PORTFOLIO THEORY WITH MULTI-INDEX MODELS

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For illustrative purposes, consider a two-index model. The results clearly generalize

Assume the following return generating process:

\[ R_{it} = a_i + \beta_{i1} R_{mt} + \beta_{i2} I_{lt} + e_i \]

Where:

(1) \( R_{mt} \) is return on market.

(2) \( I_{lt} \) is an inflation index.
Assume investor in question cares about inflation and that the market cares about inflation.

Implications:

(1) Sensitivity to inflation should be priced and we should observe an APT model like:

\[
\bar{R}_i = R_{Ft} + \beta_{i1} \lambda_m + \beta_{i2} \lambda_I
\]

(2) Investors should be willing to trade off inflation hedging for expected return.
If the investor does not care about $\beta_1$ per se, but does care about $\beta_2$, the choice set is three-dimensional. It can be represented with these three axes:

$$\overline{R}_p, \sigma_p^2, \beta_2$$

Separation Theorem.

(1) The efficient frontier can be obtained using three efficient portfolios.
If the investor cares about the $\beta_1$, then we have a four-fund theorem and the efficient frontier becomes a combination of:

1. The riskless asset.
2. Two portfolios that have a beta of one and minimum risk, e.g., factor replication portfolios.
3. A special portfolio that maximizes $\alpha$ for any residual risk.

When we have a multi-index model, it is often sensible to assume that residuals are un-correlated. In this case, an extremely easy version of simple rules exists and the optimal proportion in any asset is proportional to:

$$Z_i = \frac{\alpha_i}{\sigma^2}$$

Now assume the investor does not care about inflation, but the market does. Then the market's equilibrium model is the capm model and the investor does not pay a cost in expected return by adjusting sensitivity to it.