YIELD TO MATURITY
ACCRUED INTEREST
QUOTED PRICE
INVOICE PRICE

September 1999
Quoted Rate Treasury Bills
[Called Banker's Discount Rate]

\[ d = \left[ \frac{P_1 - P_0}{P_1} \right] \times \left[ \frac{360}{N} \right] \]

\( d = \text{Bankers discount yield} \)

\( P_1 = \text{face value} \)

\( P_0 = \text{Price} \)

\( N = \text{number of days until maturity} \)

\[ \frac{P_1}{P_0} \times \frac{N}{360} \times d = \frac{P_1}{P_0} - 1 \]

\[ \left[ 1 + \frac{P_1}{P_0} \frac{d \times N}{360} \right] = \frac{P_1}{P_0} = \text{holding period return} \]
The invoice on a bond (what you pay) is quoted price plus accrued interest.
Calculating Accrued Interest

Accrued interest is actual days/actual days

<table>
<thead>
<tr>
<th>Last coupon date</th>
<th>settlement coupon date</th>
<th>next coupon date</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

$$\frac{x}{y}$$ times interest payment

Example:

1. 10% coupon
2. $100 interest annually or $50 semi-annual per $1000 face

$$y = 183$$  $$x = 100$$

$$\frac{100}{183} \times 50 = \$27.32$$
Yield To Maturity

1. One coupon left
   [like T bill]

2. At coupon paying date

   \[
   \begin{array}{cccccc}
   0 & 1 & 2 & 3 & 4 \\
   C & P_0 & C & C & C & C + \text{Prin}
   \end{array}
   \]

   \[
P_0 = \frac{C}{[1 + \frac{r}{2}]} + \frac{C}{[1 + \frac{r}{2}]^2} + \frac{C}{[1 + \frac{r}{2}]^3} + \frac{C + \text{Prin}}{[1 + \frac{r}{2}]^4}
   \]

3. Between Coupon Dates

   \[
P_0 + A = \frac{C}{[1 + \frac{r}{2}]^w} + \frac{C}{[1 + \frac{r}{2}]^{l+w}} + \frac{C}{[1 + \frac{r}{2}]^{2+w}} + \frac{C + \text{prin}}{[1 + \frac{r}{2}]^{3+w}}
   \]
\( w = \text{fraction of year to first coupon actual days over actual days} \)

\[
\begin{array}{c}
0 \\
1 \\
2 \\
3 \\
4 \\
\end{array}
\]

\( w \rightarrow \)

Note:

Ignore that intervals may be of uneven size because of Saturdays or end of month.
Eurobonds

- will examine only bonds issued in dollars

- interest paid annually usually

Calculating accrued interest

uses 30 day months 360 day years

Example 1:

1. Issue date January 28
2. Settlement date March 5

<table>
<thead>
<tr>
<th>days</th>
<th>January</th>
<th>February</th>
<th>March</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>[29, 30]</td>
<td>30 day</td>
<td>[1,2,3,4,5]</td>
</tr>
</tbody>
</table>

\[
\frac{37}{360} \times \text{interest} = \text{accrued interest}
\]
Example 2:

1. May 14  issue date
2. Sept 17 settlement date

May 16
J, Ju Aug 90
Sept 17
123

The 31st is the same as the 30th.

\[
\frac{123}{360} \times \text{interest} = \text{accrued interest}
\]
Computing Yield To Maturity

1. With one payment remaining like T-bill but usually with 30-day 360 calendar.

2. Multiple payments

\[
\text{Price + accrued} = \frac{C}{[1 + r]^v} + \frac{C}{[1 + r]^{v+1}} + \cdots \frac{C + \text{Prin}}{[1 + r]^{n-1+v}}
\]

\( r \) is an annual rate

\( v \) is fraction of year until payment and is done on 360 days in year 30 day month calendar

Most have options. We will discuss this later.
GNMA

Definition: bonds issued with mortgages as their backing.

Guaranties

1. Issuer. If borrower fails to make a scheduled amortization payment in any month, issuer makes good. If borrower defaults, issuer must promptly remit remaining mortgage.


Rate is mortgage rate - 50 basis points

e.g. 13% mortgages 12.5% pool rate

44 basis points to issuer
6 basis points to gov.
Timing of Payments

January       February       March

[Home owners]              [Home owners]       Pass through   Pass through

If. No Prepayments constant amount paid each month

determining constant amount

\[ 100 = \frac{M}{[1 + \frac{r}{12}]} + \frac{M}{[1 + \frac{r}{12}]^2} + ... + \frac{M}{[1 + \frac{r}{12}]^{360}} \]

M is scheduled amortized payment

\[
\begin{align*}
\frac{r}{8} & \quad \frac{m}{.7338} \\
12 & \quad 1.0286
\end{align*}
\]
Quoted Prices on GNMA

Quoted as % of remaining principal balance

Assume quoted price is 95

\[
\text{1 million original Principal value} \times 0.8 \times 0.95 \quad \text{Quoted Price}
\]

\[
\text{$760,000 \; Price}
\]

Accrued interest on GNMA

Settlement day. Two business days after trade date but first settlement is usually third Wednesday for GNMA less than 9.5% and following Monday for GNMA 9.5%.

Reason: Pool factors not available until 10th of month.

Accrued interest

\[
\frac{\text{coupon}}{12} \times \frac{\text{number of days from 1st until settlement}}{30}
\]
Example:

13% GNMA

5 million face

Feb. 15 settlement date

.8 Pool factor

Accrued Interest \[= \frac{14}{30} \times [.8 \times 5 \times \frac{13}{12}] = \$20,222\]

Note:

Always use 30 days irrespective of days in the month.

Yield to Maturity

\[Price + Accrued = \frac{M}{(1 + \frac{r}{12})} + \frac{M}{(1 + \frac{r}{12})^2} + \frac{M}{(1 + \frac{r}{12})^3} + \ldots\]
LIBOR

- Interest rates are annual using “simple interest”

\[
\text{Interest payment} = \text{principal} \times \text{LIBOR} \times \frac{\text{actual days to payment}}{360}
\]

Example: One million dollar deposit made June 22 at 5.93750. Until December 22 we get

\[
\text{actual days to payment} = 183
\]

\[
\text{interest payment} = 1,000,000 \times 0.0593750 \times \frac{183}{360}
\]

\[
= 30,182
\]

Later need semiannual rate. Adjustment is

\[
\left(1 + \frac{r}{2}\right)^2 = \left(1 + \text{Libor} \times \frac{\text{actual days of life}}{360}\right)^{\frac{365}{\text{actual days}}}
\]

Pound LIBOR is quoted on 365-day year
WHY PROBLEM WITH
YIELD TO MATURITY

September 1999
Reinvestment Assumption
(Implicit in Yield to Maturity)

Example:

\[
\begin{array}{cccc}
0 & 1 & 2 & 3 & 4 \\
10 & 10 & 10 & 110 & \\
& & & & \\
\end{array}
\]

\[
\text{Price} = \frac{10}{1.05} + \frac{10}{[1.05]^2} + \frac{10}{[1.05]^3} + \frac{110}{[1.05]^4} = 117.74
\]

Assume 12%

\[
\begin{array}{c}
10 \times [1.06]^3 & = 11.91 \\
10 \times [1.06]^2 & = 11.24 \\
10 \times [1.06] & = 10.60 \\
110 & = 110.00 \\
\end{array}
\]

\[
143.75
\]

\[
143.75/(1.05)^4 = 118.26
\]
Assume 10% Reinvestment

Ending Value

\[
\begin{align*}
10(1.05)^3 & \quad 11.58 \\
10(1.05)^2 & \quad 11.03 \\
10(1.05) & \quad 10.50 \\
11 & \quad 110 \\
\frac{143.11}{(1.05)^4} & \quad = \$117.74
\end{align*}
\]

Analytics:

\[
\begin{align*}
Price & = \frac{CF(1)}{(1 + \frac{r}{2})} + \frac{CF(2)}{(1 + \frac{r}{2})^2} + \frac{CF(3)}{(1 + \frac{r}{2})^3} + \frac{CF(4)}{(1 + \frac{r}{2})^4} \\
& = \frac{CF(1)(1 + \frac{y}{2})^3 + CF(2)(1 + \frac{y}{2})^2 + CF(3)(1 + \frac{y}{2}) + CF(4)}{(1 + \frac{r}{2})^4}
\end{align*}
\]

Unless \( y = r \), these don't match.
Different Bonds Assume Different Reinvestment Rates

Relative desirability depends on assumption about reinvestment

<table>
<thead>
<tr>
<th></th>
<th>Bond A</th>
<th>Bond B</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Coupon</strong></td>
<td>10%</td>
<td>3%</td>
</tr>
<tr>
<td><strong>Principal</strong></td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td><strong>Price</strong></td>
<td>$138.90</td>
<td>$70.22</td>
</tr>
<tr>
<td><strong>Maturity</strong></td>
<td>15 years</td>
<td>15 years</td>
</tr>
<tr>
<td><strong>Frequency of payment</strong></td>
<td>Annual</td>
<td>Annual</td>
</tr>
<tr>
<td><strong>Yield to maturity</strong></td>
<td>6%</td>
<td>6.1%</td>
</tr>
</tbody>
</table>
Ending Value

Reinvestment Rate

5.8%

Usually one crossing point.
Yield to maturity on portfolio not weighted average on bonds that are in portfolio.

Illustrating the Nonadditivity of Yields

<table>
<thead>
<tr>
<th>Bond</th>
<th>Outlay (Price)</th>
<th>Periods</th>
<th>Yield to Maturity</th>
<th>Weighted Average Yield</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>1</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>A</td>
<td>-100</td>
<td>15</td>
<td>15</td>
<td>115</td>
</tr>
<tr>
<td>B</td>
<td>-100</td>
<td>6</td>
<td>106</td>
<td>115</td>
</tr>
<tr>
<td>C</td>
<td>-92</td>
<td>9</td>
<td>9</td>
<td>109</td>
</tr>
<tr>
<td>A + B</td>
<td>-200</td>
<td>21</td>
<td>121</td>
<td>115</td>
</tr>
<tr>
<td>B + C</td>
<td>-192</td>
<td>15</td>
<td>115</td>
<td>109</td>
</tr>
<tr>
<td>A + C</td>
<td>-192</td>
<td>24</td>
<td>24</td>
<td>224</td>
</tr>
</tbody>
</table>
Review

I. Terms

   a. Accrued interest
   b. Quoted price
   c. Invoice price
   d. Yield to maturity

II. Concepts

   a. That accrued interest calculations differ across types of bonds
   b. That yield to maturity is a poor measure of relative value

III. Calculations

   a. Accrued interest
   b. Yield to maturity
Problems

Assume that the settlement date is December 1, 1997 and that the bond pays interest on September 15 and March 15. Further assume that the bond matures the following September. The coupon on the bond is ten percent and the face value is $1,000.

1. If the bond is a U.S. Treasury bond, what is the accrued interest?

Answer: For Treasuries, accrued interest is actual over actual

Since last interest payment
September 15
October 31
November 30
December 1
77

In Period
September 15
October 31
November 30
December 31
January 31
February 28
March 15
181
Accrued interest

\[ \frac{77}{181} \times 50 = 21.27 \]

2. Assume the bond is a corporate. What is the accrued interest?

Answer: For corporate bonds, months are 30 days and 360-day years.

Since last payment
September 15
October 30
November 30
December 30

In period
September 15
October 30
November 30
December 30
January 30
February 30
March 15

\[ \frac{76}{180} \times 50 = 21.27 \]
3. Assume bond is Italian government bond. What is the accrued interest?

Answer: Same as 2.

4. What is the yield to maturity for the bond in one if its price is 99?

Answer: Invoice Price = 990 + 21.27 = 1011.27
\[ r = 11.317\% \]

\[
1011.27 = \frac{50}{(1 + \frac{r}{2})^{\frac{104}{181}}} + \frac{1050}{(1 + \frac{r}{2})^{\frac{104}{181}}} 
\]
Accrued Interest Calculations

Conventions

Different markets have different conventions for calculating accrued interest. The method used in each market is denoted by specifying the method of calculating two values:

\[ d \] is the number of days from the previous coupon payment date to settlement date (or from issue date to settlement date, if the next coupon payment if the first); that is, the number of days over which interest has accrued.

\[ A_y \] is the assumed number of days in one year.

The particular convention used can be Actual/Actual, Actual/365, Actual/360, or 30E/360.

In the name of the convention, the first part of the name denotes the method of computing \( d \); and the second part of the name denotes the method of calculating \( A_y \).

Computing the Accrued Interest

Once \( d \) and \( A_y \) have been calculated, accrued interest is computed by the formula:

\[ I_A = C \frac{d}{A_y} \]

\( I_A \) is the accrued interest
\( C \) is the annual coupon payment

Calculating \( d \)

The three methods of calculating \( d \) are:

1. "Actual" - Calculate the actual number of days from the previous coupon payment date to the settlement date.
Examples:

- There are 10 days from 1/3/93 to 1/13/93
- There are 41 days from 1/3/93 to 2/13/93
- There are 31 days from 1/1/93 to 2/1/93
- There are 28 days from 2/1/93 to 3/1/93
- There are 29 days from 2/1/92 to 3/1/92

2. “30” - Calculate the number of days from the previous coupon payment date to the settlement date by assuming 30-day months, as follows:

   Let the two dates by M_1/D_1/Y_1 and M_2/D_2/Y_2.
   If D_1 is 31, change it to 30.
   If D_2 is 31, change it to first of next month.
   Then \( d = 360(Y_2 - Y_1) + 30(M_2 - M_1) + (D_2 - D_1) \).

   (Note: According to this formula, February will always appear to be a 30-day month).

Examples:

- There are 10 days from 1/3/93 to 1/13/93
- There are 30 days from 1/1/93 to 2/1/93
- There are 30 days from 2/1/93 to 3/1/93
- There are 30 days from 2/1/92 to 3/1/92
- There are 4 days from 2/27/93 to 3/1/93
- There are 1 days from 5/30/93 to 5/31/93
- There are 2 days from 5/29/93 to 5/31/93

This “30” method is used in the U.S. agencies, corporates, and municipals markets.
3. "30E" - Calculate the number of days from the previous coupon payment date to the settlement date by assuming 30-day months, as follows:

Let the two dates be \( M_1/D_1/Y_1 \) and \( M_2/D_2/Y_2 \).
If \( D_1 \) is 31, change it to 30.
If \( D_2 \) is 31, change it to 30.
Then 
\[
d = 360(Y_2-Y_1) + 30(M_2-M_1) + (D_2-D_1).
\]
(Note: According to this formula, February will always appear to be a 30-day month.)

Examples:

There are 30 days from 2/1/93 to 3/1/93
There are 30 days from 2/1/92 to 3/1/92
There are 4 days from 2/27/93 to 3/1/93
There are 0 days from 5/30/93 to 5/31/93
There is 1 day from 5/29/93 to 5/31/93

This method, slightly different from the "30" method above, is used in the Eurobond market and many European government and corporate markets.

Calculating \( A_y \)

The three method of calculating \( A_y \) are:

(1) "365" - \( A_y \) is equal to 365.

(2) "360" - \( A_y \) is equal to 360.

(3) "Actual" - \( A_y \) is equal to the number of days in the current coupon period times the number of coupon payments per year. For a semi-annual coupon, the number of days in the coupon period can range from 181 to 184, so \( A_y \) can range from 362 to 368.

Examples:

(1) A 6.75% bond paying semi-annually is traded to settle on July 2, 1993. The previous coupon paid on March 15, 1993, and the next coupon pays on September 15, 1993. If the bond accrues according to the Actual/Actual convention, what is the accrued interest at settlement?

There are 184 actual days in the current coupon period, and there are 109 actual days from the last coupon payment to settlement. Therefore, the accrued interest is:

\[
\frac{109}{2 \times 184} \times 6.75 = 1.999321 \text{ per 100 of face value}
\]

(2) Assume that the bond in example (1) accrues according to the 30/360 convention, instead of Actual/Actual. What would be the accrued interest?

There are 107 "30" days from the last coupon payment to settlement. Therefore, the accrued interest is:
\[
\frac{107}{360} \times 675 = 2.00625 \text{ per 100 of face value}
\]

**Accrued Interest - Market Conventions**

<table>
<thead>
<tr>
<th>Instrument Type</th>
<th>Accrual Convention</th>
<th>Coupons per Year</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Domestic:</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>US Govt. Treasury Bonds</td>
<td>Actual/Actual</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>US Govt. Agency Bonds</td>
<td>30/360</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>Corporate Bonds</td>
<td>30/360</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>Municipal Bonds</td>
<td>30/360</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>GNMA/FNMA/FLHMC</td>
<td>Actual/360</td>
<td>12</td>
<td></td>
</tr>
<tr>
<td><strong>International:</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Eurobonds</td>
<td>30E/360</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>German Govt. and Corp. Bonds</td>
<td>30E/360</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>Italian Govt. and Corp. Bonds</td>
<td>30E/360</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>Japanese Govt. and Corp. Bonds</td>
<td>Actual/365</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>Swiss Govt. and Corp. Bonds</td>
<td>30/360</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>UK Govt. and Corp. Bonds</td>
<td>Actual/365</td>
<td>2</td>
<td></td>
</tr>
</tbody>
</table>

**Notes:**

1. In the Japanese markets, February 29 is never counted.