A. CAPM

Building Blocks

(1) Law of one price.

(2) Markets dominated by investors with well-diversified portfolios.

Law of one price = two identical items must sell at same price.

Markets dominated by well-diversified portfolios.

(1) Major portion of trading
(2) Prices set by information traders
(3) Small investors have well-diversified portfolios

Consider three assets:

<table>
<thead>
<tr>
<th>#</th>
<th>( \bar{R} )</th>
<th>( \beta )</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>21</td>
<td>2</td>
</tr>
<tr>
<td>B</td>
<td>13</td>
<td>1</td>
</tr>
<tr>
<td>C</td>
<td>16</td>
<td>1.5</td>
</tr>
</tbody>
</table>
How do all combinations of A & B plot

\[
R_c = X_A (21) + \left(1 - X_A\right) 13 = 13 + 8X_A \quad (1)
\]

\[
\beta_c = X_A 2 + \left(1 - X_A\right) = 1 + X_A \quad (2)
\]

Rearranging (2)

\[
X_A = \beta_c - 1
\]

\[
\bar{R_c} = 13 + 8(\beta_c - 1) = 5 + 8\beta_c
\]

How does it plot?
Any assets off line have violation of law of one price?

General form:

\[ \bar{R}_i = C_0 + C_1 \beta_i \]

If you have riskless asset \( \beta_i = 0 \) and you have

\[ \bar{R}_i = C_0 = R_F \]

\[ \therefore R_F = C_0 \]
Consider market portfolio for this portfolio $\beta_1 = 1$ thus

$$\bar{R}_m = C_0 + C_1(1) \text{ and } C_1 = \bar{R}_m - C_0 = \bar{R}_m - R_F$$

and

$$\bar{R}_i = R_F + \beta_i (\bar{R}_m - R_F)$$

Standard CAPM

$$\beta_i = \frac{\text{Cov}(R_i, \bar{R}_m)}{\text{Var}(\bar{R}_m)}$$
\[
\overline{R}_i = R_F + \frac{\text{Cov}(R_i, R_m)}{\text{Var}(R_m)} (\overline{R}_m - R_F)
\]

\[
\overline{R}_i = R_F + \lambda \text{Cov}(R_i, R_M)
\]

\[
\lambda = \frac{\overline{R}_m - R_F}{\text{Var}(R_m)}
\]

reward for time

reward for risk

market reward risk ratio
If riskless asset does not exist then define zero beta portfolios \( \overline{R}_Z \)

\[
\overline{R}_Z = C_0 + C_1(0)
\]

and

Zero Beta Form

\[
\overline{R}_i = \overline{R}_Z + \beta_i (\overline{R}_m - \overline{R}_Z)
\]
ARBITRAGE PRICING THEORY

A multi-index model explains security returns

\[ R_{it} = C_o + \beta_{ip} \left( \text{unexplained production changes} \right) + \beta_{iT} \left( \text{Term premium} \right) + \beta_{iD} \left( \text{Default premium} \right) + e_{it} \]

Systematic risk factors \( \left[ \beta_{ip}, \beta_{iT}, \beta_{iD} \right] \)
\[ \bar{R}_c = 8X_1 + 6X_2 + (1-X_1-X_2)^9 \]
\[ \beta_1 = 2X_1 + 1X_2 + (1-X_1-X_2)^3 \]
\[ \beta_2 = 1X_1 + 1X_2 + (1-X_1-X_2)^2 \]

Eliminating \( X_1 \) and \( X_2 \)

\[ \bar{R}_c = 5 + 2\beta_1 - \beta_2 \]

and security 4 is mis-priced
Same argument (e.g., Law of One Price) two securities with same risk characteristics must have same expected return. With 3 risk variables, we can match the characteristics of any security with a portfolio of three other securities. Since Betas and expected returns are linear, the relationship is linear.

### Example

<table>
<thead>
<tr>
<th>Sec</th>
<th>$\bar{R}_i$</th>
<th>$\beta_1$</th>
<th>$\beta_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>8</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>6</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>9</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>4</td>
<td>11</td>
<td>4</td>
<td>1</td>
</tr>
</tbody>
</table>
\[ R_c = 8X_1 + 6X_2 + (1 - X_1 - X_2)^9 \]
\[ \beta_1 = 2X_1 + 1X_2 + (1 - X_1 - X_2)^3 \]
\[ \beta_2 = 1X_1 + 1X_2 + (1 - X_1 - X_2)^2 \]

Eliminating \( X_1 \) and \( X_2 \)

\[ R_c = 5 + 2\beta_1 - \beta_2 \]

and security 4 is mis-priced.
In general, if return generating process:

\[ R_{it} = C_0 + \beta_{i1} f_1 + \beta_{i2} f_2 + \beta_{i3} f_3 + \epsilon_{it} \]

Expected return is:

\[ \bar{R}_i = R_F + \beta_{i1} \lambda_1 + \beta_{i2} \lambda_2 + \beta_{i3} \lambda_3 \]

\[ \lambda_i = \bar{R}_i - R_F \]