OBJECTIVE

FUNCTIONS

Fall 2002
Objective Functions

I. Classical Markowitz

Trade-off is not explicitly made.
II. Trade-off explicit:

A. Use utility functions

Problem is specifying utility.
B. Specify risk tolerance.

By tradition, divide variance by risk tolerance.

Mean Return

- risk penalty = \( \frac{\text{variance}}{\text{risk tolerance}} \)

risk adjusted expected return

Example:

\[
\bar{r} = 12 \\
\sigma = 15 \\
\text{Tolerance} = 50
\]

Risk adjusted expected return: \( 12 - \frac{225}{50} = 7 \frac{1}{2} \)

Same issue is how tolerance specified but maybe easier to work with investor to determine range.
III. Safety first criteria (emphasis is on avoidance of risk).

A. Roy's Criteria:

Minimize \( \text{Prob} \left( R_p < R_L \right) \)

B. Katoka's Criteria

Maximize \( R_L \)

Subject to: \( \text{Prob} \left( R_p < R_L \right) \leq \alpha \)
C. Telser's Criteria

Max $\overline{R}_P$

Subject to: $\text{Prob} \left( R_P \leq R_L \right) \leq \alpha$
Analysis of criteria:

The following analysis assumes normal returns.

A. Consider Roy's criteria:

\[
\text{Min Prob } \left( \frac{R_P}{R_L} < 1 \right)
\]
Thus, want to maximize:

\[
\frac{\bar{R}_P - R_L}{\sigma_P}
\]
\( R_L \) serves as role of \( R_F \)

B. Katoka's criteria

Maximize \( R_L \)

Subject to:

\[
\text{Prob } \left( R_P < R_L \right) \leq \alpha
\]

\[
R_L \leq \bar{R}_P - K \sigma_P
\]

Where \( K \) is set to match above constraint - example 1.65.
$$R_L \leq \overline{R}_P - 1.65\sigma_P$$

Expression of straight line

Note if riskless lending and borrowing get funny results.
Consider Telser's criteria:

\[
\max \bar{R}_p
\]

Subject to

\[
\text{Prob } \left( R_p \leq R_L \right) \leq \alpha
\]

Constraint is:

\[
R_L \leq \bar{R}_p - K\sigma_p
\]