INTERNATIONAL

DIVERSIFICATION

Fall 2002
Examining international diversification serves two purposes:

a. Interesting by itself

b. It's a nice application of prior discussion.

Return and risk of buying foreign securities
Some Underlying Relationships

1. Effects of Exchange Risk

<table>
<thead>
<tr>
<th>Time</th>
<th>Cost of 1 Mark</th>
<th>Value of German Bond</th>
<th>Value in Dollars</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>.50</td>
<td>920 DM</td>
<td>.50 x 920 = 460</td>
</tr>
<tr>
<td>1</td>
<td>.40</td>
<td>1000 DM</td>
<td>.4 x 1000 = 400</td>
</tr>
</tbody>
</table>

Return in Germany:

\[(1+R_H) = \frac{1000}{920} \quad \text{or} \quad R_H = 8.70\%\]

Return in U.S.:

\[(1+R_{US}) = \frac{400}{600} \quad \text{or} \quad R_{US} = -13.04\%\]

\[= \left( \frac{.40}{.50} \right) \left( \frac{1000}{920} \right)\]
Define Exchange Return As:

\[
\left(1 + R_X\right) = \frac{.40}{.50}
\]

\[R_X = -0.20\%
\]

\[
\left(1 + R_{US}\right) = \left(1 + R_X\right) \left(1 + R_H\right)
\]

\[1 + R_{US} = 1 + R_X + R_H + R_X R_H
\]

\[R_{US} = 1 - 0.20 + 0.0870 + (-0.20)(0.0870)
\]
Normally drop $R_X R_H$ term arguing it's small.

$$\bar{R}_{US} \approx \bar{R}_X + \bar{R}_H$$

$$\bar{R}_{US} \approx -11.30\%$$
\[ \text{Var}(R_{US}) \approx \text{Var}(R_{X} + R_{H}) \]

\[ \text{Var}(R_{US}) = \text{Var}(R_{X}) + \text{Var}(R_{H}) + \text{Cov}(R_{X} R_{H}) \]

Normally drop covariance arguing it's small.

\[ \text{Var}(R_{US}) \approx \text{Var}(R_{X}) + \text{Var}(R_{H}) \]
Effect on variance different than on standard deviation

\[
\left[ \text{Var}(R_{US}) \right]^{1/2} \approx \left[ \text{Var}(R_X) + \text{Var}(R_H) \right]^{1/2}
\]

\[
\text{Var}(R_X) = .25 \quad \text{Std.Dev.}(R_H) = .5
\]

\[
\text{Var}(R_X) = .16 \quad \text{Std.Dev.}(R_X) = .4
\]

\[
\text{Var}(R_{US}) = [ .25 + .16 ]^{1/2}
\]

\[
\text{Std.Dev.}(R_{US}) = .64
\]
Condition for non-U.S. fund to be held by U.S. investors.

Consider constant correlation model:

\[
Z_N = \frac{1}{\rho \sigma_N} \left[ \frac{\bar{R}_N - R_F}{\sigma_N} - \rho \frac{\Sigma}{1 + \rho - M \rho} \frac{\bar{R}_R - R_F}{\sigma_R} \right]
\]

Consider two security cases. \( Z_i = 0 \) if term in brackets is greater than zero.

\[
\frac{\bar{R}_N - R_F}{\sigma_N} - \rho \left[ \frac{\bar{R}_N - R_F}{\sigma_N} + \frac{\bar{R}_A - R_F}{\sigma_A} \right] > 0
\]

\[
\frac{\bar{R}_N - R_F}{\sigma_N} \left[ 1 - \frac{\rho}{1 - \rho} \right] > \frac{\rho}{1 - \rho} \left[ \frac{\bar{R}_A - R_F}{\sigma_A} \right]
\]
\[
\frac{\bar{R}_N - R_F}{\sigma_N} \left[ \frac{1}{1-\rho} \right] > \rho \frac{\bar{R}_A - R_F}{\sigma_A}
\]

\[
\left( \frac{\bar{R}_N - R_F}{\sigma_N} \right) > \left( \frac{\bar{R}_A - R_F}{\sigma_A} \right) \rho_{NA}
\]
Concepts:

1. Mean return security & portfolio
2. Correlation security & portfolio
3. Standard deviation security & portfolio
4. Efficient frontier:
   a. No lending & borrowing
   b. Lending & borrowing at same rate
   c. Lending & borrowing at different rates
   d. Capital market line
5. Estimating correlations or covariances:
   a. Single-index model
   b. Multi-index model
   c. Correlation models
6. Solving simple problems:
   a. General solution
   b. Simple rules
7. Added problems when assets in different countries