Interest-Rate Forwards and Futures

0. Overview

- Leading Futures Contracts
- Forward Contracts
- Futures Contracts
- Bond Futures
- Eurocurrency Futures
1. Leading Futures Contracts

- Contracts ranked by dollar volume:

<table>
<thead>
<tr>
<th>Contract</th>
<th>Open Interest Contracts (mm)</th>
<th>Monthly Volume Contracts (mm)</th>
<th>Dollars (bb)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Eurodollar</td>
<td>1.325</td>
<td>5.044</td>
<td>5,044</td>
</tr>
<tr>
<td>Euroyen</td>
<td>0.439</td>
<td>1.247</td>
<td>1,122</td>
</tr>
<tr>
<td>10-yr JGB</td>
<td>0.132</td>
<td>0.989</td>
<td>890</td>
</tr>
<tr>
<td>3-m sterling</td>
<td>0.212</td>
<td>0.941</td>
<td>724</td>
</tr>
<tr>
<td>Euromark</td>
<td>0.370</td>
<td>1.014</td>
<td>628</td>
</tr>
<tr>
<td>30-yr US T-bond</td>
<td>0.305</td>
<td>5.834</td>
<td>583</td>
</tr>
<tr>
<td>PIBOR</td>
<td>0.146</td>
<td>0.536</td>
<td>492</td>
</tr>
<tr>
<td>10-yr Notionnel</td>
<td>0.231</td>
<td>2.584</td>
<td>237</td>
</tr>
<tr>
<td>S&amp;P 500</td>
<td>0.157</td>
<td>1.035</td>
<td>232</td>
</tr>
<tr>
<td>German Bund</td>
<td>0.139</td>
<td>1.134</td>
<td>176</td>
</tr>
<tr>
<td>Nikkei 225</td>
<td>0.149</td>
<td>0.950</td>
<td>172</td>
</tr>
<tr>
<td>10-yr US T-note</td>
<td>0.177</td>
<td>0.935</td>
<td>93</td>
</tr>
</tbody>
</table>


- Remarks
  - Fixed income contracts dominate
  - Eurocurrencies first, then government bonds
  - Bond contracts have greater volume to open interest than euros: short-term trading v buy-and-hold
2. **Forward Contracts**

- A *forward contract* is an agreement to exchange assets at a future date for a price arranged now.

- Terminology: trade date is when trade is made, settlement date is when assets are exchanged:

  \[
  \begin{array}{ccc}
  \text{trade} & \text{settlement} \\
  t & t + n \\
  \end{array}
  \]

- With interest rate contracts, a third date is the maturity of the asset being exchanged (typically for cash):

  \[
  \begin{array}{ccc}
  \text{trade} & \text{settlement} & \text{maturity} \\
  t & t + n & t + n + m \\
  \end{array}
  \]

  Convention (usually): \( m \) is maturity at settlement.

- Interest rate forwards and futures differ primarily in the magnitudes of \( n \) and \( m \):
  
  - Forward rate agreements: \( n \) and \( m \) are typically single-digit months.
  
  - Bond futures: short settlement and long maturity (eg, \( n \) of 3 months and \( m \) of 10 years).
  
  - Eurocurrency futures: long settlement and short maturity (\( n \) out to 10 years and beyond, \( m \) of 3 months).
2. Forward Contracts (continued)

- Forward contract on a zero: agree at \( t \) to buy \( m \)-period zero at \( t + n \) for price \( F \). (This should look familiar!)

- Cash flows:

<table>
<thead>
<tr>
<th></th>
<th>( t )</th>
<th>( t + n )</th>
<th>( t + n + 1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 0 )</td>
<td>(-F)</td>
<td>100</td>
<td></td>
</tr>
</tbody>
</table>

- Replication with zeros (\( p_n \) = price of \( n \)-period zero):

<table>
<thead>
<tr>
<th></th>
<th>( t )</th>
<th>( t + n )</th>
<th>( t + n + m )</th>
</tr>
</thead>
<tbody>
<tr>
<td>(-p_{n+m})</td>
<td></td>
<td>100</td>
<td></td>
</tr>
<tr>
<td>(xp_n)</td>
<td></td>
<td>(-100x)</td>
<td></td>
</tr>
</tbody>
</table>

Choose \( x \) to replicate cash flows of forward contract:

\[
0 = -p_{n+m} + xp_n
\]

- Solution:

\[
x = \frac{p_{n+m}}{p_n} \quad \Rightarrow \quad F = \frac{p_{n+m}}{p_n} \times 100
\]
2. Forward Contracts (continued)

- Forward contract on a bond: agree at $t$ to buy an $m$-period bond at $t + n$ for price $F$.

- Coupons complicate the analysis, but the idea is the same as the zero: replicate with long position in the bond and short positions sufficient to offset the purchase price and (in this case) the coupons between now and settlement.

- Key ingredient: replication includes a long position in the bond, so you’re indirectly gaining exposure to the bond.
2. **Forward Contracts (continued)**

- Floating rate agreements (FRAs)
- **Contract terms**
  - In an “$n \times m$” (both quoted in months), $m$ is what we’ve called $m+n$
  - Fixed “contract” rate ($C$)
  - Floating “reference” rate ($r$) (fixed at settlement)
  - Notional principal
  - Cash flow at settlement:

$$\text{Payment} = \frac{(C-r)(m-n)/12}{1 + r(m-n)/12} \times \text{Principal}$$

  (plus the usual eurocurrency day count adjustments)

- **Example:** $6 \times 12$, 1mm notional
  - Contract rate = 6%
  - Reference rate = 6-month LIBOR
  - If 6-month LIBOR is 5% at settlement,

$$\text{Payment} = \frac{(.06-.05)/2}{1+.05/2} \times 1,000,000 = 4,878$$

- Contract rate is essentially the forward rate between $t+6$ and $t+12$
2. **Forward Contracts (continued)**

- Using forwards to modify interest sensitivity
  - Duration not defined: duration is the proportional change in the price and is not defined for contracts (like swaps, forwards, and futures) that have no net value.
  - Quantify interest sensitivity as we did with swaps:
    * Use the DV01
    * Compute duration for long and short positions separately
  - Bottom line:
    * Forward contracts on long-maturity bonds are useful tools for modifying duration: you add (say) a long position in a long bond, and short a position of equal value in a short bond.
    * Forward contracts on short bonds — FRAs, for example — are useful protection against near-term changes in short rates.
3. Essential Features of Futures Contracts

- Similar to forward contracts
- Easy to short: big advantage over bonds
- Fixed contract terms
- Liquid: often easier/cheaper to trade than bonds
- Low credit risk: guaranteed by the exchange
- Trades public: good source of market information
- Cash flows:
  - No payment due on trade date
  - . . . but money is set aside in margin account
  - Margin account varies from
    * Daily changes in contract price
    * Interest on the account
    * Margin calls
- Daily “mark to market”
  - Reduces credit exposure of exchange
  - Complicates cash flows and valuation (slightly)
4. Treasury Futures

- US treasury bond/note contracts (CBOT)  
  (foreign government bond contracts are similar)

- Standard features
  - Contract size: $100,000 face value
  - Contracts expire quarterly (Mar/Jun/Sep/Dec)
  - Delivery: short position
    * Can deliver any time in the contract month
    * Delivers $100,000 face value of bonds, gets cash.

- Eligible (contract grade) bonds:
  * 30-year bond contract: US treasury bonds with maturity at least 15 years from delivery date
  * 10-year note contract: US treasury notes with maturity 6.5 to 10 years from delivery date
  * 5-year note contract: US treasury notes with original maturity no more than 5.25 years and maturity on first day of delivery month of at least 4.25 years

- Wild-card option: price at close (2pm) good till 8pm
- Timing option: futures price fixed on last trading day
4. Treasury Futures (continued)

- Valuing contracts
  - Summary: if \( F \) is quoted futures price,
    \[
    \text{Value} = F \times \text{Conversion Factor} + \text{Accrued Interest}
    \]
  - Conversion factor mitigates tendency to deliver lowest-coupon bond (the cheapest one eligible)
  - Compute conversion factor as follows:
    * Compute maturity at first delivery date, round down to nearest 3-month interval
    * Compute invoice price by discounting coupon and principal at 8% (refer to set 1, yield-to-maturity)
    * Subtract accrued interest
    * Divide by 100 to get conversion factor
  - Examples for Dec 97 contracts
    * 10.75 due Feb 2016: \( n = 36, \) Invoice = 126.00, Accrued = 0 , CF = 1.2600.
    * 10.375% due December 2020: \( n = 46, \) Invoice = 124.80, Accrued = 0 , CF = 1.2480.
    * 10.375% due April 2021: \( n = 47, w = 0.5, \) Invoice = 127.46, Accrued = 2.59 , CF = 1.2487.
4. Treasury Futures (continued)

- Interest sensitivity: standard approach
  - Easy way to add/subtract exposure to long bond
  - In earlier terminology: $n$ is small and $m$ is large
  - DV01 based on the cheapest to deliver bond:
    \[ \text{DV01 of Futures} = \frac{\text{DV01 of Bond}}{\text{Conversion Factor}} \]
  - Remark: don’t ask!

- Complications
  - Sensitivity to specific part of the yield curve
  - Difference between cash and futures price may vary over time, esp when volatility changes
5. Eurocurrency Futures

- Contracts on 3-month LIBOR in major currencies (CME, LIFFE, SIMEX)

- Standard features
  - Contract size: interest on $1,000,000
  - Contracts expire quarterly (Mar/Jun/Sep/Dec) out ten years or more (third Wednesday)
  - Quoted index:
    \[ \text{Index} = 100 \times (1 - \text{Yield}) \]
  - Effective price of a contract:
    \[ \text{Price} = 1\text{mm} \times (1 - \text{Yield}/4) \]
  - Cash settlement at
    \[ \text{Settlement Price} = 1\text{mm} \times (1 - 3\text{-m LIBOR}/4) \]
    Note: no strange delivery options!
  - Strips: combinations of contracts with different maturities

- Uses
  - Helpful for hedging FRNs, swaps, etc
  - Source of market information on forward rates
  - In earlier terminology: \( n \) can be large, but \( m \) is small
Summary

- Forward contracts for bonds are equivalent to a combination of a long position in a long bond and a short position of equal value in a short bond.

- Futures contracts differ from forwards in the daily mark to market.

- Fixed income futures include government bond and eurocurrency contracts.

- Bond futures are truly ugly contracts.

- Like other derivatives, forwards and futures offer leverage: you can arrange great exposure with less money down than buying the underlying instrument.