FORWARD

DEFINE: AGREEMENT TO TRANSFER SOME ITEM AT A FUTURE DATE AT A FIXED PRICE.

EXAMPLE: ONE YEAR TREASURY BILL AT 88 FOR DELIVERY ON DECEMBER 18

PROFIT or LOSS ASSUME ONE YEAR TREASURY BILLS ARE $90 AS OF DECEMBER 18. LOSS/GAIN: BUYER +2, SELLER -2

NOTES: 1. ZERO SUM GAME
2. BOTH CAN BE RISK AVERSE BUYER AND SELLER AND HAVE SAME EXPECTATIONS

MARKETS WHERE DEFAULT RISK IS SMALL
**FUTURE**

**DIFFERENCE:** SAME AS FORWARD WITH DAILY SETTLEMENT. DAILY SETTLEMENT ELIMINATES DEFAULT RISK.

**MARK TO MARKET**

<table>
<thead>
<tr>
<th>Future</th>
<th>-3</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
</tr>
</thead>
<tbody>
<tr>
<td>Price</td>
<td>88</td>
<td>87</td>
<td>89</td>
<td>90</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Day</th>
<th>Buyer</th>
<th>Seller</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2</td>
<td>-1</td>
<td>+1</td>
</tr>
<tr>
<td>-1</td>
<td>+2</td>
<td>-2</td>
</tr>
<tr>
<td>0</td>
<td>+1</td>
<td>-1</td>
</tr>
<tr>
<td></td>
<td>+2</td>
<td>-2</td>
</tr>
</tbody>
</table>

**NOTE:** SAME DIFFERENCE AS FORWARD
FUTURES CONTRACTS INSTITUTIONAL DETAILS

1. MARKET TO MARKET

2. MARGIN

3. MOVE LIMITS

4. CHEAPEST TO DELIVER
   (SMALL OPTION VALUE)

5. RELATIONSHIP OF PRICE TO ARBITRAGE MODEL
TABLE 2

EFFECT OF MARKING TO THE MARKET
(for $1MM 3 Month Contracts)

<table>
<thead>
<tr>
<th>DOLLAR SIZE</th>
<th>PERCENT OF OCCURRENCES</th>
</tr>
</thead>
<tbody>
<tr>
<td>-$44 to -$39</td>
<td>1.00</td>
</tr>
<tr>
<td>-39 to -29</td>
<td>.67</td>
</tr>
<tr>
<td>-29 to -19</td>
<td>5.00</td>
</tr>
<tr>
<td>-19 to -9</td>
<td>8.90</td>
</tr>
<tr>
<td>-9 to +1</td>
<td>7.89</td>
</tr>
<tr>
<td>+1 to 11</td>
<td>29.37</td>
</tr>
<tr>
<td>11 to 21</td>
<td>31.81</td>
</tr>
<tr>
<td>21 to 31</td>
<td>7.45</td>
</tr>
<tr>
<td>31 to 41</td>
<td>4.11</td>
</tr>
<tr>
<td>41 to 51</td>
<td>.11</td>
</tr>
<tr>
<td>51 to 61</td>
<td>.00</td>
</tr>
<tr>
<td>61 to 71</td>
<td>.56</td>
</tr>
<tr>
<td>71 to 81</td>
<td>1.11</td>
</tr>
<tr>
<td>81 to 91</td>
<td>.44</td>
</tr>
<tr>
<td>91 to 102</td>
<td>1.56</td>
</tr>
</tbody>
</table>

AVERAGE EFFECT = $10.00
Which preferred - forward or future (depends on receipt of cash and the reinvestment rate).

1. If interest rates rise, bond price falls and Buyer loses

   Future better to Seller and high reinvestment rate

2. If interest rates decrease, bond prices rises and Buyer gains

   Future worse to seller and low reinvestment rate

Therefore, if increase and decrease equally likely, Buyer prefers forward

\[ P_{\text{fut}} < P_{\text{for}} \]

\[ \therefore \] Model somewhat overstate price
1. PRICE IS PRIMARILY DETERMINED BY ARBITRAGE
EXAMPLE 1

STRATEGY A

BUY 45-DAY T-BILL

STRATEGY B

1. BUY 136-DAY T-BILL

2. WRITE A FUTURES CONTRACT FOR DELIVERY OF A 91-DAY T-BILL IN 45 DAYS

3. DELIVER THE T-BILL IN 45 DAYS

PRICING

1. KNOW T-BILL PRICES. SOLVE FOR FUTURES PRICE.
PRICING

\( R_{45} \) = RATE OF RETURN ON 45-DAY T-BILL

\( P_{136} \) = PRICE OF 136-DAY T-BILL

\( f \) = FUTURE PRICE OF 91-DAY T-BILL TO BE DELIVERED IN 45 DAYS

\[
\frac{f - P_{136}}{P_{136}} = R_{45}
\]

\[
\frac{f}{P_{136}} - 1 = \frac{100}{P_{45}} - 1
\]

\[
f = \frac{P_{136}}{P_{45}} \times 100
\]
SOME INSTITUTIONAL DETAILS

1. Contract size 1 million

2. Delivery any of 3 days as 89, 90, 91 day bill

3. Price is quoted as 100 - BDY in dollars

EXAMPLE:

a. Price of 136-day T bill is 98

b. Price of 45-day T bill is 99.4

Futures Price is

\[ f = 100 \left( \frac{98}{99.4} \right) = 98.5916 \]
QUOTED FUTURES PRICE

\[ BDY = \frac{360}{91} \left( \frac{100 - 98.5916}{100} \right) = 0.055717 \]

or in dollars 5.5717

Thus theoretical quoted futures price is

100 - 5.5717 = 94.42831
EXAMPLE 2

(FUTURES ON CURRENCY)

STRATEGY A

BUY 91-DAY U.S. GOVERNMENT T-BILL

STRATEGY B

1) BUY 91-DAY BRITISH GOVERNMENT BILL

2) ENTER INTO FUTURES CONTRACT TO EXCHANGE POUNDS FOR DOLLARS IN 91 DAYS
DEFINE

1) \( r_{US} = \) RETURN ON U.S. T-BILL

2) \( r_{\text{£}} = \) RETURN ON BRITISH T-BILLS

3) \( F = \) FUTURES PRICE OF 1 POUND (e.g. $1.49)

4) \( S = \) CURRENT PRICE OF 1 POUND (e.g. $1.42)

LAW OF ONE PRICE => SAME RETURN

ASSUME INVEST $1

\[
(1 + r_{us}) = \frac{1}{S} (1 + r_{\text{£}}) \cdot F
\]

SOLVE FOR \( F \)
EXAMPLE 3

I. STRATEGY A

BUY ONE-YEAR GOVERNMENT BILL

II. STRATEGY B

1. BUY LONG-TERM GOVERNMENTS

2. WRITE FUTURES CONTRACT FOR DELIVERY OF LONG GOVERNMENTS IN ONE YEAR

LAW OF ONE PRICE ⇒ SAME RETURN

\[ r_1 = \text{RETURN ON ONE YEAR GOVERNMENT} \]

\[ F = \text{FUTURES PRICE} \]

\[ PL = \text{PRICE OF LONG GOVERNMENT LESS PRESENT VALUE OF COUPONS} \]
\[ r_1 = \frac{F - P_L}{P_L} \]

\[ r_1 = \frac{F}{P_L} - 1 \]

\[ F = P_L(1 + r_1) \]
SOME INSTITUTIONAL CONSIDERATIONS

∙ cheapest to deliver
∙ some options
∙ effect of accrued
∙ conversion factor

Prior formula used invoice prices futures are quoted without accrued interest and on a standard bond.

STEPS

1. Calculate invoice price on cheapest to deliver.
2. Use formula to get invoice price on future.
3. Subtract off accrued interest from future price.
4. Divide by conversion factor to get quoted futures price.
Example:

1. Cheapest to deliver is 10% coupon bond with conversion factor of 1.2 and a price of 110.

2. Future is for delivery in 240 days.

3. Interest rate to delivery day is 4% and 1.65% to next coupon date.

4. Time scale on cheapest to deliver bond is shown below.

5. Coupon payable in 100 days and 282 days.

<table>
<thead>
<tr>
<th>now</th>
<th>coupon</th>
<th>delivery</th>
<th>coupon</th>
</tr>
</thead>
<tbody>
<tr>
<td>&lt;--- 83 ---&gt;</td>
<td>&lt;--- 100 ---&gt;</td>
<td>&lt;--- 140 ---&gt;</td>
<td>&lt;--- 42 ---&gt;</td>
</tr>
</tbody>
</table>
Invoice price on cheapest to deliver is

\[ 110 + \frac{83}{183} \times 5 = 112.268 \]

Present value of interest is

\[ \frac{5}{1.0165} = 4.919 \]

Calculate futures invoice price

\[ F = (112.268 - 4.919) \times 1.04 = 111.6429 \]
Remove accrued interest

\[ 111.6429 - \left( \frac{83}{183} \right) \times 5 = 110.626 \]

Use conversion factor to get Quoted Futures Price

Theoretical Quoted Price =

\[ \frac{110.626}{1.2} = 92.189 \]
USES

1. MATURITY CHANGE (superior way of timing)

2. RISK REDUCTION

3. RETURN PICK UP

4. DURATION LENGTHENING

5. ALPHA FUND

6. RISK HEDGE
EXAMPLE MATURITY CHANGE

CURRENT PORTFOLIO

LONG-TERM GOVERNMENTS

DESIRE

SHORTEN MATURITY

ACTION

SELL FUTURES FOR DELIVERY OF LONG-TERM GOVERNMENTS IN ONE YEAR

RESULTS

MATURITY EFFECTIVELY ONE YEAR
EXAMPLE RISK REDUCTION

POSITION

YOU ARE DEALER
OWN 1 BILLION OF GOVERNMENT BONDS

RISK

ADVERSE INTEREST RATE MOVEMENTS BEFORE SELL

RISK REDUCTION

SELL FUTURES
RETURN PICKUP

FOR MOST PERIODS RETURN ON LONG T-BILL PLUS SHORT FUTURE > RETURN ON SHORT T-BILL

WANT SHORT T-BILL HOLD "PSEUDO" T-BILL
DURATION LENGTHENING

• OWNING BOND FUTURES IN CONJUNCTION WITH BONDS ↑ THE DURATION

• IN SOME PERIODS ONLY WAY TO IMMUNIZE
DURATION OF FUTURE

RECALL

(1) DURATION OF PORTFOLIO IS WEIGHTED AVERAGE OF DURATION OF BONDS THAT COMPRISE IT.

(2) FUTURE CAN BE REPLICATED AS PORTFOLIO OF TWO BONDS (ONE SOLD SHORT).

(3) THUS DURATION ON FUTURE MUST BE SAME AS ITS REPLICATING PORTFOLIO.

(4) SAME WITH CONVEXITY.
I. Terms

a. Forward

b. Future

I. Concepts

a. What affects difference in price of a forward and future?

b. Pricing futures by arbitrage.

c. Uses of futures.

d. How to calculate duration of a future.

II. Calculations


c. Calculating the duration of a future.
EXAMPLE 2

1. BUY LONG BOND

2. BUY FUTURE AND SUFFICIENT T BILLS TO TAKE DELIVERY

3. ASSUME PERIOD SO SHORT NO COUPON

A                  B

LONG BOND          LONG T BILLS
                 FUTURE

OR

A                  B

LONG BOND          FUTURE
SHORT T BILLS

\[ D_f = D_{CTD} - D_{TB} \]