Interest-Rate Options

0. Overview

- Fixed Income Options

- Option Fundamentals

- Caps and Floors

- Options on Bonds

- Swaptions
1. Fixed Income Options

- Options imbedded in bonds:
  - Callable bonds
  - Putable bonds
  - Convertible bonds

- Options on futures
  - Bond futures
  - Eurocurrency futures

- OTC options
  - Caps, floors, and collars
  - Swaptions
2. Option Basics

- Big picture
  - Options are everywhere
    * Stock options for CEOs and others
    * Corporate equity: call option on a firm
    * Mortgages: the option to refinance
  - Options are like insurance
    * Premiums cover the down side, keep the up side
    * Customers like this combination
    * Insurer bears or shares risk (reinsurance, diversification)
  - Managing cost of insurance
    * Out-of-the-money options are cheaper
      (insurance with a big deductible)
    * Collars: sell some of the up side
    * Aggregate: basket option cheaper than basket of options
  - Managing option books
    * Customer demands may result in exposed position
    * Particular exposure to volatility:
      puts and calls both rise with volatility
    * Hedging through replication is another route
2. Option Basics (continued)

- Option terminology
  - Options are the right to buy (a **call**) or sell (a **put**) at a fixed price (**strike price**)
  - The thing being bought or sold is the **underlying**
  - This right typically has a fixed **expiration** date
  - A short position is said to have **written** an option
  - **European** options can be exercised only at expiration
  - **American** options can be exercised any time
  - **Bermuda** options can be exercised at specific dates (eg, bonds callable only on coupon dates)
2. Option Basics (continued)

- Features of options
  - Leverage (cheap source of exposure)
  - Nonlinear payoffs
    * Translates into variable duration
      (convexity rears its ugly head)
    * Creates risk management hazards
  - Volatility has positive effect on puts and calls
    * Another risk management hazard!
  - A state-contingent claim
    (no way ’round it, but nothing new either)
3. Modeling Approaches

- Why pricing models? No choice.
- What pricing model? Good question.
- Interest rate trees
  - Been there . . . (and it hasn’t changed)
- The Black-Scholes benchmark
  - Parameters: \( n \)-period European call on \((n+m)\)-period zero at strike price \( k \).
  - Formula:
    \[
    \text{Call Price} = p_{n+m} N(x_1) - d_n k N(x_2)
    \]
    with
    \[
    \begin{align*}
    p_n &= \text{price of } n\text{-period zero} \\
    d_n &= \text{\( n \)-period discount factor} \\
    N &= \text{normal cdf} \\
    x_1 &= \left[ \log(f/k) + \frac{n\sigma^2}{2} \right] / n^{1/2} \sigma \\
    x_2 &= x_1 - n^{1/2} \sigma \\
    f &= 100 \times p_{n+m}/p_n \text{ (forward price)}
    \end{align*}
    \]
3. Modeling Approaches (continued)

- Black-Scholes (cont’d): Properties
  - The Delta:
    \[
    \text{Delta} = \frac{d\text{Call Price}}{dp_{n+m}} = N(x_1),
    \]
    which varies between zero and one (nonlinear).
  - Volatility is the only unobservable
    (so we use call prices to infer it)
  - Price increases in volatility

- Remarks on Black-Scholes for fixed income
  - Obviously interest rates aren’t fixed
  - But formula based on log-normal zero prices
    \Rightarrow normal (continuously compounded) spot rates
    \Rightarrow possibility of negative spot rates
  - Volatility varies systematically with \((m, n)\)
    (the “term structure of volatility”)
  - Despite problems, a common benchmark
4. Caps, Floors, and Collars

- An interest rate cap pays the difference between the cap rate and a reference rate, if positive.

- Contract terms:
  - Cap rate
  - Reference rate (typically LIBOR)
  - Frequency of payment
  - Notional principal (amount on which interest is paid)

- Example: two-year, semi-annual, 7% cap on 6-m LIBOR, Notional principal 250mm.

Effective Cash Flow at $t+n$:

$$(r_{t+n} - 0.07)^+ \times \frac{1}{1 + r_{t+n}/2} \times \frac{\text{Notional Principal}}{2}$$
4. Caps, Floors, and Collars (continued)

- Rate tree (our usual)

```
6.036  6.587  7.869  8.913
  4.587  5.869  6.913
  3.869  4.913
     2.913
```

- Cash flow tree

```
0.000  0.000  1.045  2.289
  0.000  0.000  0.000
  0.000  0.000
     0.000
```

- Price path

```
0.504  1.039  2.146  2.289
  0.000  0.000  0.000
  0.000  0.000
     0.000
```
4. Caps, Floors, and Collars (continued)

- A floor pays the difference between the floor rate and a reference rate, if positive
  eg, a 3% floor pays 0.8% if LIBOR is 2.2%

- A collar is a combination of a cap and a floor

- Collars on FRNs limit interest payments,
  reduce cost by returning part of the “up” side

- Options on eurocurrency futures can be used to hedge caps (calls) and floors (puts)
5. Options on Bonds

- Earliest and most common interest-rate option?

- Examples of corporate callable bonds
  - Intel Overseas Corporation’s 8-1/8s, issued April 1, 1987 (really), due March 15, 1997, callable at par. Par in this situation means par plus accrued interest: the firm pays the relevant interest as well as the face value. The bonds were called March 15, 1994, at 100.
5. Options on Bonds (continued)

- Example: call option on 2-year 5% bond
  (the usual rate tree)

  Price path of bond:

  \[
  \begin{array}{c}
  100.94 \\
  100.00 \\
  102.81 \\
  103.58 \\
  103.53 \\
  102.50 \\
  102.50 \\
  102.50 \\
  102.50 \\
  102.50 \\
  102.50 \\
  \end{array}
  \]

- 18-month European option (callable at 102.5)

  Price path is:

  \[
  \begin{array}{c}
  0.134 \\
  0.010 \\
  0.227 \\
  0.525 \\
  0.042 \\
  1.028 \\
  0.000 \\
  0.000 \\
  0.000 \\
  0.000 \\
  \end{array}
  \]

  Nodes in boxes indicate cash flows from exercise, other nodes the value in earlier periods.
5. Options on Bonds (continued)

- American option has greater value
  (can exercise at expiration, or earlier if better)

- Approach: start at expiration, work backwards
  At each node, choose better of “exercise” or “hold”

- Cash flows from immediate exercise:

  \[
  \begin{array}{cccc}
  0.000 & < & 0.000 & < 0.000 \\
  0.314 & < & 1.080 & < 1.028 \\
  \end{array}
  \]

- Node with box:
  - If hold:
    \[
    \text{Value} = \frac{0.5}{1 + 0.03869/2} \times (0.042 + 1.028) = 0.525
    \]
  - If exercise:
    \[
    \text{Value} = 1.080
    \]
    This is better: we take it.
5. Options on Bonds (continued)

- American option (continued)

- Node (0.1) (one down move from start):
  - If hold:
    \[
    \text{Value} = \frac{0.5}{1 + 0.04587/2} \times (0.000 + 1.080) = 0.538
    \]
  - If exercise:
    \[
    \text{Value} = 0.314
    \]
    Hold is better in this case, so we write 0.538 here.

- Complete price path:

```
  0.266    0.010    0.000    0.000    0.000
  0.538    0.021    0.000    0.000    0.000
  1.080    0.042    0.000    0.000    0.000
  1.028
```

Boxes indicate nodes where option is exercised.

- Summary
  - Worth more than European call
  - Valued recursively
6. Swaptions

- Options on swaps

- Just like bond options
  (swap = long bond, short FRN that trades at par)

- European, American, Bermuda

- Currently the OTC standard

- Good source of implied volatilities (%):

<table>
<thead>
<tr>
<th>Swap Option</th>
<th>1 yr</th>
<th>2 yr</th>
<th>5 yr</th>
<th>10 yr</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 m</td>
<td>15.50</td>
<td>16.00</td>
<td>16.75</td>
<td>15.25</td>
</tr>
<tr>
<td>3 m</td>
<td>17.50</td>
<td>18.50</td>
<td>18.25</td>
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<tr>
<td>1 yr</td>
<td>21.50</td>
<td>21.25</td>
<td>19.25</td>
<td>16.50</td>
</tr>
<tr>
<td>5 yr</td>
<td>20.00</td>
<td>19.00</td>
<td>17.50</td>
<td>15.50</td>
</tr>
</tbody>
</table>

Summary

- Options are ubiquitous.

- Their nonlinear payoffs pose challenges to valuation and risk management.

- Nonlinearity translates in this context into nonlinear price-yield relations — convexity, in other words.

- Black-Scholes is less well suited for fixed income than other securities, but remains a common benchmark nonetheless.

- American options are valued recursively: at each node, we decide whether to hold or exercise.

- Shortcuts don’t work: yield-to-call is meaningless.