^3) \quad 4.25 + d^{w+3}100 \]
\[ \Rightarrow \quad d = 0.97021 \]
\[ y = 6.14\% \]

(The last step is easier with a computer)
10. Other Day Count Conventions

- US Corporate bonds (30/360 day count convention)
  (roughly: count days as if every month had 30 days)

Example: Citicorp’s 7 1/8s

<table>
<thead>
<tr>
<th>Settlement</th>
<th>June 16, 1995</th>
</tr>
</thead>
<tbody>
<tr>
<td>Matures</td>
<td>March 15, 2004</td>
</tr>
<tr>
<td>Coupon Freq</td>
<td>Semianual</td>
</tr>
<tr>
<td>Quoted Price</td>
<td>101.255</td>
</tr>
</tbody>
</table>

Calculations:

\[ n = \]
\[ u = \]
\[ v = \]
\[ w = \]

Accrued Interest =

Invoice Price =

\[ \text{Invoice Price} = d^w \left( \frac{1 - d^n}{1 - d} \right) \text{Coupon} + d^{w+n-1} \times 100 \]

\[ \Rightarrow y = 6.929\% \]

Remark: term in brackets is a formula
10. Other Day Count Conventions (continued)

- Eurobonds (30E/360 day count convention)
  (ie, count days as if every month has 30 days)

Example: IBRD 9s, dollar-denominated

<table>
<thead>
<tr>
<th>Settlement</th>
<th>June 20, 1995</th>
</tr>
</thead>
<tbody>
<tr>
<td>Matures</td>
<td>August 12, 1997</td>
</tr>
<tr>
<td>Coupon Freq</td>
<td>Annual</td>
</tr>
<tr>
<td>Quoted Price</td>
<td>106.188</td>
</tr>
</tbody>
</table>

Calculations:

\[ n = \]
\[ u = \]
\[ v = \]
\[ w = \]

Accrued Interest  =
Invoice Price  =

\[ \text{Invoice Price} = d^w \left( \frac{1 - d^n}{1 - d} \right) \text{ Coupon} + d^{w+n-1}100 \]
\[ \Rightarrow y = 5.831\% \]

Remark: annual compounding, \( d = 1/(1 + y) \)
10. Other Day Count Conventions (continued)

- Eurocurrency deposits
  (generally actual/360 day count convention)

Example: 6-month dollar deposit in interbank market

<table>
<thead>
<tr>
<th>Settlement</th>
<th>June 22, 1995</th>
</tr>
</thead>
<tbody>
<tr>
<td>Matures</td>
<td>December 22, 1995</td>
</tr>
<tr>
<td>Rate (LIBOR)</td>
<td>5.375</td>
</tr>
</tbody>
</table>

Cash flows: pay (say) 100, get 100 plus interest

Interest computed by

\[
\text{Interest} = \text{LIBOR} \times \frac{\text{Actual Days to Payment}}{360}
\]

\[
= 5.375 \times \left(\frac{183}{360}\right) = 3.018.
\]

Remarks
- Can be denominated in any currency
- “Interest” is analogous to \( y/2 \) in treasury formulas
11. Forward Rates

- A one-period forward rate $f_n$ at date $t$ is the rate paid on a one-period investment arranged at $t$ (“trade date”) and made at $t + n$ (“settlement date”)

<table>
<thead>
<tr>
<th></th>
<th>trade</th>
<th>settlement</th>
<th>maturity</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$t$</td>
<td>$t + n$</td>
<td>$t + n + 1$</td>
</tr>
</tbody>
</table>

- Representative cash flows (scale is arbitrary)

<table>
<thead>
<tr>
<th></th>
<th>$t$</th>
<th>$t + n$</th>
<th>$t + n + 1$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0</td>
<td>$-F$</td>
<td>100</td>
</tr>
</tbody>
</table>

with $F = 100 / (1 + f_n / 2)$ (verify that rate is $f_n$)

- Replication with zeros

<table>
<thead>
<tr>
<th></th>
<th>$t$</th>
<th>$t + n$</th>
<th>$t + n + 1$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$-p_{n+1}$</td>
<td></td>
<td>100</td>
</tr>
<tr>
<td></td>
<td>$xp_n$</td>
<td>$-100x$</td>
<td></td>
</tr>
</tbody>
</table>

Choose $x$ to replicate cash flows of forward contract

$$0 = -p_{n+1} + xp_n$$

$$\Rightarrow \quad 1 + f_n / 2 = \frac{p_n}{p_{n+1}}$$
11. Forward Rates (continued)

- Sample forward rate calculations:

<table>
<thead>
<tr>
<th>Maturity</th>
<th>Price ($)</th>
<th>Spot Rate (%)</th>
<th>Forward Rate (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>97.09</td>
<td>5.99</td>
<td>5.99</td>
</tr>
<tr>
<td>1.0</td>
<td>94.22</td>
<td>6.05</td>
<td>6.10</td>
</tr>
<tr>
<td>1.5</td>
<td>91.39</td>
<td>6.10</td>
<td>6.20</td>
</tr>
<tr>
<td>2.0</td>
<td>88.60</td>
<td>6.15</td>
<td>6.29</td>
</tr>
</tbody>
</table>

- Forward rates are the marginal cost of one more period:

\[
\begin{array}{c|c|c|c}
    f_0 & f_1 & f_2 \\
    \hline
    y_3 & y_3 & y_3 \\
\end{array}
\]

- Spot rates are (approximately) averages:

\[
y_n \approx n^{-1}(f_0 + f_1 + \cdots + f_{n-1}) \\
\approx n^{-1} \sum_{j=1}^{n} f_{j-1}
\]
11. Forward Rates (continued)

- Forward and spot rates in May 1995:
12. Yields and Returns on Zeros

- Example: Six-month investments in two zeros

<table>
<thead>
<tr>
<th>Zero</th>
<th>Maturity</th>
<th>Price</th>
<th>Spot Rate (%)</th>
<th>Forward Rate (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0.5</td>
<td>97.56</td>
<td>5.00</td>
<td>5.00</td>
</tr>
<tr>
<td>B</td>
<td>1.0</td>
<td>94.26</td>
<td>6.00</td>
<td>7.00</td>
</tr>
</tbody>
</table>

- Scenarios for spot rates in six months

<table>
<thead>
<tr>
<th>Spot Rates (%)</th>
<th>Scenario 1</th>
<th>Scenario 2</th>
<th>Scenario 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y_1 )</td>
<td>8.00</td>
<td>5.00</td>
<td>2.00</td>
</tr>
<tr>
<td>( y_2 )</td>
<td>9.00</td>
<td>6.00</td>
<td>3.00</td>
</tr>
</tbody>
</table>

“up 3” “no change” “down 3”

- One-period returns on zeros

\[
1 + \frac{h}{2} = \frac{\text{Sale Price}}{\text{Purchase Price}}
\]

\( h \) for holding period, which is six months here

- Scenario 2 returns

(A) \( 1 + \frac{h}{2} = \frac{100}{97.56} = 1.025 \Rightarrow h = 0.0500 \)

(B) \( 1 + \frac{h}{2} = \frac{97.56}{94.26} = 1.035 \Rightarrow h = 0.0700 \)
12. Yields and Returns on Zeros (continued)

- Six-month returns \((h)\):

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Return on A (%)</th>
<th>Return on B (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5.00</td>
<td>4.02</td>
</tr>
<tr>
<td>2</td>
<td>5.00</td>
<td>7.00</td>
</tr>
<tr>
<td>3</td>
<td>5.00</td>
<td>10.08</td>
</tr>
</tbody>
</table>

- Remarks
  - Return on A is the same in all scenarios
  - Standard result when holding period equals maturity:
    \[
    (1 + h/2)^n = \frac{100}{P_n} = (1 + y/2)^n
    \]
  - Return on B depends on interest rate movements
13. Yields and Returns on Coupon Bonds

- One-period returns

\[ 1 + \frac{h}{2} = \frac{\text{Sale Price} + \text{Coupon}}{\text{Purchase Price}} \]

(buy and sell just after coupon payment)

- Return when held to maturity
  Needed: return \( r \) on reinvested coupons
  - Three-period example

\[ (1 + \frac{h}{2})^3 = \frac{(1 + r)^2C + (1 + r)C' + C' + 100}{\text{Purchase Price}} \]

  - Return depends on reinvestment rate \( r \) (arbitrary)
  - No simple connection between return and yield

- Bottom line: yields are not returns
Summary

- Bond prices and discount factors represent the time-value of money

- Spot rates do, too

- Conventions govern the calculation of spot rates from discount factors

- Yields on coupon bonds are a common way of representing prices, but are not otherwise very useful

- Cash flows of coupon bonds can be “replicated” with zeros, and vice versa

- Replication and arbitrage relations apply to frictionless markets, but hold only approximately in practice

- Yields and returns aren’t the same thing